

## GM2Calc - 2 for the 2HDM

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GM2Calc is a leading tool for calculating precise contributions to  $a_\mu$  in the Minimal Supersymmetric Standard Model. In this proceeding we detail GM2Calc version 2 where it is extended so it can calculate two-loop contributions to  $a_\mu$  in the Two-Higgs Doublet Model (2HDM), based on the work in Ref. [1]. The 2HDM is a simple model, yet it is one of the few single field extensions of the Standard Model which is able to explain the muon  $g - 2$  anomaly. We demonstrate the powerful and flexible 2HDM capabilities of GM2Calc2, which include the most precise contributions in the literature and allow the user to work in their favourite type of the 2HDM as well as use complex and lepton flavour violating couplings. With its multiple interfaces and input flexibility, GM2Calc2 is a powerful tool both as a standalone code and as part of a larger code toolchain.

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## 1. Introduction

Currently there exists a  $4.2\sigma$  discrepancy between the Standard Model (SM) and experimental values of  $a_\mu$ . This discrepancy was confirmed by the Fermilab experiment [2], who measured  $a_\mu$  to be

$$a_\mu^{\text{Exp}} = (11659206.1 \pm 4.1) \times 10^{-10}, \quad (1)$$

after combining their results with those from Brookhaven National Laboratory [3]. This is higher than the SM prediction from the Muon  $g - 2$  Theory Initiative White Paper [4] (which itself builds on results done in Refs. [5–28])

$$a_\mu^{\text{SM}} = (11659181.0 \pm 4.3) \times 10^{-10}, \quad (2)$$

by the amount

$$a_\mu^{\text{BSM}} = (25.1 \pm 5.9) \times 10^{-10}. \quad (3)$$

The size of this discrepancy, which is on order of the weak contributions to  $a_\mu$  [29], suggests new physics at work in the value of  $a_\mu^{\text{BSM}}$ . Many models have been proposed as an explanation of the  $a_\mu$  discrepancy (for a review see [30]), including the Two-Higgs Doublet Model (2HDM).

The 2HDM involves the simple addition of a 2nd Higgs doublet to the SM with identical properties to the first. For a simple extension, it has been shown to be adept at explaining many different physics anomalies. Type-II 2HDM is motivated by the Minimal Supersymmetric Standard Model (MSSM) (see e.g. Ref. [31]). The general or type-III 2HDM has been shown to be able to explain the flavour changing charged current for  $R(D)$ , whilst respecting constraints from the lepton flavour violating (LFV) decays  $\tau \rightarrow \mu\gamma$  and  $\tau \rightarrow 3\mu$  [32]. The 2HDM may also provide an explanation for things such as LHC data [33–37], B-anomalies [38–42], and decays of Higgs bosons [43–51], to name a few.

There have several demonstrations of the ability of the 2HDM to explain the value of  $a_\mu^{\text{BSM}}$ , and a detailed list has been included in the original work in Ref. [1]. Both type-X and flavour-aligned 2HDM are shown to be able to explain the value of  $a_\mu^{\text{BSM}}$  in the review in Ref. [30]. Ref. [52] studied the type-X 2HDM and found that while it was possible to produce an explanation of  $a_\mu^{\text{BSM}}$  that was consistent with the electron anomalous magnetic moment and collider searches, it was not consistent with  $\tau$  and  $Z$  decays. Ref. [53] also investigated the simultaneous explanation of the muon and electron magnetic moment anomalies in the type-X 2HDM and found it was possible with a light CP-even scalar. Ref. [54] also found that the type-X 2HDM can explain the  $a_\mu$  anomaly whilst allowing for highscale completion in scenarios with a low-mass CP-odd scalar. Refs. [55, 56] use LFV one-loop contributions in the General 2HDM to explain the value of  $a_\mu^{\text{BSM}}$ , while Ref. [32] showed the inconsistency of a simultaneous explanation of the  $a_\mu$  anomaly with Flavour-Changing Neutral Currents in the General 2HDM.

