PROCEEDINGS OF SCIENCE

Multiplicity moments at the LHC: how bad is the negative binomial distribution ?

G. R. Germano[*] and F. S. Navarra

Departamento de Física Nuclear, Instituto de Física, Universidade de São Paulo,
Rua do Matão, 1371, CEP 05508-090, São Paulo, SP, Brasil

E-mail: guilherme.germano@usp.br, navarra@if.usp.br

In this work, we compare the first C-moments of the multiplicity distributions recently measured in proton-proton collisions at the LHC with the predictions of the Bialas-Praszalowicz model. In this model the multiplicity distribution is given by a negative binomial distribution (NBD). In our comparison, we try to identify the regions of the phase space where the NBD fails. We divide the data into three sets according to their phase space coverage: I: $p_T > 100$ MeV and $|\eta| < 0.5$; II: $p_T > 100$ MeV and $|\eta| < 2.4$ and II: $p_T > 500$ MeV and $|\eta| < 2.4$. The mean multiplicity grows with the energy according to a power law and the power is different for each set. The C_n moments grow continuously with the energy, slowly in set I and faster in the other sets. We find that the NBD gives a very good description of the measured moments C_2, C_3 and C_4 and slightly overestimates C_5 in all data sets. The negative binomial parameter k decreases continuously with the energy and there is no sign of change in this behavior.

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 - 17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

[*]Speaker

PoS(XVHadronPhysics)063

1. Introduction

It is well known [1, 2] that multiplicity distributions measured in proton-proton collisions at energies up to $\sqrt{s} = 0.54$ TeV can be well described by a negative binomial distribution (NBD), which is defined by two parameters $\langle n \rangle$ and k. Deviations from the NBD were discovered at $\sqrt{s} = 0.9$ TeV and later confirmed at the Tevatron at $\sqrt{s} = 1.8$ TeV. More recently, these deviations were also observed at the LHC at $\sqrt{s} = 7$ TeV [3]. In this work we check whether deviations from the NBD predictins appear in the C_n moments of the multiplicity distribution moments measured at LHC energies up to 13 TeV.

We use the Bialas - Praszalowicz (BP) Model [4]. In this model multiparticle production is a two-step process, in which particle sources are produced following a certain distribution function and then each source emits particles according to a Poisson distribution. The final particle number distribution is NBD. With the BP model it is easy to find analytical expressions for the first C_n moments.

In the most recent experimental papers, the multiplicity distributions $P(n)$ were presented but the moments C_n were not. In [5] we computed the moments from the multiplicity tables, which we took from the hepdata.net databasis.

The predictions of the BP model will be compared with the most recent data from the LHC on non single diffractive pp collisions, which can be grouped into three sets:

- Set I: $p_T > 100$ MeV, $|\eta| < 0.5$, and energies $\sqrt{s} = 900$, 2360 and 7000 GeV.
- Set II: $p_T > 100$ MeV, $|\eta| < 2.4$, and energies $\sqrt{s} = 900$, 7000, 8000 and 13000 GeV.
- Set III: $p_T > 500$ MeV, $|\eta| < 2.4$, and energies $\sqrt{s} = 900$, 7000, 8000 and 13000 GeV.

These data sets may contain particles produced through different production mechanisms. The particles measured in set I are produced mainly from gluons; those measured in set II come also from the fragmentation region (larger rapidities) and hence are produced also from the valence quarks. Due to the larger transverse momentum cut-off, set III contains more particles which are produced perturbatively. These differences might lead to a different behavior of the multiplicity distributions.

2. The Bialas - Praszalowicz Model

In the BP model the multiplicity distribution is given by:

$$P_{BP}(n) = \int_0^\infty dt\, F(t)\, e^{-\bar{n}t} \frac{(\bar{n}t)^n}{n!} \qquad \text{with} \qquad F(t,k) = \frac{k^k}{\Gamma(k)} t^{k-1} e^{-kt} \tag{1}$$

where t is a fraction of the average multiplicity, and $F(t)$ the distribution of sources that contribute a fraction t to the multiplicity. With the above choice for F, P_{BP} turns out to be the negative binomial distribution (NBD). Moreover, for this choice of F we have $\langle n \rangle = \bar{n}$. Distribution (1) depends on one parameter k, which depends on the collision energy. The analysis of lower energy data shows that k decreases with increasing energy. When $k = 1$ the probability distribution P_{NBD} becomes a geometrical distribution. When k is large ($1/k \to 0$), the distribution P_{NBD} tends to a Poisson distribution. The mean multiplicity is an input to calculate P_{NBD} and can be parametrized as [5] :

