

Unsupervised machine learning correlations in EoS of neutron stars

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Neutron stars are compact objects of large interest in the nuclear astrophysics community. The extreme conditions present in such systems impose big challenges to our current microscopic models of nuclear structure. Equation of states (EoS) are frequently derived from sophisticated quantum mechanical models, such as: relativistic, non-relativistic and many mean-field approaches. Every single model, in general, contains many parameters such as the NN interaction strength, particle compositions, etc. These are particular features of each model and can be represented by numbers and categories in a machine learning context. Different choices of features will affect EoS properties leading to different macroscopic properties of the star. In this work we analyze a selection of EoS containing a variety of different physics models. One of our objectives is to develop tools that enable a better understanding of the correlations among the different model features and the outcome produced by them when employed to model neutron stars.

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1. Introduction

Neutron stars (NS) are supernova remnants with strong gravitational fields and rapid rotation. They consist of the highest density objects in the Universe with numbers ranging from a few g/cm^3 at their surface to more than $10^{15} \text{ g}/\text{cm}^3$ at their center [1].

The microscopic description of these systems have been extensively studied in the last few decades and yet, a unified and complete theoretical understanding is missing. It is still difficult to exclude the many possible scenarios and converge to a single set of parameterization and constraints on the EoS even when both astrophysical observations and nuclear physics experiments are considered [2].

Part of the challenge here relates to extreme physical environments, e.g. large matter-energy densities, and the limits of our models that contain parameters adjusted to reproduce, at their best, nuclear properties on natural conditions present on Earth. These challenges open up space for many new functional parameterization and models that can reproduce the physics of matter from Earth-like conditions and be extrapolated to the high density stellar environment. In this work, we demonstrate how simple unsupervised machine learning techniques can help us to identify important correlations among the various EoS of dense matter, commonly used to model NS. To understand the outcomes of different equations of states, we employ dimensional reduction algorithms such as: Multi Dimensional Scaling (MDS), Principal Value Component Analysis (PCA), and t-distributed Stochastic Neighbor Embedding (t-SNE). The dimensional reduction provided by such tools helps to discover underlying structures present in the different physics models. We have selected a set of popular EoS that represent different models to demonstrate our approach. In Section 2 we give a short description of the physics of equation of state. Section 3 presents a brief description of the ML methods employed in this study, as well as our results and discussions. Final remarks are provided in Sec. 4.

2. Equation of state

The high regime/densities of the EoS describing the NS interior has not been fully constrained, leaving an open question in nuclear astrophysics. Only a few microscopic physics constraints are currently possible when one considers neutron stars: electric neutrality, beta equilibrium, positive pressure, $p \geq 0$, and $dp/d\rho > 0$ from the Chateliers' principle and finally causality, i.e., the speed of the sound v_s must be less than the speed of light c . The uncertainty in the NS interior leads to a large variety of EoS, roughly distinguished by the compressibility of the nuclear matter (i.e., softness and stiffness, which is associated to the speed of sound in the matter) and its behavior in large energy density regimes. There are several methods to calculate the EoS: Perturbation expansion within the Brueckner-Bethe-Goldstone-[Hartree-Fock] theory (BHF), perturbation expansion within the Green's-function theory, variational method (VF), energy density functional (EDF) theory, relativistic mean-field (RMF) models [1, 3–7]. Point-coupling and non-relativistic models such as Skyrme and Gogny Hartree-Fock Bogoliubov (HFB) theories are also used [8–14]. Skyrme and RMF models span more than 500 parameterization possibilities, which raises doubt whether they can reproduce different density environments simultaneously. Some studies try to constrain all these parameters in the vicinity of the nuclear saturation density [15–

17], while others consider the binary neutron star merger observation released by LIGO-VIRGO collaborations [18–20] as constrain for the mass-radius of NS and consequently for the EoS which generates them [21–27].

One can expect correlations among all the different existing models, but their variety and large set of parameters involved poses barriers for a more detailed study. Here we selected a small set of EoS derived from different physics models, the most commonly used in the literature [28–31]. We summarize their characteristics in Table 1. Models with same composition but with non-relativistic and relativistic approaches are included.

