PROCEEDINGS OF SCIENCE

The Green function of the NJL–SU(2) model lagrangian with the AMM effect

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Magnetars and heavy-ion-collisions (HIC) are environments where only a robust theory of strong interactions can extract reasonable quantities according to the recent astronomical observations and HIC-experiments. Because of the sign problem, the Lattice Quantum Chromodynamics (LQCD) isn't treatable in important cases where there are finite chemical potentials. The Nambu–Jona-Lasinio (NJL) model, one of the most prominent among the effective models for QCD, allowed a lot of questions in low energy hadron physics to be studied. The thermodynamic properties of the QCD phase diagram can be altered in a non-negligible way under an electromagnetic field and taking a phenomenological term of anomalous magnetic moment (AMM). Here, we will present the general Quantum Electrodynamics (QED) formalism which this work is based and so we will obtain the Green function associated with the NJL–SU(2) density lagrangian.

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 -17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

[*] Speaker

PoS(XVHadronPhysics)050

1. Introduction

It's well known that strong magnetic fields are generated in a class of neutron stars, the so called magnetars [1], and both electric and magnetic fields are created in non-central heavy-ion collisions [2–4]. Strong magnetic fields on LQCD (Lattice Quantum Chromodynamics) have an important place in recent publications. The sign problem in the LQCD results is a limiting factor to the understanding of the finite chemical potential region of the QCD phase diagram. In that way, the Nambu–Jona-Lasinio (NJL) model [5–8] has been taking a leading role in the effective approach for QCD. The NJL model abides by the symmetries of the QCD lagrangian and for finite temperatures, e.g., it can emulate effects like the QCD confinement with the Polyakov loop.

The key to the anomalous magnetic moment (AMM) effect in the QCD phase diagram has not been understood. From some works [10–13], we can see that several properties of the QCD phase diagram can be altered. In a path to obtain a form of the density lagrangian to afterward implement some regularization scheme, we can calculate the Green function to this model with the AMM effect.

In Section 2 we present the model, in Section 3 we calculate explicitly the Green function to the density lagrangian of the model and finally in Section 4 we analyze the general framework in this matter and talk about future perspectives.

2. Nambu–Jona-Lasinio – SU(2) model with AMM

The density lagrangian is given by

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu(i\partial^\mu - e\hat{Q}A^\mu) - \hat{m} + \frac{1}{2}\hat{a}\sigma_{\mu\nu}F^{\mu\nu}\right]\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \tag{1}$$

where $\psi = (\psi_u \;\; \psi_d)^T$ is the quark fermion field, $\hat{m} = \mathrm{diag}(m_u, m_d)$ is the bare quark mass matrix, $\hat{Q} = \mathrm{diag}(q_u, q_d) = \mathrm{diag}(2/3, -1/3)$ is the quark charge matrix, $\hat{a} = \mathrm{diag}(a_u, a_d)$ is the AMM factor and G is the coupling constant. The letter u and d stand for the quark flavors up and down. The anomalous magnetic moment (AMM) term is in the first square brackets, related to the sigma matrices and the electromagnetic tensor. It is considered a phenomenological term, like was shown in [10–12].

We take the isospin symmetry $m_u = m_d \equiv m$ and in a mean-field approach, the lagrangian is given by Eq. (2)

$$\mathcal{L}^{MFA} = \bar{\psi}\left[\gamma_\mu(i\partial^\mu - e\hat{Q}A^\mu) - M + \frac{1}{2}\hat{a}\sigma_{\mu\nu}F^{\mu\nu}\right]\psi - \frac{(M-m)^2}{4G}\,, \tag{2}$$

where $M = m - 2G\langle\bar{\psi}\psi\rangle$ is the effective mass. Defining $\hat{a} = \hat{Q}\hat{\alpha}\mu_B$, where $\hat{\alpha} = \mathrm{diag}(\alpha_u, \alpha_d)$ and $a_f = q_f\alpha_f\mu_B$, we can write

$$\therefore\;\; \mathcal{L}^{MFA} = \sum_{f=u,d}\bar{\psi}_f\left[\gamma_\mu\left(i\partial^\mu - e_f A^\mu\right) - M + \frac{1}{2}a_f\sigma_{\mu\nu}F^{\mu\nu}\right]\psi_f - \frac{(M-m)^2}{4G}\,, \tag{3}$$

where we have an explicit sum over the quark flavors up and down.

In order to deal with this lagrangian in a way that we can avoid the complex usual methods, like the second quantization, or applying statistical mechanics, we use some tradicional Quantum Electrodynamics (QED) works. This can be justified in the way which QED is a regularizable theory (i.e., unlike NJL model that has to be regularized in some scheme), so we can take integral lagrangian forms, easily to be dealt with, as can be seen in [14, 15].

3. Green function

From Eq. (3), we can take a tradicional step and use the Euler–Lagrange equations. In this way, we obtain

$$\left[\gamma_\mu\left(i\partial^\mu - e_f A^\mu\right) - M + \frac{1}{2}a_f\sigma_{\mu\nu}F^{\mu\nu}\right]\psi_f = 0\,,\ \ f = u, d\,. \tag{4}$$

So we have two similar equations for each flavor. Defining $\gamma_\mu\partial^\mu \equiv \gamma\cdot\partial$, $\gamma_\mu A^\mu \equiv \gamma\cdot A$, $\sigma_{\mu\nu}F^{\mu\nu} \equiv \sigma\cdot F$, i.e., here the " $\cdot$ " represent the contraction of the 4-vectors. Then

$$\left[M + \frac{1}{i}\gamma\cdot\partial + e_f\gamma\cdot A - \frac{1}{2}a_f\sigma\cdot F\right]\psi_f = 0\,, \tag{5}$$

where $e_f = eq_f$.