Previously, GM2Calc was a tool solely for calculating the contributions to  $a_\mu^{\text{SUSY}}$  in the MSSM. It has been used to analyze the MSSM in a variety of different scenarios (see e.g. [57–70]). It was also included in GAMBIT [71] as a backend, where it was used in several global fits of the MSSM shown in Refs. [30, 72–74]. We recently extended it to enable it to calculate contributions in the 2HDM [1]. We incorporated state-of-the-art calculations in GM2Calc, allowing it to calculate contributions and return the uncertainty at up to the two-loop level. In this proceeding we provide a quick overview of the GM2Calc2's new 2HDM capabilities.

## 2. Usage

GM2Calc version 2 features  $a_\mu$  contributions at the one-loop level taken from [32] and two-loop contributions taken from [75]. See [1] for in-depth details on the contributions included. Here we give an essential overview.

When extending GM2Calc to include the 2HDM we aimed to make the internal calculations as general as possible, so that it is possible to use the software with any of the popular types of the 2HDM. The general potential of the two Higgs doublets  $\Phi_{1,2}$  in the lambda basis is given by

$$\begin{aligned}
 -\mathcal{L}_{\text{Scalar}} = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h. c.} \right] \\
 & + \frac{1}{2} \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) \\
 & + \left[ \frac{1}{2} \lambda_5 \left( \Phi_1^\dagger \Phi_2 \right)^2 + \lambda_6 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_1^\dagger \Phi_2 \right) + \lambda_7 \left( \Phi_2^\dagger \Phi_2 \right) \left( \Phi_1^\dagger \Phi_2 \right) + \text{h. c.} \right].
 \end{aligned} \quad (4)$$

In addition to  $\tan(\beta)$ , we can use the lambdas  $\lambda_{1,\dots,5}$  and  $m_{12}^2$  as input parameters for our GM2Calc calculation, or we can transform the components of the two Higgs doublets into their mass eigenstates, and use the masses  $m_{h,H,A,H^\pm}$ ,  $m_{12}^2$ , and the alignment parameter  $\sin(\beta - \alpha)$  instead. The quartic couplings  $\lambda_{6,7}$  are not often non-zero in the literature, however GM2Calc allows them to be for generality.

In general the two Higgs doublets may have any of the below couplings to the SM fermions

$$-\mathcal{L}_{\text{Yuk}} = \Gamma_d^0 \overline{q_L^0} \Phi_1 d_R^0 + \Gamma_u^0 \overline{q_L^0} \Phi_1 u_R^0 + \Gamma_l^0 \overline{l_L^0} \Phi_1 e_R^0 + \Pi_d^0 \overline{q_L^0} \Phi_2 d_R^0 + \Pi_u^0 \overline{q_L^0} \Phi_2 u_R^0 + \Pi_l^0 \overline{l_L^0} \Phi_2 e_R^0 + \text{h. c.}, \quad (5)$$

where  $\Phi_{1,2}^c = i\sigma_2 \Phi_{1,2}^*$ . The above  $3 \times 3$  Yukawa matrices are allowed to take any complex value in the general 2HDM. We are able obtain the other types by setting the Yukawa matrices to the values shown in Table 1. If the user should choose one of the types-I, II, X or Y, then the Yukawa matrices are set according to these matrices (and the remaining ones according to the SM masses), so then the user does not need to specify any Yukawa couplings. In the flavour-aligned 2HDM (FA2HDM) the ratio of the Yukawa matrices  $\Pi_f^0$  and  $\Gamma_f^0$  are fixed to be  $\xi_f^{(*)}$  (if  $f=u$ ). The user may input the strength of the Yukawa couplings in this case through the variable  $\zeta_f$ , which is related to the alignment parameter by

$$\xi_f = \frac{\zeta_f + \tan \beta}{1 - \zeta_f \tan \beta} \quad (6)$$

In the traditional FA2HDM there is no flavour violation (outside of the CKM matrix  $V_{\text{CKM}} = V_u V_d^\dagger$ , where  $M_f^D = V_f M_f U_f^\dagger$  diagonalize the mass matrices), however, in GM2Calc we allow the user to input a matrix of deviations from the diagonal  $\Delta_f$ . Finally for the general 2HDM we allow the user to directly input the matrices  $\Pi_f = V_f \Pi_f^0 U_f^\dagger$ . This completes the list of parameters we need to calculate the contributions to  $a_\mu^{\text{BSM}}$  in GM2Calc.