EoS	Composition	Model	Approach	Potential
APR1-4 [32]	npem	non-relativistic	Variational	Two-three body
BBB2 [33]	npem	non-relativistic	BHF	Two-three body
FPS [34]	npem	non-relativistic	Variational	Two-three body
SLy4 [35]	npem	non-relativistic	EDF	Two-body
PAL6 [36]	npem	non-relativistic	Schematic potential	Two-body
WWF1-3 [37]	npem	non-relativistic	Variational	Two-body
ENG [38]	npem	relativistic	Dirac-BHF	Meson exchange
MPA1 [39]	npem	relativistic	Dirac-BHF	Meson exchange
MS1-2,1b[40]	npem	relativistic	MF	Meson exchange
BPAL12 [41]	npem	relativistic	Dirac-BHF	Two-body
PS [42]	meson	non-relativistic	Potential	Two-body
GS1-2 [43]	meson	relativistic	MF	Meson exchange
GNH3 [44]	meson	relativistic	MF	Two-body
H1-7 [45]	hyperon	relativistic	MF	Meson exchange
PCL2 [46]	hyperon	relativistic	MF	Meson exchange
ALF1-4 [47]	quark	relativistic	MIT	Gluons (QCD)

Table 1: Summary of selected EoS. The composition npem stands for nucleonic matter in β -equilibrium. Meson, hyperon and quarks are models collective known as K/ π /H/q models.

3. Correlations and Data Grouping

The first unsupervised machine learning method that we have used is the Principal Components Analysis (PCA) [48]. This algorithm is part of the *dimension reduction*. It transforms the characteristics of a dataset into a new set of features called Principal Components. By doing this, many variables across the full dataset are effectively compressed in fewer feature columns. This reduction creates a new set of 'uncorrelated' variables as functions of the old features. We use this approach in a multidimensional scaling (MDS) [49] technique to find a low-dimensional graphical projection, a 2D data in our case. The resulting reduced data is presented in a grouped form rule by their best similarities obtained with data point distances. The second method used in this work is the t-distributed Stochastic Neighbor Embedding (t-SNE) [50]. In this case, we again reduce the dimensions of the data, trying at the same time to keep similar (EoS) distances close and dissimilar

instances apart. Unlike PCA, which is a linear technique, t-SNE is nonlinear, and it permits to separate data that cannot be separated by any straight line.

3.1 Using the MDS

In figure 1 we have a multidimensional scaling projection considering the PCA reduction in our dataset from the EoS table 1. In each denser color, we display the maximum masses for the EoS sample: in blue we have the less massive stars and in yellow the most massive ones. In geometric forms, we have the approach utilized to generate the EoS. We can see the formation of clusters, the first pattern is due to the parameterization within each EoS, i.e., one can see a cluster formation in ALF1-4, H1-7, APR3-4, APR1-2; which is obvious at a first glance. However, what is remarkable is that the EoS responsible for the highest/lowest mass are scattered: One cannot see a clusterization due this physical quantity because there is no clear-colored region.

