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Low-Q^2 parametrizations of the $\gamma^* N \to N^*$ transition amplitudes

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The electromagnetic structure of the nucleon resonances N^* are usually parametrized by $\gamma^* N \to N^*$ helicity amplitudes at the resonance rest frame. Those amplitudes are, however, constrained by kinematic conditions in the limit where the photon three-momentum vanishes (pseudothreshold limit). Although the pseudothreshold limit is below the photon point ($Q^2 = 0$) it has an impact on the structure of the helicity amplitudes at low Q^2. Most of the empirical parametrizations of the data ignore those constraints. In this work we study the effect of the pseudothreshold constraints on some analytic parametrizations of the data, performing analytic continuations of the parametrizations to the $Q^2 < 0$ region. We conclude that the pseudothreshold constraints are fundamental for some resonances, particularly for the $\Delta(1232)$ and the $N(1535)$.

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1. Status of the problem

The structure of the $\gamma^* N \to N^*$ transitions can be described in terms of transverse amplitudes ($A_{1/2}$ and $A_{3/2}$) and a longitudinal amplitude $S_{1/2}$, functions of the square transfer momentum $q^2 = -Q^2$, deppending on the polarization of the photon and the spin projections of the resonance. These helicity amplitudes are defined in the resonance rest frame.

However, the helicity polarization amplitudes, are not independent functions. They can be represented in terms of independent kinematic constraint-free form factors defined by the gauge-invariant form of the transition current [1, 2]. The explicit dependence varies with the resonance state J^P, where J is the spin and P is the parity. When we represent the helicity amplitudes in terms of the kinematic constraint-free form factors, we conclude that the helicity amplitudes have a particular dependence on the magnitude of the photon three-momentum $|\mathbf{q}|$ at the resonance rest frame. The leading order dependence on $|\mathbf{q}|$ for small $|\mathbf{q}|$ for the states J=$\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$ are presented in the left side of Table 1.

In addition, the scalar amplitude $S_{1/2}$ is correlated with the electric amplitude E, a combination of transverse amplitudes, according to $S_{1/2} \propto E|\mathbf{q}|$, in the limit where $|\mathbf{q}| \to 0$, also known as the pseudothreshold. The pseudothreshold is the point where the nucleon and the resonance are both at rest, and q was only the energy component $q = (M_R - M_N, 0, 0, 0)$, where M_R and M_N are the resonance and nucleon masses, respectively. At the pseudothreshold one has $Q^2 = -(M_R - M_N)^2$. The previous result is more commonly known as Siegert's theorem [1, 3–6]. The explicit correlation between amplitudes are presented on the right side of the Table 1.

Parametrizations of the data ignore, in general, the correlations between the amplitudes and threat the amplitudes as independent functions [5, 7–9]. The purpose of this work is to emphasize that the correlations between the helicity amplitudes cannot be ignored at low Q^2, and that they have an impact on the shape of the helicity amplitudes near $Q^2 = 0$.

We propose a method that can be used to modify any smooth analytic parametrization of the data, valid for finite Q^2, in order to be consistent with the pseudothreshold constraints, below a certain point $Q_P^2 > 0$.

$\frac{1}{2}^+$	$A_{1/2} \propto \lvert\mathbf{q}\rvert$,	$S_{1/2} \propto \lvert\mathbf{q}\rvert^2$	
$\frac{1}{2}^-$	$A_{1/2} \propto 1$,	$S_{1/2} \propto \lvert\mathbf{q}\rvert$	$S_{1/2} \propto A_{1/2}\lvert\mathbf{q}\rvert$
$\frac{3}{2}^+$	$A_{1/2} \propto \lvert\mathbf{q}\rvert$,	$S_{1/2} \propto \lvert\mathbf{q}\rvert^2$	$S_{1/2} \propto (A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2})\lvert\mathbf{q}\rvert$
	$A_{3/2} \propto \lvert\mathbf{q}\rvert$,		
$\frac{3}{2}^-$	$A_{1/2} \propto 1$,	$S_{1/2} \propto \lvert\mathbf{q}\rvert$	$S_{1/2} \propto (A_{1/2} + \sqrt{3}A_{3/2})\lvert\mathbf{q}\rvert$
	$A_{3/2} \propto 1$,		$(A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2}) \propto \lvert\mathbf{q}\rvert^2$

Table 1: On the right: Leading order dependence of the amplitudes of $|\mathbf{q}|$. **On the left:** Correlations between amplitudes.