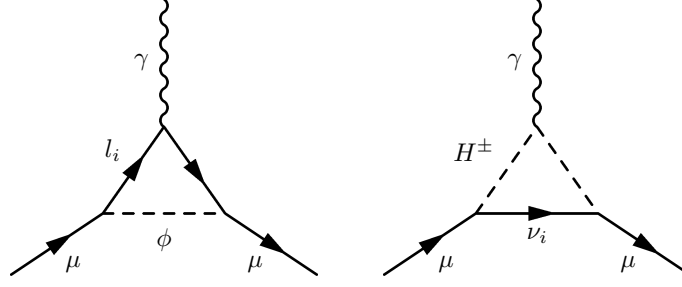
We divide the contributions into two parts, one-loop and two-loop

$$a_\mu^{\text{BSM}} = a_\mu^{1\ell} + a_\mu^{2\ell}. \quad (7)$$

The one-loop contributions  $a_\mu^{1\ell}$  included in GM2Calc are the full BSM contributions and are shown in Fig. 1, and are defined in Eq. (40) of [1]. They include diagrams with the SM-like Higgs, since

**Table 1:** Values of the Yukawa matrices to recover the types-I, II, X, Y, and FA2HDM.

Type I	$\Gamma_u^0 = \Gamma_d^0 = \Gamma_l^0 = 0$
Type II	$\Gamma_u^0 = \Pi_d^0 = \Pi_l^0 = 0$
Type X	$\Gamma_u^0 = \Gamma_d^0 = \Pi_l^0 = 0$
Type Y	$\Gamma_u^0 = \Pi_d^0 = \Gamma_l^0 = 0$
Flavour-Aligned	$\Pi_u^0 = \xi_u^* \Gamma_u^0, \quad \Pi_{d,l}^0 = \xi_{d,l} \Gamma_{d,l}^0$

**Figure 1:** One-loop contribution included within GM2Calc, where  $\phi = h, H, A$  and  $i = 1, 2, 3$ .

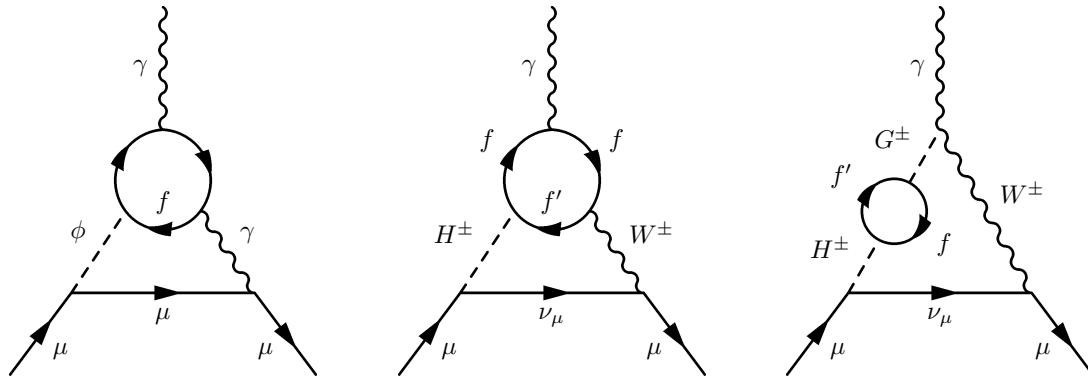
in the 2HDM it can have non-SM effects such as lepton flavour violation (which can occur if we include off-diagonal couplings in the matrix  $\Delta_l$  in the FA2HDM or  $\Pi_l$  in the general 2HDM) and Higgs alignment. To make sure that we are only including BSM contributions we subtract off the SM-like Higgs contributions that would be included in the SM prediction.