The Green equation is given in Eq. (6).

$$\left[M + \frac{1}{i}\gamma\cdot\partial + e_f\gamma\cdot A - \frac{1}{2}a_f\sigma\cdot F\right]G(x,x'|A) = \delta(x-x')\,. \tag{6}$$

So, when we apply the dynamical equation on the Green function $G(x,x'|A)$, we get the Dirac delta function. Setting some definitions,

$$\Pi_\mu \equiv \frac{1}{i}\partial_\mu + e_f A_\mu\,,\ \ M' \equiv M - \frac{1}{2}a_f\sigma\cdot F, \tag{7}$$

the Eq. (6) can be written simply as

$$\left[M' + \gamma\cdot\Pi\right]G(x,x'|A) = \delta(x-x')\,. \tag{8}$$

We suppose an ansatz to the Green function

$$G(x,x'|A) = \left[M' - \gamma\cdot\left(\frac{1}{i}\partial + e_f A'\right)\right]\Delta(x-x'|A')\,. \tag{9}$$

In the Fock–Schwinger gauge

$$A'^\mu \equiv A^\mu_{SF}(x) = -\frac{1}{2}F^{\mu\nu}(x-x')_\nu\,. \tag{10}$$

Setting this gauge and calculating the explicit form of these operator we get

$$\Big\{\Box + \left(M - \frac{1}{2}a_f\sigma\cdot F\right)^2 + \frac{e_f}{2}\sigma\cdot F + \frac{e_f^2}{4}(x-x')^\mu F^2_{\mu\nu}(x-x')^\nu + \\ -i\frac{e_f}{2}F_{\mu\nu}\left[(x-x')^\nu\partial^\mu - (x-x')^\mu\partial^\nu\right]\Big\}\Delta(x-x'|A') = \delta(x-x')\,, \tag{11}$$

where $\Box$ is the d'Alembert operator and it is possible to show that the last term on the key brackets can be eliminated due to rotational symmetry. We can set more new quantities and rewrite Eq. (11) as

$$\mathcal{K}^2 = \left(M - \frac{1}{2}a_f\sigma\cdot F\right)^2 + \frac{e_f}{2}\sigma\cdot F\,,$$
$$X^\mu = (x-x')^\mu\,, \tag{12}$$

hence

$$\left[\Box + \mathcal{K}^2 + \frac{e_f^2}{4}X^\mu F^2_{\mu\nu}X^\nu\right]\Delta(X|A') = \delta(X)\,. \tag{13}$$

Here we take a Fourier transformation to the 4-momentum space and an ansatz [14, 16, 17]

$$\Delta(p|A') = \int_0^\infty ds\;\, e^{\mathcal{M}(s)}e^{-s(\mathcal{K}^2-i\epsilon)}, \tag{14}$$

where

$$\mathcal{M}(s) = p^\alpha \mathcal{X}_{\alpha\beta}(s)p^\beta + \mathcal{Y}(s)\,, \tag{15}$$

and $\mathcal{X}_{\alpha\beta}$ is a symmetric tensor. There are some boundaries conditions for this work, so when the external electromagnetic field is turned off $\mathcal{X}_{\alpha\beta}(s) \to s g_{\alpha\beta}$ and $\mathcal{Y}(s) \to 0$, where $g_{\alpha\beta}$ is the metric tensor. (Different from Dittrich and collaborators' works, here we are using the non west-coast metric, i.e., the usual Minkowski metric tensor with signature "+ − − −".)

Using the boundary conditions to the external field we solve the problem with

$$\mathcal{X}(s) = \frac{\tan(e_f F s)}{e_f F}\,,$$
$$\mathcal{Y}(s) = -\frac{1}{2}\,\mathrm{Tr}\ln\left[\cos(e_f F s)\right]\,. \tag{16}$$

From these solutions, we can find the find $\Delta(p|A')$. Then, taking the inverse Fourier transform backing to the 4-vectors space, we get

$$\Delta(x-x'|A') = \frac{i}{(4\pi)^2}\int_0^\infty ds\exp\left\{-s\left(\mathcal{K}^2 - i\epsilon\right) + \frac{1}{4}X^t\cdot(\mathcal{X})^{-1}\cdot X + \right.$$
$$\left. + \,\mathcal{Y}(s) - \frac{1}{2}\,\mathrm{Tr}\ln\left(s^{-1}\mathcal{X}(s)\right)\right\}. \tag{17}$$

where

$$\mathcal{Y}(s) - \frac{1}{2}\,\mathrm{Tr}\ln\left(s^{-1}\mathcal{X}(s)\right) = -\frac{1}{2}\,\mathrm{Tr}\ln\left(\frac{\sin(e_f F s)}{e_f F s}\right). \tag{18}$$

Therefore, with Eqs. (9, 17, 18) we have the Green function related to the density lagrangian from Sec. 2, Eq. (1), i.e., the lagrangian of the NJL–SU(2) model with the AMM contribution.

4. Conclusions

We have calculated the Green function of the Nambu–Jona-Lasinio SU(2) model including the phenomenological anomalous magnetic moment using a formalism based on previous QED works. Applying general methods of quantum field theory, like the Euler–Lagrange equations and proposing some ansatz based on the literature, we were able to obtain an explicit integral form for the Green function.

In a recent work [13] was obtained the thermodynamic potentials and the gap equation for this model, an important feature of the NJL based models. Compared to a non-AMM model, some parameters, e.g., the critical temperature decreases with the AMM effect and the magnetic catalysis holds for low temperatures without the oscillations effects. Therefore, we intend to continue to explore the effects that AMM can portray.

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