2. Methodology

For the purpose of the discussion, we consider A a generic amplitude ($A_{1/2}$, $A_{3/2}$ and $S_{1/2}$). We assume then that the parametrization is known for $Q^2 \geq Q_P^2$, and can be expanded according to

$$A(Q^2) = A^{(0)} + A^{(1)}(Q^2 - Q_P^2) + \frac{A^{(2)}}{2!}(Q^2 - Q_P^2)^2 + \frac{A^{(3)}}{3!}(Q^2 - Q_P^2)^3 + ...,$$

We consider now the analytical continuation for $-(M_R - M_N)^2 \leq Q^2 \leq Q_P^2$

$$A = |\mathbf{q}|^n(\alpha_0 + \alpha_1|\mathbf{q}|^2 + \alpha_2|\mathbf{q}|^4 + \alpha_3|\mathbf{q}|^6),$$

where n can take the values $n = 0, 1, 2$ (see Table 1). The coefficients α_l ($l = 0, .., 3$) are determined using the constraints on the amplitudes and the continuity of A, the first derivative (A'), and the second derivative (A''). In a few cases, when necessary, we consider also the continuity of the third derivative (A''').

To study the sensibility of the original parametrization to the pseudothreshold constraints, we use different values of Q_P^2 near $Q^2 = 0$. We consider in particular $Q_P^2 = 0.1$, 0.3 and 0.5 GeV^2.

To exemplify the method, we use the Jefferson Lab parametrizations of the data [8, 9], which is successful in the description of the CLAS data associated with the resonances $\Delta(1232)\frac{3}{2}^+$, $N(1440)\frac{1}{2}^+$, $N(1520)\frac{3}{2}^-$ and $N(1535)\frac{1}{2}^-$, among others, at intermediate and large Q^2 [10].

3. Analytic extension of parametrizations for low Q^2

The method described above was implemented to the available J=$\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$ parametrizations of the data [8]. The details are discussed in Ref. [1]. Here we highlight the results for the resonances $\Delta(1232)\frac{3}{2}^+$ and $N(1535)\frac{1}{2}^-$.

The results for the $\Delta(1232)\frac{3}{2}^+$ are presented in Fig. 1. Notice that all the extensions induce in the amplitudes a shape consistent with the expected result at the pseudothreshold (the amplitudes vanish). From the figure, one concludes also that the parametrizations with $Q_P^2 = 0.1$ and 0.5 GeV^2 deviate more from the low-Q^2 data, but in different directions. Only the parametrization with $Q_P^2 = 0.3$ GeV^2 describe well the data. The original parametrization has a shape incompatible with the threshold constraints. Additional discussions about the $\Delta(1232)$ form factors and Siegert's theorem can be found in Refs. [11–14].

In Fig. 2, we present the extensions of the parametrizations for the resonance $N(1535)$. For the resonance $N(1535)$, however, we need to consider two distinct cases. In the first case, we use the amplitude $A_{1/2}$ as reference and modify the amplitude $S_{1/2}$ in order to satisfy Siegert's theorem. On that account, $A_{1/2}$ is a smooth function, and the solutions are represented by the thicker lines. The consequence of this choice is that the amplitude $S_{1/2}$ has a sharp variation near $Q^2 = Q_P^2$ (the derivatives of $S_{1/2}$ are large).

