PROCEEDINGS OF SCIENCE

Thermodynamics of the three flavor PNJL0 model

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The Polyakov-Nambu-Jona-Lasinio (PNJL) effective quark model incorporates the confinement/deconfinement phase-transition in the original Nambu-Jona-Lasinio model (NJL) by inserting the Polyakov Loop (Φ) in the equations of state (EOS) at finite temperature. However, at zero temperature regime, the PNJL model loses all contributions from Φ and the EOS return to the original NJL ones at $T = 0$. In this work, we present the SU(3) PNJL model at zero temperature, called SU(3) PNJL0. The model is based on the modification of coupling constants, by making them dependent on Φ and adding one term function of Φ in the grand canonical potential density, which limits the loop within $0 \leq \Phi \leq 1$ and favors a nonvanishing Φ solution. We impose that in the free quark regime (deconfinement region) all interactions vanish. We investigate the first order phase transition and how the strange quark favors the restoration of chiral symmetry. Another point discussed is how the constants G_V and a_3 (a parameter in the Polyakov potential) affect the quarkyonic phase of this model.

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 -17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

[*] Speaker

PoS(XVHadronPhysics)046

1. Introduction

The NJL [1] model is an effective model widely used and based on the dynamical breaking of the chiral symmetry, which incorporates the mechanism of mass generation caused by a non-vanishing quark condensate [2]. However, it is not able to reproduce the dynamics of the quark deconfinement [3]. The Polyakov-Nambu-Jona-Lasinio model (PNJL) [4] is a extended version of the NJL model, that includes the dynamics of gluons in an effective way through a background field, namely, the Polyakov loop (Φ). However, at zero temperature the PNJL model can not sustain the modifications involving Φ and the equations of state (EOS) are reduced to the ones found in the NJL model at $T = 0$. In order to circumvent this issue, it was proposed in references [5, 6] a modification of the couplings in the NJL model, by making them dependent on Φ. As a consequence of such approach, the Polyakov loop does not vanish at $T = 0$. The model built with this modification, named as PNJL0, is now able to describe the confinement/deconfinement phase-transition at $T = 0$. In this work, we will describe the $SU(3)$ version of the PNJL0 model and show its applications.

2. The Polyakov-Nambu-Jona-Lasinio model at $T = 0$ (PNJL0)

The idea of the PNJL0 model is to eliminate some of the problems that the NJL and PNJL models exhibit. Inspired by the entanglement-PNJL (EPNJL) model [7], we have proposed a change in the coupling constants of the scalar (G_s), vector (G_V) and flavor mixing (K) channels, which are now functions of the Polyakov loop Φ, in order to allow the confinement/deconfinement phase-transition at the zero temperature regime. The couplings are modified as $G_s \to \mathcal{G}_s(G_s, \Phi) = G_s(1 - \Phi^2)$, $G_V \to \mathcal{G}_V(G_V, \Phi) = G_V(1 - \Phi^2)$ and $K \to \mathcal{K}(K, \Phi) = K(1 - \Phi^2)$. The quark condensate and quark density are given by

$$\rho_{sf} = \langle\bar{\psi}_f\psi_f\rangle = -\frac{\gamma M_f}{2\pi^2}\int_{k_{Ff}}^{\Lambda}\frac{dk\,k^2}{(k^2 + M_f^2)^{1/2}}, \quad \rho_f = \langle\bar{\psi}_f\gamma_0\psi_f\rangle = \frac{\gamma}{6\pi^2}k_{Ff}^3, \tag{1}$$

where k_{Ff} is the Fermi momentum for the quark f, $\gamma = N_c \times N_S = 6$ from the color and spin numbers $N_c = 3$ and $N_S = 2$, respectively. The constituent quark mass, M_f, originates from the dynamical mass generation mechanism with dependence on the chemical potential[1], and are given, respectively, by

$$M_f = m_f - 2G_s\left(1 - \Phi^2\right)\rho_{sf} - 2K\left(1 - \Phi^2\right)\prod_{f \neq f'}\rho_{sf'}\,, \tag{2}$$

$$\mu_f = \sqrt{k_{Ff}^2 + M_f^2} + 2G_V\left(1 - \Phi^2\right)\rho_f\,, \tag{3}$$

differently, in the PNJL model at $T = 0$ the gluon sector does not influence the quark properties, i.e., the Polyakov loop does not contributes to the value of M_f and μ_f. Here, we have a full back-reaction, in other words, the two sectors interfere with each other, exactly as in the $SU(2)$ version of the PNJL0 model [5]. In the SU(3) version of the PNJL0 model, the Polyakov loop