The 2HDM's state-of-the-art two-loop contributions in GM2Calc are the taken from [75]. The two-loop contributions are divided into fermionic and bosonic parts

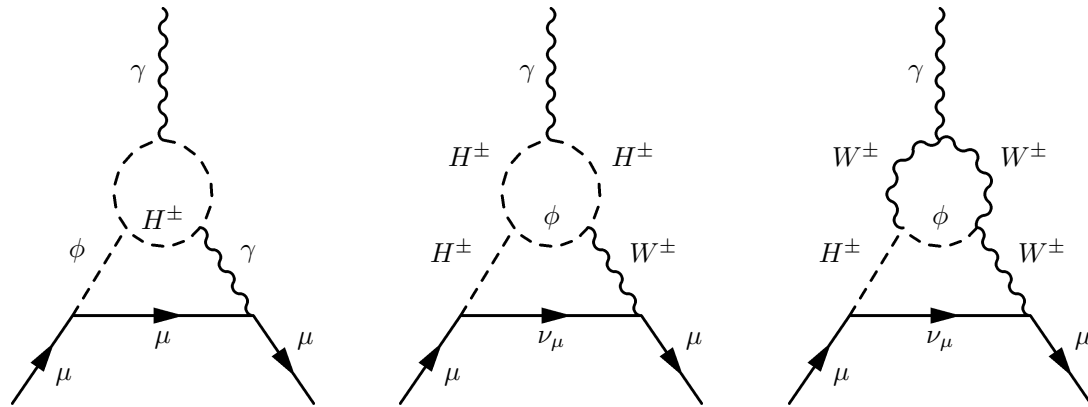
$$a_\mu^{2\ell} = a_\mu^F + a_\mu^B. \quad (8)$$

The fermionic contributions are shown in Fig. 2, and defined in Eq. (50) of [1]. In GM2Calc2 we allow for CKM mixing, so in the middle and right Barr-Zee diagrams in the case of  $f = u, d$  the fermion loops may have two  $SU(2)_L$  partners from different generations. Again, just as for the one-loop contributions we must remember to subtract off contributions from the SM-like Higgs to avoid double counting. The bosonic contributions shown in Figs. 3 and 4 are defined in Eq. (66) of [1], and can be divided into Barr-Zee and three boson diagrams respectively. These contributions have been extended compared to their original mention in [75] to allow for a non-zero  $\lambda_{6,7}$  in the general scalar potential.

GM2Calc is available for download from a git [<https://github.com/GM2Calc/GM2Calc>] and a hepforge [<https://gm2calc.hepforge.org>] repository. Its requirements have been kept to a minimum, only requiring a C++14 or a C11 compiler as well as the Boost [76] (version 1.37.0 from [<http://eigen.tuxfamily.org>]) and Eigen (version 3.1 from [<http://www.boost.org>]) libraries. To build GM2Calc enter the below commands in the directory where GM2Calc was downloaded



**Figure 2:** Two-loop fermionic contribution included within GM2Calc, where  $\phi = h, H, A$ ,  $f = u, d, l$ , and  $f'$  is the  $SU(2)_L$  partner of  $f$ . We can replace the photon in the left diagram with a Z boson, although these contributions are suppressed.



**Figure 3:** Two-loop bosonic Barr-Zee contribution included within GM2Calc, where  $\phi = h, H, A$ . We can replace the photon in the left diagram with a Z boson, although these contributions are suppressed.

```

1   mkdir build; cd build
2   cmake ..
3   make

