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Magnetic transitions in ultraperipheral collisions

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In non-central relativistic heavy ion collisions a very strong magnetic field is formed. There are several studies of the effects of this field, where $\vec{B}$ is calculated with the expressions of classical electrodynamics. A quantum field may be approximated by a classical one when the number of field quanta in each field mode is sufficiently high. This may happen if the field sources are intense enough. In heavy ion physics the validity of the classical treatment was never investigated. In a previous work we proposed a test of the quality of the classical approximation. We calculated an observable quantity using the classical magnetic field and also using photons as input. We focused on the process in which a nucleon is converted into a delta resonance, which then decays into another nucleon and a pion, i.e., $N \to \Delta \to N'\pi$. In ultraperipheral relativistic heavy ion collisions this conversion can be induced by the classical magnetic field of one of the ions acting on the other ion. Alternatively, we can replace the classical magnetic field by a flux of equivalent photons, which are absorbed by the target nucleons. We calculated the cross sections in these two independent ways and found that they differ from each other by $\simeq 10$ % in the considered collision energy range. This suggests that the two formalisms are equivalent and that the classical approximation for the magnetic field is reasonable.

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1. Introduction

The magnetic field produced in relativistic heavy ion collisions is extremely strong [1, 2]. A natural place to study the effects of this field is in ultraperipheral relativistic heavy ion collisions (UPC's) [3]. Since there is no superposition of hadronic matter, the strong interaction is suppressed and the collision becomes essentially a very clean electromagnetic process.

In [4] it was argued that forward pions are very likely to be produced by magnetic excitation (ME) of the nucleons in the nuclei. The strong classical magnetic field produced by one nucleus induces magnetic transitions, such as $N \rightarrow \Delta$ (where N is a proton or a neutron), in the nucleons of the other nucleus. The produced Δ keeps moving together with the nucleus (or very close to it) and then decays almost exclusively through the reaction $\Delta \rightarrow N + \pi$. The produced pion has a very large longitudinal momentum and very large rapidity. Since there is no other competing mechanism for forward pion production in UPC's the observation of these pions would be a signature of the magnetic excitation of the nucleons and also an indirect measurement of the magnetic field. In [4] it was shown that ME has a very large cross section. In [5] we proposed a way to test the classical approximation for the magnetic field. The process discussed in [4], $N \rightarrow \Delta \rightarrow N'\pi$, was recalculated. In the quantum formalism the transition was induced by photons and not by the classical magnetic field. We computed the same process using a different formalism where the quanta of the field play the important role. We then compared the results obtained with the two formalisms. In this contribution we review the content of [4] and [5] and expand the discussion.

2. Formalism

2.1 The semi-classical formalism

A strong magnetic field can convert a hadron into another one with a different spin, by "flipping the constituent quark spins". Let us consider an ultraperipheral $Pb - p$ collision, where the proton is at rest, as shown in Fig. 1a. Under the influence of the strong magnetic field generated by the moving nucleus, the nucleon is converted into a Δ. For the sake of definiteness let us consider the transition $|p \uparrow\rangle \rightarrow |\Delta^+ \uparrow\rangle$. The amplitude for this process is given by [4]:

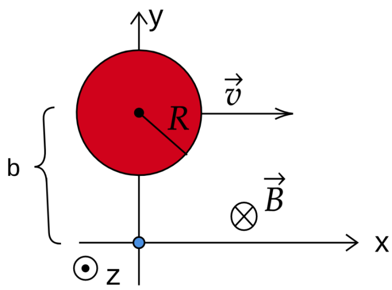
$$a_{fi} = -i \int_{-\infty}^{\infty} e^{iE_{fi}t'} \langle \Delta^+ \uparrow | H_{int}(t') | p \uparrow \rangle \, dt' \tag{1}$$

where $\hbar = 1$ and $E_{fi} = (m_\Delta^2 - m_n^2)/2m_n$, where m_Δ and m_n are the Δ and nucleon masses respectively. The interaction Hamiltonian is given by:

