PROCEEDINGS OF SCIENCE

Asymptotic solution of the full next-to-leading order Balitsky-Kovchegov equation

Y. Bandeira,[a,*] **V. P. Gonçalves**[a] **and J. T. de Santana Amaral**[b]

[a] *Instituto de Física e Matemática, Universidade Federal de Pelotas (UFPel), Caixa Postal 354, CEP 96010-900, Pelotas, RS, Brazil*

[b] *Instituto de Matemática, Estatística e Física, Universidade Federal do Rio Grande (FURG), Caixa Postal 474,CEP 96201-900, Rio Grande, RS, Brazil*

E-mail: yan.bandeira@ufpel.edu.br, barros@ufpel.edu.br, jtamaral@furg.br

The increasing of the gluon distribution at small - x implies that the hadrons are characterized by a high partonic density and that non - linear (saturation) effects in the QCD dynamics cannot be disregarded at high energies. The evolution with the rapidity of the high density system is described by the Balitsky - Kovchegov (BK) equation. In this contribution, we consider the recent results for the next - to - leading order corrections to the BK equation and derive its asymptotic solution in the saturation regime assuming that the hard scale is the saturation scale. A detailed comparison with the solutions obtained assuming other prescriptions for the hard scale is presented.

XV International Workshop on Hadron Physics (XV Hadron Physics) 13 -17 September 2021
Online, hosted by Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil

*Speaker

PoS(XVHadronPhysics)075

1. Introduction

The high energy regime of Quantum Chromodynamics (QCD) has been under intense investigation in electron-hadron and hadron-hadron collisions. In this regime, saturation effects associated with the high gluonic density are expected to modify the QCD dynamics, implying the presence of non-linear effects that reduce the growth of the gluon distribution at small-x. The description of the saturation region is given by the Color Glass Condensate (CGC), which is an effective theory for the high-density regime. The CGC theory implies that the dipole-hadron scattering amplitude satisfies the Balitsky-Kovchegov (BK) equation in the mean-field approximation, which allows us to estimate the contribution of the non-linear effects.

In recent years, several authors have obtained the numerical solution of the BK equation at leading order in the full kinematical range, as well as its analytical solutions in the linear and saturation regimes [1–4]. On the other hand, the description of the next-to-leading order (NLO) corrections for the BK equation is still a theme of intense debate in the literature. Currently, there are different treatments for the NLO corrections and distinct prescriptions for the hard scale of the running coupling constant. One of them is the full next-to-leading order correction which consists of the implementation of the quark and gluon loops to the BK equation, which is called the full Next-to-Leading Order BK equation (fNLO - BK) [7]. In this work, we derive a solution to the NLO BK equation on the saturation regime assuming that the hard scale is given by the saturation scale Q_s^2, and present a comparison of our result with other analytics solutions present in the literature which were derived assuming different prescriptions to the hard scale.

One of the motivations for this study is associated to the fact that the asymptotic solutions of the BK equation, derived at leading order, has been considered in the construction of phenomenological models for the dipole - hadron scattering amplitude (See e.g. Ref. [13]), which are used to predict the high energy behavior of the observables at HERA, RHIC and LHC. In principle, a similar procedure can be performed at NLO if the solutions of the fNLO-BK equation are precisely determined. Such is the main goal of this contribution.

2. The full next-to-leading order Balitsky-Kovchegov equation

The full next-to-leading order Balitsky-Kovchegov equation for the S - matrix is given by [7]

$$\begin{aligned}\frac{\partial S(r,Y)}{\partial Y} = &\int \mathrm{d}^2 z K_1[S(r_1,Y)S(r_2,Y) - S(r,Y)] \\ &+ \int \mathrm{d}^2 z \mathrm{d}^2 r_2' K_2\left[S(r_1,Y)S(r_3,Y)S(r_2',Y) - S(r_1,Y)S(r_2,Y)\right] \\ &+ \int \mathrm{d}^2 z \mathrm{d}^2 r_2' K_3\left[S(r_1',Y)S(r_2,Y) - S(r_1,Y)S(r_2,Y)\right],\end{aligned} \tag{1}$$