[1] Here $\mu_u = \mu_d = \mu_s = \mu$, being μ the common chemical potential for quarks

potential is written as

$$\mathcal{U}(\rho_f,\rho_{sf},\Phi) = G_V\Phi^2\sum_f \rho_f^2 - G_s\Phi^2\sum_f \rho_{sf}^2 - 4K\Phi^2\prod_f \rho_{sf} + \mathcal{U}_0(\Phi), \quad \text{with} \tag{4}$$

$$\mathcal{U}_0(\Phi) \equiv a_3T_0^4 \ln\left(1-6\Phi^2+8\Phi^3-3\Phi^4\right), \tag{5}$$

where $T_0 = 190$ MeV [5] and a_3 is a dimensionless parameter, which controls the dynamics of the Polyakov loop at $T = 0$. By considering the term $\mathcal{U}_0$ in the potential, $\Phi \neq 0$ solutions are possible and at the same time it restricts the Polyakov loop in the range $0 \leq \Phi \leq 1$ [8]. The grand canonical potential for the model reads

$$\begin{aligned}\Omega_{\mathrm{PNJL0}} &= G_s\sum_f \rho_{sf}^2 + 4K\prod_f \rho_{sf} - G_V\sum_f \rho_f^2 - \frac{\gamma}{2\pi^2}\sum_f\int_0^\Lambda dk\,k^2(k^2+M_f^2)^{1/2} \\ &- \frac{\gamma}{6\pi^2}\sum_f\int_0^{k_{Ff}} dk\frac{k^4}{(k^2+M_f^2)^{1/2}} + \mathcal{U}\left(\rho_{sf},\rho_f,\Phi\right) - \Omega_{(\mathrm{vac})}\,,\end{aligned} \tag{6}$$

where the constant $\Omega_{(\mathrm{vac})} = \Omega_{\mathrm{PNJL0}}(\rho_u = \rho_d = \rho_s = 0)$ is added in the EOS to ensure $\Omega_{\mathrm{PNJL0}}(\rho_u = \rho_d = \rho_s = 0) = 0$.

3. Thermodynamics of the PNJL0 model

For the quark sector of the model we use the RKH parametrization [2], that has $\Lambda = 602.3$ MeV, $G_s\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$, $m_u = m_d = 5.5$ MeV, and $m_s = 140.7$ MeV. Λ is the cutoff parameter, adopted to avoid divergences in the momentum integrals [2]. For the vacuum we have $M_u^{(\mathrm{vac})} = M_d^{(\mathrm{vac})} = 367.6$ MeV, $M_s^{(\mathrm{vac})} = 549.5$ MeV. In the PNJL0 model, the presence of a $G_V \neq 0$ is essential to produce nonvanishing solutions for Φ [6]. The results for the SU(3) PNJL0 model are obtained by solving Eqs. (1) and (2) with the condition that $\partial\Omega_{\mathrm{PNJL0}}/\partial\Phi = 0$. Since Ω_{PNJL0} from Eq. (6) is a multi-valued function, we remove the unstable and metastable branches from the final curve by the requirement of thermodynamic stability [9]. As shown in Fig. 1, that results in two first-order phase transitions of the quark matter, one being for the restored/broken chiral symmetry and another for the quark confinement/deconfinement. For the former, the phase transition occurs at $\mu_{\mathrm{chiral}} = 372.1$ MeV for the choice of G_V and a_3 values given in the figure label.

As we can see, the non-strange quark sector presents a drastic decrease in the value of $M_u = M_d$, while the M_s is little reduced. This indicates that the restoration of chiral symmetry is stronger for the u and d quarks in comparison with the strange quark. The chemical potential for the phase transition of confinement/deconfinement is $\mu_{\mathrm{conf}} = 507.7$ MeV. For this case, we see that the strange quark has a strong reduction in its mass (almost 50%). This happens due to nonvanishing values of Φ close to the deconfined phase. The decrease of the coupling constants $\mathcal{G}_s$, $\mathcal{G}_V$ and $\mathcal{K}$ when Φ takes non-zero values induces this reduction. Notice that this is not a feature observed in the original SU(3) NJL model, as we can see in the figure 1.

Another aspect investigated here is the region between the two phase transitions, $\mu_{\mathrm{chiral}} < \mu < \mu_{\mathrm{conf}}$, identified as the quarkyonic phase [10]. This is a particular thermodynamical phase in which quarks present a low mass compared with its vacuum values, but remain confined ($\Phi = 0$). As we can see in Fig. 1 above $\mu_{\mathrm{chiral}} = 372.1$ MeV, where the chiral phase is restored, the quarkyonic phase