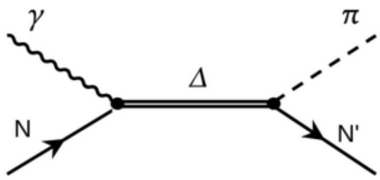
$$H_{int}(t) = -\vec{\mu}.\vec{B}(t) \qquad \text{with} \qquad \vec{\mu} = \sum_{i=u,d} \vec{\mu}_i = \sum_{i=u,d} \frac{q_i}{m_i} \vec{S}_i \tag{2}$$

The magnetic dipole moment of the nucleon is given by the sum of the magnetic dipole moments of the corresponding constituent quarks, q_i and m_i are the charge and constituent mass of the quark of type i and $\vec{S}_i$ is the spin operator acting on the spin state of this quark. In Fig. 1a we show the system of coordinates, the moving projectile nucleus and target proton at the origin of the coordinates. We assume that the projectile-generated field is the same produced by a point charge. The field is given by [4]:

$$B_z(t) = \frac{1}{4\pi} \frac{q v \gamma (b - y)}{((\gamma(x - vt))^2 + (y - b)^2 + z^2)^{3/2}} \tag{3}$$



(a)



(b)

Figure 1: a) Classical magnetic transition. b) Quantum version of the transition.

In the above expression γ is the Lorentz factor, b is the impact parameter along the y direction, $v \simeq 1$ is the projectile velocity and the projectile electric charge is $q = Ze$. The interaction Hamiltonian acts on spin states. The relevant ones are:

$$|p\uparrow\rangle = \frac{1}{3\sqrt{2}}[udu(\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow -2\uparrow\downarrow\uparrow) + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow -2\downarrow\uparrow\uparrow) + uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow -2\uparrow\uparrow\downarrow)] \tag{4}$$

$$|\Delta^+\uparrow\rangle = \frac{1}{3}(uud + udu + duu)(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \tag{5}$$

With these ingredients we can compute the matrix element $\langle\Delta^+\uparrow|H_{int}|p\uparrow\rangle$. It can be obtained by substituting Eq. (3) into Eq. (2) and then calculating the sandwiches of H_{int} with the spin states given above. The cross section for a single $N \to \Delta$ transition is given by:

$$\sigma = \int |a_{fi}|^2\, d^2b = 2\pi \int |a_{fi}|^2\, b\, db \tag{6}$$

Inserting the matrix elements into (1) and using it in the above expression we find:

$$\sigma = \frac{Z^2e^4}{9\pi m^2}\left(\frac{E_{fi}}{v\gamma}\right)^2 \int_R^\infty \left[K_1\left(\frac{E_{fi}b}{v\gamma}\right)\right]^2 b\, db \tag{7}$$

where K_1 is the modified Bessel function.

2.2 The quantum formalism

In the quantum formalism, the electromagnetic field produced by an ultrarelativistic electric charge is replaced by a flux of photons [3]. Now, in a high energy UPC, the projectile becomes a source of almost real photons and we replace the classical field by a collection of quanta. The cross section of the process can be written in a factorized form in terms of the photon flux produced by the projectile and the photon-nucleon cross section [3]:

$$\sigma = \int \frac{d\omega}{\omega}\, n(\omega)\, \sigma_{\gamma N\to N\pi}(\omega) \tag{8}$$

In the above expression $n(\omega)$ represents the photon spectrum generated by the source [3]:

$$n(\omega) = \frac{Z^2\alpha}{\pi}\left[2\xi K_0(\xi)K_1(\xi) - \xi^2[K_1^2(\xi) - K_0^2(\xi)]\right], \qquad \xi = \frac{\omega(R_1+R_2)}{\gamma} \tag{9}$$

where ω is the photon energy, R_1 and R_2 are the radii of the projectile and the target respectively. From the above expression it follows that the average energy carried by an emitted photon increases with γ and hence with the collision energy $\sqrt{s}$. In the LHC energy region $\gamma \simeq 1000$ and the photon energy can be as high as $\simeq 10$ GeV.