Another possibility is to impose, instead, that is the function $S_{1/2}$ that should be smooth (small derivatives). In this situation, it is the amplitude $A_{1/2}$ that is determined from Siegert's theorem. We obtain then the solutions represented by the thinner lines (also for $Q_P^2 = 0.1$ 0.3 and 0.5 GeV^2). In this last case, one can observe the sharp variation of the amplitude $A_{1/2}$ at low Q^2.

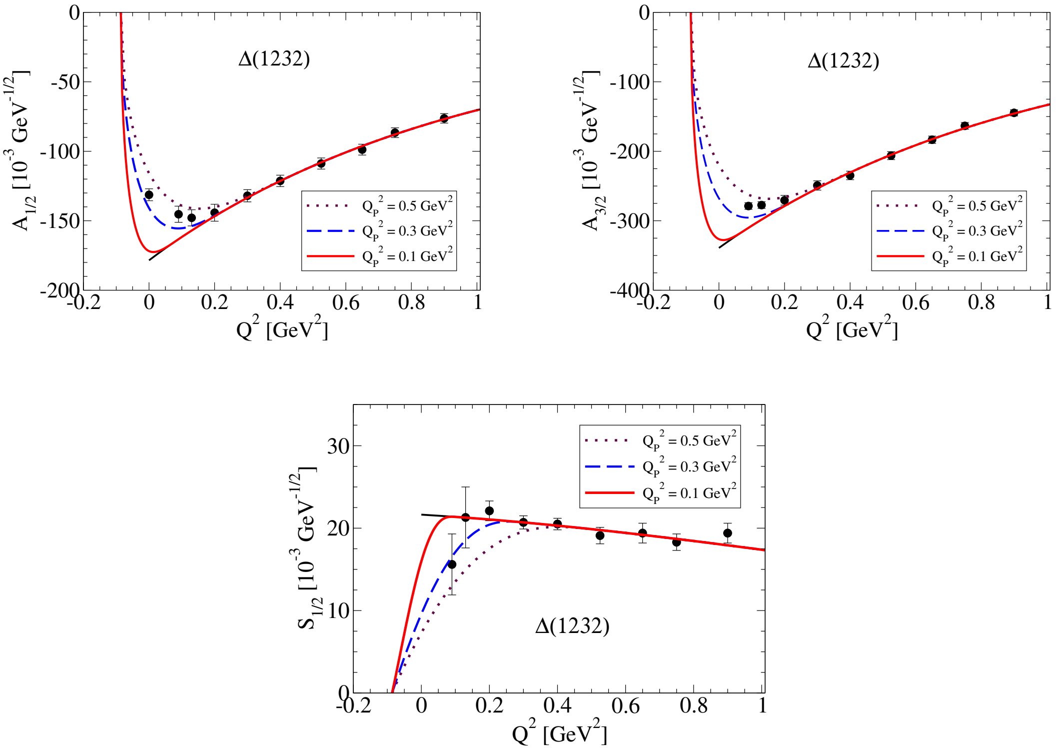

Figure 1: $\gamma^* N \to \Delta(1232)\frac{3}{2}^+$ transition amplitudes. The original parametrization is the black solid line [8]. The data are from CLAS [10] and Refs. [13, 14].

Which solution: sharp amplitude $S_{1/2}$ (thicker lines) or sharp amplitude $A_{1/2}$ (thinner lines) is closer to the physical solution? At the moment, there is no way to discriminate between the solutions, since they are both compatible with the data. Notice that the data are nonexistent below $Q^2 = 0.3$ GeV^2, except for the results for $A_{1/2}(0)$, affected by significant uncertainties. Only more data for $A_{1/2}$ and $S_{1/2}$ below $Q^2 = 0.3$ GeV^2, or a more precise measurement of $A_{1/2}(0)$ can decide.

Notice also that the data for $A_{1/2}(0)$ from the Particle Data Group (PDG) are changing along the time. From the left to the right, one has PDG 2013 [15], PDG 2017 [16] and PDG 2021 [17].