$$\langle n \rangle = \left(\frac{s}{q_0^2} \right)^{\Delta} \tag{2}$$

In [5] we fixed q_0 and Δ from the fit of the available experimental data and the results are shown in Table 1. The data of sets I and II have the same lower p_T cut and very different rapidity coverage. In set I, we observe particles produced in the central (rapidity) region, which is dominated by gluons. In set II there is a larger contribution coming from the fragmentation region, where the valence quarks play an important role in particle production. The separation of central and fragmentation regions was shown to be relevant for particle production in [6]. In the fragmentation region partons from the projectile and from the target collide in a very asymmetric kinematical configuration. Data in this region of the phase space are more sensitive to the low-x QCD dynamics and to saturation effects, which tame the growth of several observables [7] with the energy. Probably the weaker energy dependence of the data of set II is a manifestation of low x saturation effects.

Set	Δ	q_0 (GeV)
I	0.13	6.31
II	0.11	0.01
III	0.16	4.83

Table 1: Parameters q_0 e Δ.

The moments are defined as:

$$C_m = \frac{\langle n^m \rangle}{\langle n \rangle^m} \tag{3}$$

In the BP model the moments are obtained from (1). The first moments, C_2 and C_3 are given by:

$$C_2 = \frac{1}{\langle n \rangle} + 1 + \frac{1}{k} \quad \to \quad \frac{1}{k} = C_2 - 1 - \frac{1}{\langle n \rangle}. \tag{4}$$

$$C_3 = C_2(2C_2 - 1) - \frac{C_2 - 1}{\langle n \rangle} \tag{5}$$

The expressions for C_4 and C_5 can be found in [5]. All the moments are functions only of C_2 and $\langle n \rangle$.

3. Results and discussion

In order to calculate the C_n moments, we need to know k to compute the moment C_2 and then all the other moments. Instead of choosing values for k, we follow [4] and parametrize C_2 as

$$C_2 = a + b \log(\sqrt{s}[\text{GeV}]) \tag{6}$$

Next, we fit C_2 to the data, fixing a and b, and finally we calculate k using (4). The obtained values (a, b) are (1.68 , 0.02) for Set I; (0.97 , 0.08) for Set II and (1.30 , 0.06) for Set III. Having