```

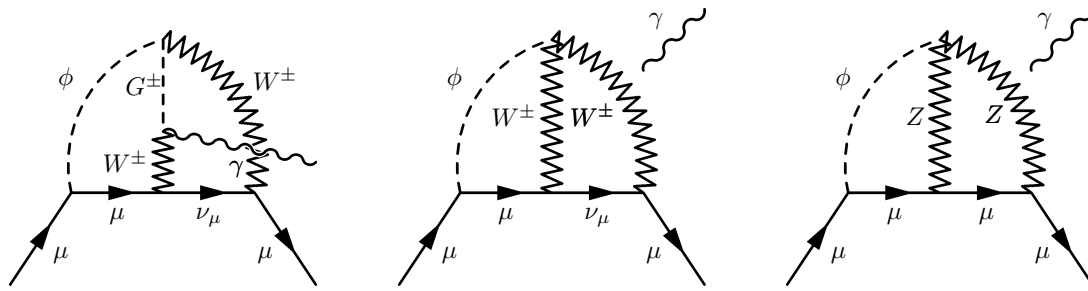
Running GM2Calc can be done straight from the command-line. Inputs are given through an SLHA-like input file. Using the provided example file, one can run the command

```

1   bin/gm2calc.x --thdm-input-file=../input/example.thdm

```

or equivalently replace `example.thdm` with the users own input file. Alternatively one can use one of GM2Calc's interfaces. The C, C++, and Mathematica [77] interfaces which worked for the MSSM



**Figure 4:** Two-loop bosonic three boson contribution included within GM2Calc, where  $\phi = h, H, A$ . In the middle and right diagrams the photon can couple to either of the vector bosons. Also in the right diagram the neutral Higgs can swap its position with one of the Z bosons.

work just as well for the 2HDM. New in GM2Calc version 2 are the Python 2 or Python 3 [78] interfaces, which requires the package cppy [79] from [https://pypi.org/project/cppy/], and attaching the flag `-DBUILD_SHARED_LIBS=ON` to the cmake build step.

### 3. Examples

In this section we examine some examples of using GM2Calc2 to calculation the contributions to  $a_\mu^{\text{BSM}}$  in the 2HDM. All of these examples are present in Ref. [1], however we use different interfaces compare to those in the original work. Each of the examples also uses a scenario taken from the literature, allowing comparison of GM2Calc2's capabilities with existing tools and techniques.

The first example follows from Ref. [80] and is a scan over  $\tan\beta$  and  $m_A$  in the types-II and X 2HDM. Below we show the code of the Python interface used for the example

```

1  #!/usr/bin/env python
2
3  from __future__ import print_function
4
5  from gm2_python_interface import *
6
7  cppy.include(os.path.join("gm2calc", "gm2_1loop.hpp"))
8  cppy.include(os.path.join("gm2calc", "gm2_2loop.hpp"))
9  cppy.include(os.path.join("gm2calc", "gm2_uncertainty.hpp"))
10 cppy.include(os.path.join("gm2calc", "gm2_error.hpp"))
11 cppy.include(os.path.join("gm2calc", "THDM.hpp"))
12
13 cppy.load_library("libgm2calc")
14
15 # Load data types
16 from cppy.gbl import std
17 from cppy.gbl import Eigen
18 from cppy.gbl import gm2calc
19 from cppy.gbl.gm2calc import SM
20 from cppy.gbl.gm2calc import THDM