where K_1, K_2, and K_3 are the equation kernels which are defined by:

$$\begin{aligned}K_1 =& \frac{\overline{\alpha}_s(r^2)}{2\pi}\frac{r^2}{r_1^2 r_2^2}\left[1 + \overline{\alpha}_s(r^2)\left(b\ln r^2\mu^2 - b\frac{r_1^2 - r_2^2}{r^2}\ln\frac{r_1^2}{r_2^2}\right.\right. \\ &\left.\left.+\frac{67}{36} - \frac{\pi^2}{12} - \frac{5N_f}{18N_c} - \frac{1}{2}\ln\frac{r_1^2}{r^2}\ln\frac{r_2^2}{r^2}\right)\right],\end{aligned} \tag{2}$$

$$K_2 = \frac{\overline{\alpha}_s^2(r^2)}{8\pi^2}\left[-\frac{2}{r_3^4} + \left(\frac{r_1^2 r_2'^2 + r_1'^2 r_2^2 - 4r^2 r_3^2}{r_3^4\left(r_1^2 r_2'^2 - r_1'^2 r_2^2\right)} + \frac{r^4}{r_1^2 r_2'^2\left(r_1^2 r_2'^2 - r_1'^2 r_2^2\right)} + \frac{r^2}{r_1^2 r_2'^2 r_3^2}\right)\ln\frac{r_1^2 r_2'^2}{r_1'^2 r_2^2}\right], \tag{3}$$

$$K_3 = \frac{\overline{\alpha}_s^2(r^2)N_f}{8\pi^2 N_c}\left(\frac{2}{r_3^4} - \frac{r_1'^2 r_2^2 + r_2'^2 r_1^2 - r^2 r_3^2}{r_3^4\left(r_1^2 r_2'^2 - r_1'^2 r_2^2\right)}\ln\frac{r_1^2 r_2'^2}{r_1'^2 r_2^2}\right), \tag{4}$$

where $r = x_\perp - y_\perp$, $r_1 = x_\perp - z_\perp$, $r_2 = y_\perp - z_\perp$, $r_1' = x_\perp - z_\perp'$, $r_2' = y_\perp - z_\perp'$ and $r_3 = z_\perp - z_\perp'$ are the dipole transverse sizes. In eq. (2) b is the first coefficient of the β function, and μ is the renormalization scale (See Ref. [7] for details).

In the saturation region the scattering amplitude approaches the unitarity, $N \sim 1$, then due to the relation between the scattering amplitude and the S-matrix, the S-matrix tends to zero. Consequently, all non - linear S-matrix terms of the full next-to-leading order BK equation, eq. (1), can be neglected. Therefore, only the term associated to kernel K_1 remains. So the fNLO - BK equation reads

$$\partial_Y S(r, Y) = -2\int d^2 z K_1 S(r, Y), \tag{5}$$

where the factor 2 on the r.h.s. of the eq. (5) comes from considering the symmetry of the two integral regions [8, 9]. Meanwhile, the kernel K_1 depends on the prescription to the running coupling hard scale, like the *parent dipole running coupling* (PDRC) prescription [9, 11, 12] in which the coupling scale is determined by the size of the dipole generating the cascade (parent dipole). However, there are other prescriptions for the scale of the running coupling. One alternative is the *smallest dipole running coupling* (SDRC) prescription in which the coupling scale is determined by the size of the smallest dipole [7, 9, 10]. Despite these two prescriptions, the most intuitive is to assume that the scale that determines the value of the coupling constant is the saturation scale Q_s, i.e., $\alpha_s(Q_s^2)$, which we can call as *saturation scale running coupling* (SSRC) prescription. In what follows we will derive, for the first time, the asymptotic solution of the fNLO-BK equation to this new prescription.