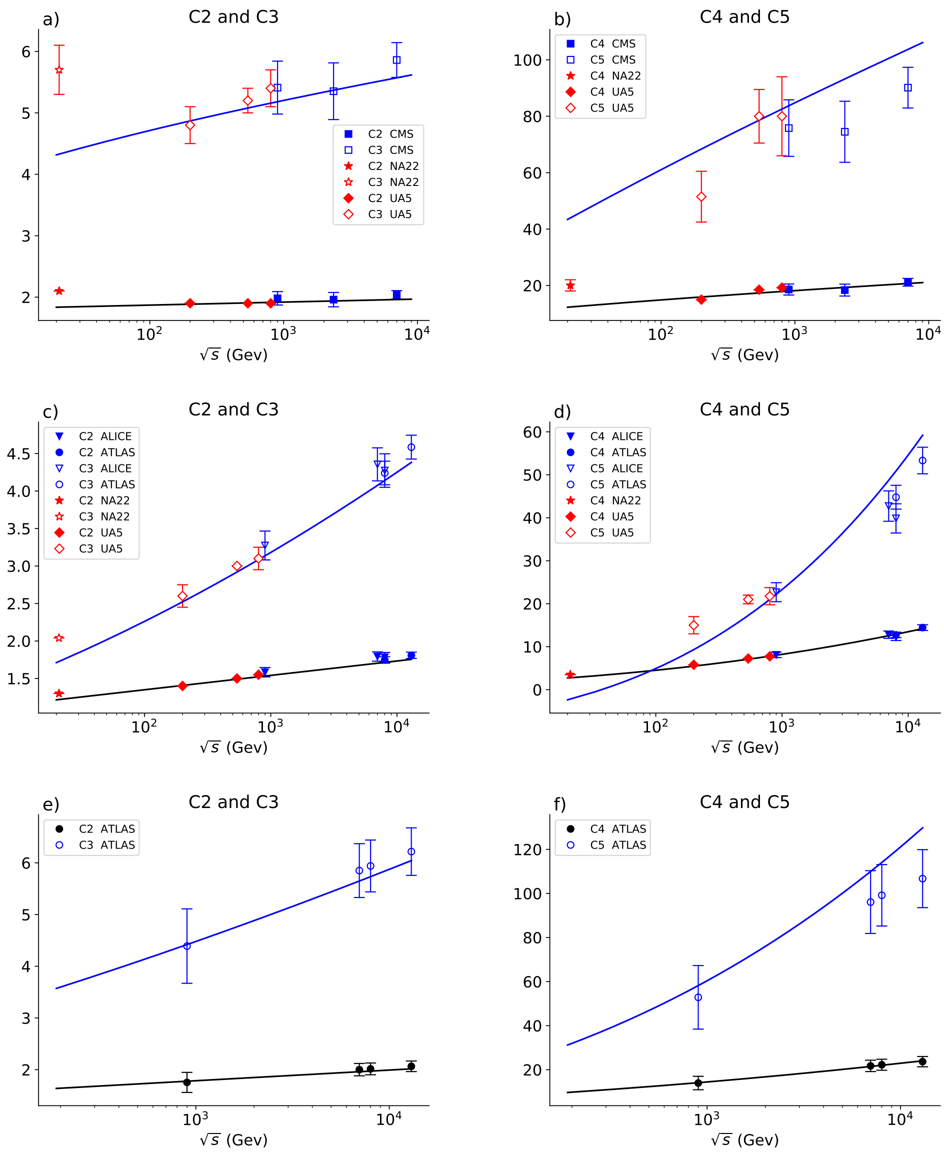

Figure 1: C_n moments. Solid lines: BP model. a) C_2 and C_3. Set I. b) C_4 and C_5. Set I. c) C_2 and C_3. Set II. d) C_4 and C_5. Set II. e) C_2 and C_3. Set III. f) C_4 and C_5. Set III.

determined the parameters a and b, we can calculate all the first C moments, compare them with data and make predictions. This is shown in Fig. 1, where we compare the BP moments with the three data sets. Looking first at the data (which were put together for the first time in [5]) we observe that in all figures the moments grow with the energy. The moments from set I grow much slower and are even compatible with a constant value. Comparing the moments obtained with sets II and III we see that the C_n's grow with energy in the same (strong) way. Having fitted $< n >$ and C_2 we may return to (4) and plot $1/k$ as a function of $\sqrt{s}$. This was done in [5] where it was found that $1/k$ is an increasing function and there is no sign of a different behavior. This result extends the conclusions found ten years ago in [4] to the present energies, which are two times higher.

Now we come back to question formulated in the title. With a negative binomial distribution (derived from the BP model) and assuming a logarithmic growth (6) of C_2 moment, we were able to reproduce the first multiplicity moments over a wide range of energies for different rapidity intervals. Large deviations from the NBD predictions might be observed for higher order moments. This conclusions is in line with the results found in [3], where the single NBD was shown to deviate from the measured multiplicity distributions only for very large values of n (where $P(n)$ is three orders of magnitude smaller than its maximum value).

To summarize: for practical purposes the single NBD captures the main features of the multiplicity distributions even at the highest LHC energies. Discrepancies appear only at the tails of these distributions. NBD is not so bad!

A final remark: the growth of C_2 with energy can be translated through (4) into a decrease of the parameter NBD k. This behavior is consistent with lower energies and does not exhibit the change predicted in [8].

Acknowledgments

We would like to thank the Brazilian funding agency Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and also the INCT-FNA for supporting this work.

References

[1] For a review see: W. Kittel, E.A. De Wolf, Soft Multihadron Dynamics, World Scientific, 2005.

[2] J. F. Grosse-Oetringhaus, K. Reygers, J. Phys. G **G37**, 083001 (2010).

[3] P. Ghosh, Phys. Rev. D **85**, 054017 (2012).

[4] M. Praszalowicz, Phys. Lett. B **704**, 566 (2011). A. Bialas, Acta Phys. Polon. B **41**, 2163 (2010).

[5] G. R. Germano and F. S. Navarra, Phys. Rev. D **105**, 014005 (2022); arXiv:2011.08912.

[6] G. N. Fowler, F. S. Navarra, M. Plumer, A. Vourdas, R. M. Weiner and G. Wilk, Phys. Rev. C **40**, 1219 (1989).

[7] V. P. Gonçalves, M. V. T. Machado, B. D. Moreira, F. S. Navarra and G. S. dos Santos, Phys. Rev. D **96**, 094027 (2017).

[8] F. Gelis, T. Lappi, L. McLerran, Nucl. Phys. A **828**, 149 (2009).