4. Conclusions

Parametrizations of the data valid for intermediate and large Q^2 ignore in general constraints at the pseudothreshold due the gauge-invariance of the transition current, which cannot be ignored We propose a method that can be used to conciliate any smooth analytic parametrizations of the data with the constraints at the pseudothreshold. The presented analysis can be extended to higher mass resonances once more low-Q^2 data became available. Depending of the resonance, we can find a point Q_P^2 which conciliate the pseudothreshold constraints and the low Q^2 data.

Evidences of the pseudothreshold constraints are manifest in some resonances. The $\Delta(1232)$, presented here, is a good example. New data are fundamental to determine the shape of the helicity amplitudes for some particular resonances. The $N(1535)$ is the best illustration of the necessity of more data. Future data will establish if it is the amplitude $A_{1/2}$ which have a sharp variation near $Q^2 = 0$, or if it is the amplitude $S_{1/2}$ which change sign near the pseudothreshold.

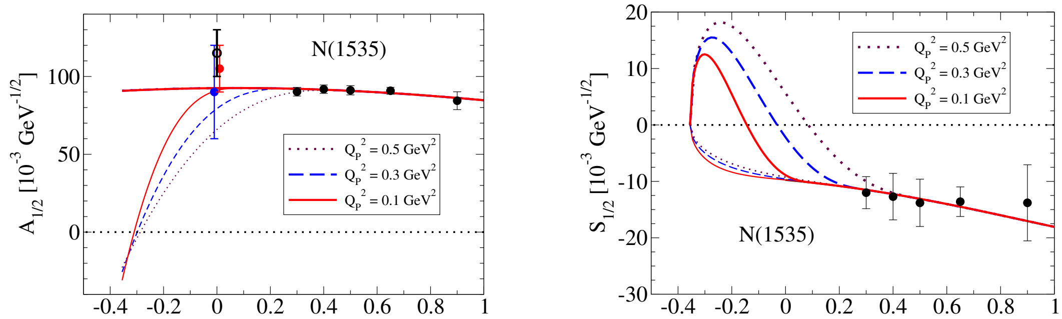

Figure 2: $\gamma^* N \to (1535)\frac{1}{2}^-$ transition amplitudes. The thinner lines describe parametrizations where $S_{1/2}$ is a smooth function, as discussed in the main text. The data for $Q^2 = 0$ are from PDG 2013 [15], PDG 2017 [16] and PDG 2021 [17] (from left to the right). The data for finite Q^2 are from CLAS [10].

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References

[1] G. Ramalho, Phys. Rev. D **100**, 114014 (2019).

[2] R. C. E. Devenish, T. S. Eisenschitz and J. G. Korner, Phys. Rev. D **14**, 3063 (1976).

[3] A. J. Buchmann, U. Meyer, A. Faessler and E. Hernandez, Phys. Rev. C **58**, 2478 (1998).

[4] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).

[5] L. Tiator, Few Body Syst. **57**, 1087 (2016).

[6] G. Ramalho, Phys. Lett. B **759**, 126 (2016)

[7] D. Drechsel, S. S. Kamalov and L. Tiator, Eur. Phys. J. A **34**, 69 (2007).

[8] `https://userweb.jlab.org/~isupov/couplings/`

[9] A. N. Hiller Blin *et al.*, Phys. Rev. C **100**, 035201 (2019).

[10] V. I. Mokeev, `https://userweb.jlab.org/~mokeev/resonance_electrocouplings/`

[11] G. Ramalho, Phys. Rev. D **93**, 113012 (2016).

[12] G. Ramalho, Phys. Rev. D **94**, 114001 (2016).

[13] G. Ramalho, Eur. Phys. J. A **54**, 75 (2018).

[14] G. Ramalho, Eur. Phys. J. A **55**, 32 (2019).

[15] J. Beringer *et al.* [Particle Data Group], Phys. Rev. D **86**, 010001 (2012).

[16] C. Patrignani *et al.* [Particle Data Group], Chin. Phys. C **40**, 100001 (2016).

[17] P. A. Zyla *et al.* [Particle Data Group], PTEP **2020**, 083C01 (2020).