Assuming the saturation scale as being the hard scale, one has that the kernel K_1 will be given by

$$K_1 = \frac{\overline{\alpha_s}(Q_s^2)}{2\pi r_1^2} + \frac{\overline{\alpha_s}^2(Q_s^2)}{2\pi r_1^2} b\ln r^2\mu^2 - \frac{\overline{\alpha_s}^2(Q_s^2)}{2\pi r^2} b\ln\frac{r_1^2}{r^2} + \frac{\overline{\alpha_s}^2(Q_s^2)}{2\pi r_1^2} b\ln\frac{r_1^2}{r^2} + \left(\frac{67}{36} - \frac{\pi^2}{12} - \frac{5N_f}{18N_c}\right)\frac{\overline{\alpha_s}^2(Q_s^2)}{2\pi r_1^2}. \tag{6}$$

Substituting this kernel in Eq. (5), assuming that $\mu^2 = Q_s^2$ and using the one loop expression for the running coupling, given by

$$\alpha_s(r_1^2) = \frac{1}{b\ln\left(\frac{1}{r_1^2\Lambda^2}\right)}, \tag{7}$$

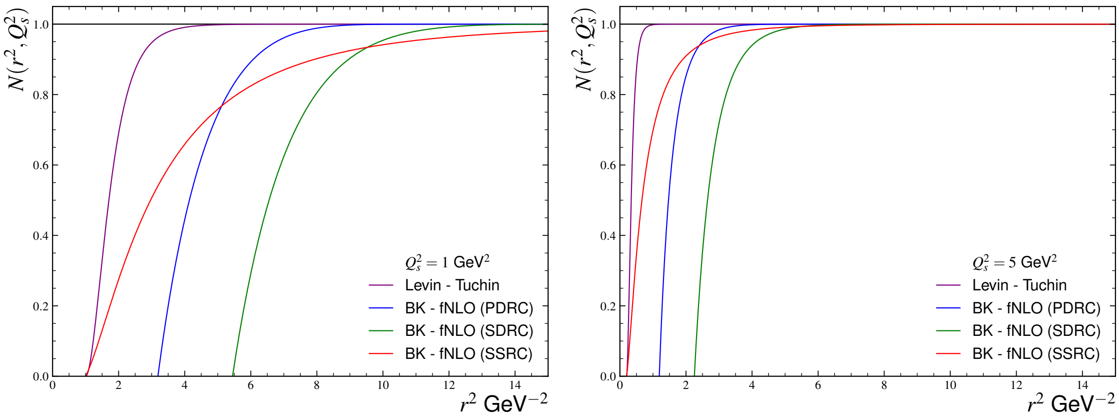

Figure 1: Comparison between the BK-LO asymptotic solution and the fNLO-BK equation asymptotic solutions for the saturation scale fixed in 1 GeV^2 (left) and in 5 GeV^2 (right).

with $b = {}^{1}/_{12\pi}(11N_c - 2N_f)$, results that

$$\partial_Y S(r,Y) = \Bigg\{ - \Bigg[\overline{\alpha_s}(Q_s^2) + \frac{1}{2}\overline{\alpha_s}^2(Q_s^2) b \ln r^2 Q_s^2 \; + \left(\frac{67}{36} - \frac{\pi^2}{12} - \frac{5N_f}{18N_c} \right) \overline{\alpha_s}^2(Q_s^2) \Bigg] \times \ln\left(r^2 Q_s^2\right) + \overline{\alpha_s}^2(Q_s^2) b \left[\frac{1}{r^2 Q_s^2} \ln\left(r^2 Q_s^2\right) - 1 + \frac{1}{r^2 Q_s^2} \right] \Bigg\} S(r,Y). \tag{8}$$