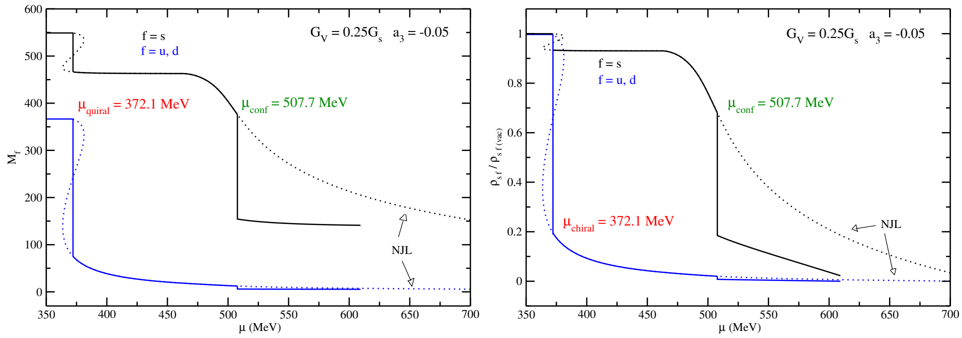

Figure 1: Left panel: constituent quark mass as function of μ. Right panel: $\rho_{sf}/\rho_{sf(\mathrm{vac})}$ as function of μ. For both panels, $G_V = 0.25G_s$ and $a_3 = -0.05$ for the PNJL0 model (solid lines) and the SU(3) NJL model (dotted lines).

is established up to the deconfinement phase transition at $\mu_{\mathrm{conf}} = 507.7$ MeV. We reinforce that this effect is not present in the original NJL model, since it lacks the confinement/deconfinement dynamics, as shown in Fig. 1.

As already mentioned, the dynamics of the Polyakov loop is determined by the constants G_V and a_3. In Fig. 2, we investigate how these constants affect the quarkyonic phase of the model. In the left panel, we fixed $G_V = 0.25G_s$ and vary a_3. For this case, a_3 does not change the value of $\mu_{\mathrm{quiral}} = 372.1$ MeV. However, the variation of a_3 modifies μ_{conf} strongly. As a consequence, the quarkyonic phase shrinks since as a_3 increases, one observes a decreasing of $\Delta\mu = \mu_{\mathrm{conf}} - \mu_{\mathrm{chiral}}$. Similar results are found for fixed a_3 and different values of G_V. As the value of G_V increases, the quarkyonic phase also shrinks since $\Delta\mu$ decreases. In this particular case, G_V impacts both, μ_{chiral} and μ_{conf}, unlike the previous case, in which a_3 only affects μ_{conf}.

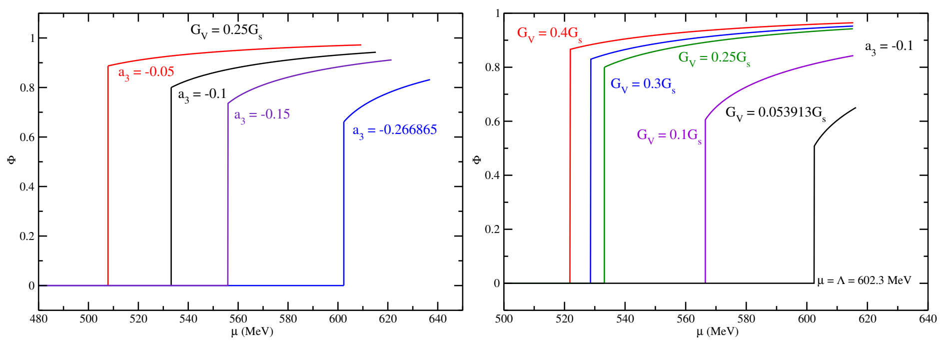

Figure 2: Φ as function of μ for the PNJL0 model. In the left, $G_V = 0.25G_s$ is fixed and a_3 have different values. In the right, $a_3 = -0.1$ is fixed and G_V have different values.

4. Summary and concluding remarks

In this work we present the three flavor version of the Polyakov-Nambu-Jona-Lasinio model at zero temperature (PNJL0). We illustrate the effects of the confinement/deconfinement at $T = 0$, where the quark-gluon infrared dynamics is implemented by making the coupling constants G_V, G_s and K dependent on the traced Polyakov loop (Φ). We show that the system presents two phase transitions, namely, one associated with the confinement/deconfinement transition and other representing the restored/broken chiral symmetry transition. For the latter, we show how chiral symmetry restoration of the strange quark is induced by the emergence of $\Phi \neq 0$. Another interesting feature presented by the PNJL0 model is the appearence of a thermodynamical phase between $\mu_{\rm chiral}$ and $\mu_{\rm conf}$, namely, the quarkyonic phase, in which the light quarks have chiral symmetry restored, but remains confined ($\Phi = 0$). We also study how the constants G_V and a_3 affect this phase.

Acknowledgments

This work is a part of the project INCT-FNA proc. No. 464898/2014-5. It is also supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under Grants No. 406958/2018-1, 312410/2020-4 (O.L.) and 308486/2015-3 (T.F.). We also acknowledge Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under Thematic Project 2017/05660-0 (O.L. and T.F) and Grant No. 2020/05238-9 (O.L.). O.A.M. thanks for the fellowship provided by CNPq - Brazil.

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