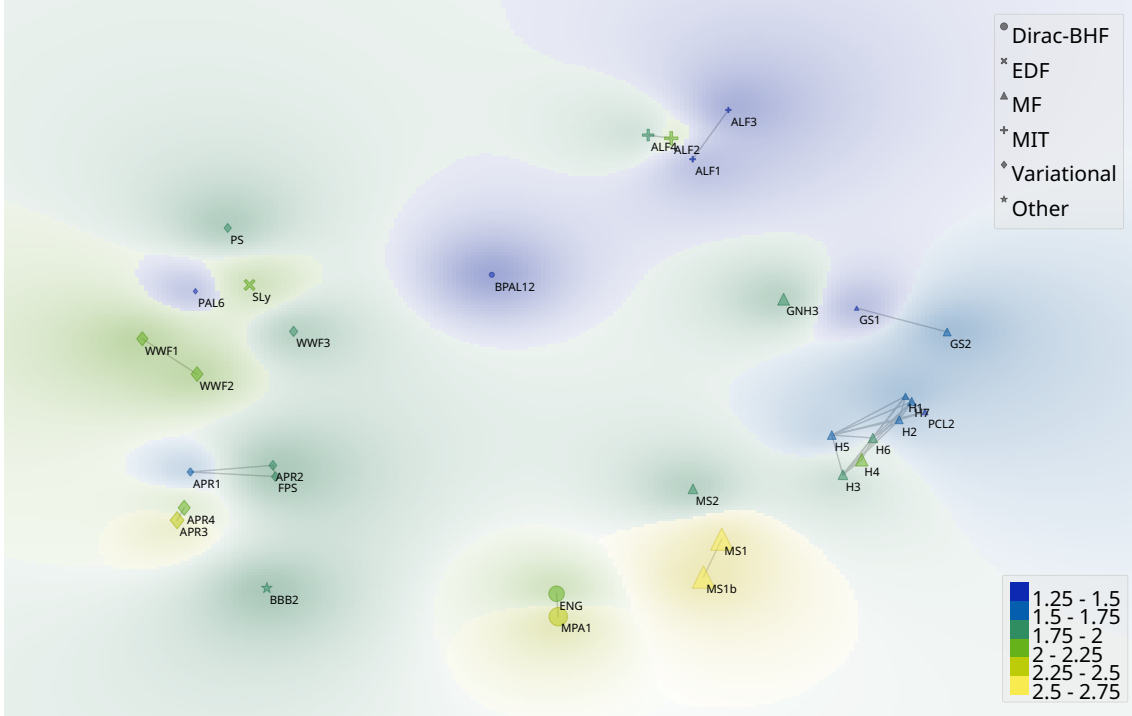


Figure 1: Multidimensional scaling projection using PCA. We use M_{\max} as colors. The different geometric forms represent the feature approach utilized to generate the EoS.

In figure 2 we have again the MDS considering PCA reduction. In this second case, we have in colors the composition of the EoS and in geometric forms if the model is relativistic or not. We have the formation of clusters as the previous case, however here it is possible to see a separation of the colored regions due to the composition of the EoS. One sees one exception, PS, and one can consider it as an outlier in terms of composition. However, there is a correlation with PAL6 and SLY4 which overrules the composition of the EoS as principal characteristic. It is also possible to see a separation among the non-relativistic and relativistic models. On the left side of the graphic we have non-relativistic nucleonic EoS and from the bottom-center, where one finds a group of nucleonic, to the upper-right side, the relativistic ones, where the $K/\pi/H/q$ models are

located. Regarding the relativistic-K/ π /H/q models, it is possible to see one EoS that is an outlier, the GS with its two parameterizations. In figure 2, we have a clear separation of the relativistic and non-relativistic models, however comparing with the previous Fig. 1, we see no correlation of the maximum mass with the nature of the models.



Figure 2: Multidimensional scaling projection using PCA. We use the composition of the EoS as colors. In geometric forms, we display if the model is non-relativistic or relativistic.

3.2 Using the t-SNE

In figure 3 it is shown the correlation of the dataset using the t-SNE which used a PCA with 6 principal components. In this figure, we have again the formation of well-defined regions according to the composition of the EoS. Again, PS is an outlier; however, this time it is less correlated with the hadronic models in comparison with Fig. 2. Using t-SNE we can see that the GS EoS is more related to the K/ π /H/q models and that they are more strongly correlated, i.e., ones can see that the nucleonic models are closer as well as the K/ π /H/q models. One needs a strong jittery to separate the EoS.

4. Concluding Remarks

In this work, we have investigated correlations in a small sample of EoS. We made use of different unsupervised machine learning algorithms in a dimensionality reduction approach. Using three algorithms (PCA, MDS and t-SNE) we were able to visualize correlations, i.e., features which are hidden when dealing with complex physical models. This is one of the key points when employing machine learning techniques, and can be used to provide feedback for the theoretical



Figure 3: t-SNE using PCA with 6 components. We use the composition of the EoS as colors. In geometric forms, we have the different models.

models. This approach can be very helpful to achieve a better description of nuclear matter at high densities and temperatures.

We want to stress that a large set of EoS and its features, as well as combination of precise data from advanced detectors (VIRGO-LIGO-KAGRA and eXTP) is required for best use of machine learning models. We believe that the use of unsupervised ML approaches and sophisticated visualization tools can help to categorize the many models available in the market. Refining the microscopic models we can find a more realistic EoS that can describe the internal structure of neutron stars.

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