In the asymptotic saturation regime one has that $r^2 Q_s^2 \gg 1$, which implies that the above equation can be simplified to

$$\partial_Y S(r,Y) = \Bigg\{ - \Bigg[\overline{\alpha_s}(Q_s^2) + \frac{1}{2}\overline{\alpha_s}^2(Q_s^2) b \ln r^2 Q_s^2 + \left(\frac{67}{36} - \frac{\pi^2}{12} - \frac{5N_f}{18N_c} \right) \overline{\alpha_s}^2(Q_s^2) \Bigg] \ln\left(r^2 Q_s^2\right) - \overline{\alpha_s}^2(Q_s^2) b \Bigg\} S(r,Y), \tag{9}$$

whose solution is

$$S(r,Y) = S_0 \exp\Bigg\{ -\frac{N_c}{cb\pi} \Bigg\{ \ln^2(r^2 Q_s^2) + \frac{N_c b}{3\pi} \alpha_s(Q_s^2) \ln^3(r^2 Q_s^2) + \frac{N_c}{\pi} \left(\frac{67}{36} - \frac{\pi^2}{12} - \frac{5N_f}{18N_c} \right) \alpha_s(Q_s) \ln^2(r^2 Q_s^2) + \frac{2N_c}{\pi} \alpha_s(Q_s^2) b \ln\left(r^2 Q_s^2\right) \Bigg\} \Bigg\} \tag{10}$$

where $\ln\left(r^2 Q_s^2\right) = \tau$ is the scaling variable. Moreover, the saturation momentum at NLO is given by

$$\ln\left(\frac{Q_s^2}{\Lambda^2}\right) = \sqrt{c(Y - Y_0)} + O(Y^{1/6}). \tag{11}$$

3. Results

In Fig. 1 we present our results (red lines) for the scattering amplitude as a function of the dipole size for two distinct values of saturation scale. For comparison, we also present the asymptotic solutions derived using the parent dipole running coupling (PDRC) and smallest dipole running coupling (SDRC) prescriptions for the hard scale, whose analytical expressions are presented in Refs. [7, 9]. Moreover, in order to analyze the impact of the NLO corrections, we also show the asymptotic solution obtained at leading order, usually denoted by Levin-Tuchin law [3]. One has that the unitarity limit $N = 1$ is approached for larger dipole sizes when the NLO corrections are taken into account, which is directly associated with the fact that these corrections imply a slower evolution of the saturation scale. Moreover, one has that the dependence on r^2 is sensitive to the prescription for the hard scale. This result highlights the sensitivity of the evolution equation on the coupling hard scale prescription. Moreover, as the cross sections in *ep* and *pp* collisions can be expressed in terms of the dipole - hadron scattering amplitude, such dependence on the prescription is also expected to be present in the observables. Such an aspect will be explored in a forthcoming study.

Acknowledgments

This work was partially financed by the Brazilian funding agencies CNPq, CAPES, FAPERGS and INCT-FNA (process number 464898/2014-5).

References

[1] A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. B **640** (2002), 331-350

[2] A. J. Askew, J. Kwiecinski, A. D. Martin and P. J. Sutton, Phys. Rev. D **49** (1994), 4402-4414

[3] E. Levin and K. Tuchin, Nucl. Phys. B **573** (2000), 833-852

[4] E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A **708** (2002), 327-352

[5] I. Balitsky, Phys. Rev. D **75** (2007), 014001

[6] Y. V. Kovchegov and H. Weigert, Nucl. Phys. A **784** (2007), 188-226

[7] I. Balitsky and G. A. Chirilli, Phys. Rev. D **77** (2008), 014019

[8] W. Xiang, Phys. Rev. D **79** (2009), 014012

[9] W. Xiang, Y. Cai, M. Wang and D. Zhou, Phys. Rev. D **101** (2020) no.7, 076005

[10] D. Bendova, J. Cepila, J. G. Contreras and M. Matas, Phys. Rev. D **100** (2019) no.5, 054015

[11] J. L. Albacete and Y. V. Kovchegov, Phys. Rev. D **75** (2007), 125021

[12] J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, Phys. Rev. D **80** (2009), 034031

[13] E. Iancu, K. Itakura and S. Munier, Phys. Lett. B **590** (2004), 199-208

[14] A. H. Rezaeian and I. Schmidt, Phys. Rev. D **88** (2013), 074016