In order to perform the calculation of the total cross section, it is necessary to know the cross section of the process $\gamma N \rightarrow N\pi$. This is a very well known process, especially in the energy region around the threshold of Δ production. We are primarily interested in the high energy region, far from this threshold. We need a formula which correctly reproduces the behavior of the cross section in the Δ resonance region and which can be extrapolated to higher energies. A simple parametrization of the π^0 photoproduction cross section can be taken from Jones and Scadron [6]:

$$\sigma_{\gamma N \rightarrow N\pi}(\omega) = 2\pi \int_0^{\pi} d\theta \, \sin\theta \, \frac{\alpha\, \omega}{12\, m_n\, W} \, \frac{m_\Delta^2 \Gamma}{|W^2 - m_\Delta^2 + i m_\Delta \Gamma|^2} \left[\, |F_+^*|^2 \, f(\theta) \, + \, |G_+^*|^2 \, g(\theta) \right] \tag{10}$$

In the above expression $\alpha = 1/137$, $W^2 = m_n^2 + 2\,\omega\, m_n$ is the photon-nucleon center of mass energy squared, m_n is the nucleon mass, Γ is the Δ decay width and $F_+^* = G_M^* - 3G_E^*$, $G_+^* = G_M^* + G_E^*$. We took the form factors G_M^* and G_E^* from [6]. The angular dependence is given by $f(\theta) = (3\cos^2\theta + 1)/2$, $g(\theta) = (9\sin^2\theta)/2$. We have adjusted (10) to the most recent data, as explained in [5]. In order to estimate the uncertainty in the extrapolation of (10) to higher photon energies we have varied the Δ width within the interval $100 < \Gamma < 120$ MeV. We have found that the high energy tail of the curve is not very sensitive to changes in Γ. Having determined $\sigma_{\gamma N \rightarrow N\pi}$, we insert it into (8) and evaluate the cross section of the quantum process.

3. Results and discussion

The two cross sections calculated in the previous section are presented in Fig. 2. The cross sections are plotted as a function of the energy per nucleon (of the projectile) in the laboratory frame $E_{Lab} = \gamma\, m_n$. In the figure we compare the curves obtained with (7) (dashed line) and with (8) (solid lines). The band in the lower curve represents the different choices of the width Γ. The difference between these two curves approaches 10 % at the highest energies. These results suggest

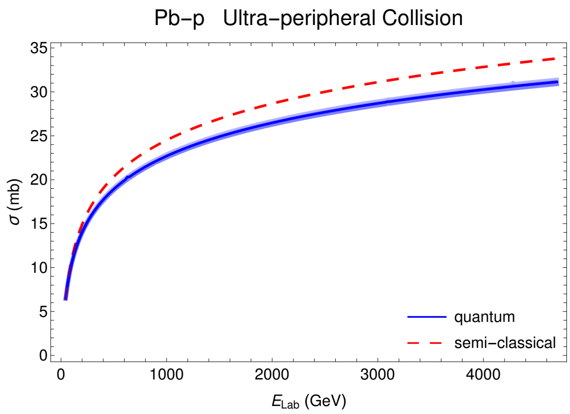


Figure 2: Comparison between classical and quantum cross sections for magnetic excitation.

that the classical approximation of the magnetic field reproduces most of the photon interaction in photoproduction in high energies.

The semi-classical amplitude (1) was based on first order perturbation theory. The calculation could be made more accurate by including second order corrections. The quantum formula (10) could also be improved. At low energies there are other resonances. To improve the accuracy of the extrapolation to higher photon energies, it would be necessary to change (10) including higher resonances or, alternatively, define some procedure to "average over the bumps", as it was done (although in a different context), for example, in [7].

To summarize: in heavy ion collisions the sources are so intense that one can treat classically the electromagnetic field. In particular, one can compute the magnetic field and use it to make a number of predictions. Although plausible, this conjecture was never tested before. In [5] we have devised a test for this idea. We have found a process which can be calculated in two different ways: one using the magnetic field and one relying solely on quantum physics. Our results give some support to the classical approximation for the magnetic field and hence give support to all the calculations done previously based on this approximation.

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