```

```

21 from cppy.gbl.gm2calc import Error
22
23 # Based on arxiv:1409.3199, examining the mA-tan(beta) plane
24 def calc_amu(mA, tb, yukawa_type):
25     sm = gm2calc.SM()
26     sm.set_alpha_em_mz(1.0/128.94579)
27     sm.set_mu(2,173.34)
28     sm.set_mu(1,1.28)
29     sm.set_md(2,4.18)
30     sm.set_ml(2,1.77684)
31
32     vev = sm.get_v()
33     lambda_max = 3.5449077018110321 # Sqrt[4 Pi[]]
34     mh = 126.
35     mH = 200.
36
37     basis = gm2calc.thdm.Mass_basis()
38     basis.yukawa_type = gm2calc.thdm.Yukawa_type.type_2
39     basis.mh = mh
40     basis.mH = mH
41     basis.mA = mA
42     basis.mHp = mH
43     basis.sin_beta_minus_alpha = 1.
44     basis.lambda_6 = 0.
45     basis.lambda_7 = 0.
46     basis.tan_beta = tb
47     basis.m122 = mH*mH/tb + (mh*mh - \
lambda_max*sm.get_v()*sm.get_v())/(tb*tb*tb);
48     basis.zeta_u = 0.
49     basis.zeta_d = 0.
50     basis.zeta_l = 0.
51     basis.Delta_u = Eigen.Matrix3d().setZero()
52     basis.Delta_d = Eigen.Matrix3d().setZero()
53     basis.Delta_l = Eigen.Matrix3d().setZero()
54     basis.Pi_u = Eigen.Matrix3d().setZero()
55     basis.Pi_d = Eigen.Matrix3d().setZero()
56     basis.Pi_l = Eigen.Matrix3d().setZero()
57
58     config = gm2calc.thdm.Config()
59     config.force_output = False
60     config.running_couplings = True
61
62     amu = None
63
64     try:
65         model = gm2calc.THDM(basis, sm, config)
66         amu = gm2calc.calculate_amu_1loop(model) + \
gm2calc.calculate_amu_2loop(model)
67     except gm2calc.Error as e:
68         pass
69
70     return amu

```

```

71 |
72 |
73 | mA_start = 1
74 | tb_start = 1
75 | mA_stop = 100
76 | tb_stop = 100
77 | N_steps = 198
78 |
79 | print("# mA/GeV           ", "# tan(beta)           ", "# amu(II)           \
    |     ", "# amu(X)           ")
80 |
81 | for i in range(0, N_steps+1):
82 |     for j in range(0, N_steps+1):
83 |         tb = tb_start + i*(tb_stop - tb_start) / N_steps
84 |         mA = mA_start + j*(mA_stop - mA_start) / N_steps
85 |         type_2 = calc_amu(mA, tb, gm2calc.thdm.Yukawa_type.type_2)
86 |         type_X = calc_amu(mA, tb, gm2calc.thdm.Yukawa_type.type_X)
87 |
88 |         print("{0:>20.8e} {1:>20.8e} {2:>20.8e} \
    | {3:>20.8e}".format(tb, mA, type_2, type_X))

```

The first 5 lines setup the above Python script so that it may be used by both Python2 and Python3, and imports the interface module `gm2_python_interface`, which is a convenient script built with GM2Calc which loads the necessary libraries for GM2Calc to work in Python. Lines 7-11 include the header files needed to run the 2HDM in Python, while line 13 loads the source code of GM2Calc. Next, lines 16-21 load the needed namespaces and classes. After that is the function `calc_amu` which calculates and returns the value of  $a_{\mu}^{\text{BSM}}$ , given the inputs  $m_A$ ,  $\tan(\beta)$  and the 2HDM type. Lines 25-30 fix the masses of the SM fermions and the electroweak constant. Next, lines 32-35 set the needed values to correctly calculate  $m_{12}^2$ . Lines 37-56 set up the basis object, and in this particular example we use the mass basis. The parameters are set to  $m_H = m_{H^\pm} = 200$  GeV,  $\beta - \alpha = \pi/2$  and are taken from Fig. 3 in Ref. [80], and on line 47 the value of  $m_{12}^2$  is set according to Eq. (14) of Ref. [80]. Since in this example we will be using the types-II and X 2HDM we do not need to input values for  $\zeta_f$ ,  $\Delta_f$ , and  $\Pi_f$ , as they are ignored, however we show how to input to them for completeness. Next lines 58-60 set the configuration options, where we choose to use running couplings for the fermion masses. Finally lines 64-68 calculate the contributions to  $a_{\mu}^{\text{BSM}}$  given the above inputs. Next in lines 73-77 the parameters of the scan are set. Then in lines 81-86 the script loops over the values of  $m_A$  and  $\tan\beta$ , calculating the contributions to  $a_{\mu}^{\text{BSM}}$  using the function `calc_amu`, and finally in line 88 it prints the contribution to  $a_{\mu}^{\text{BSM}}$  in the types-II and X 2HDM. The results of the scan in the above code are shown in Fig. 5.

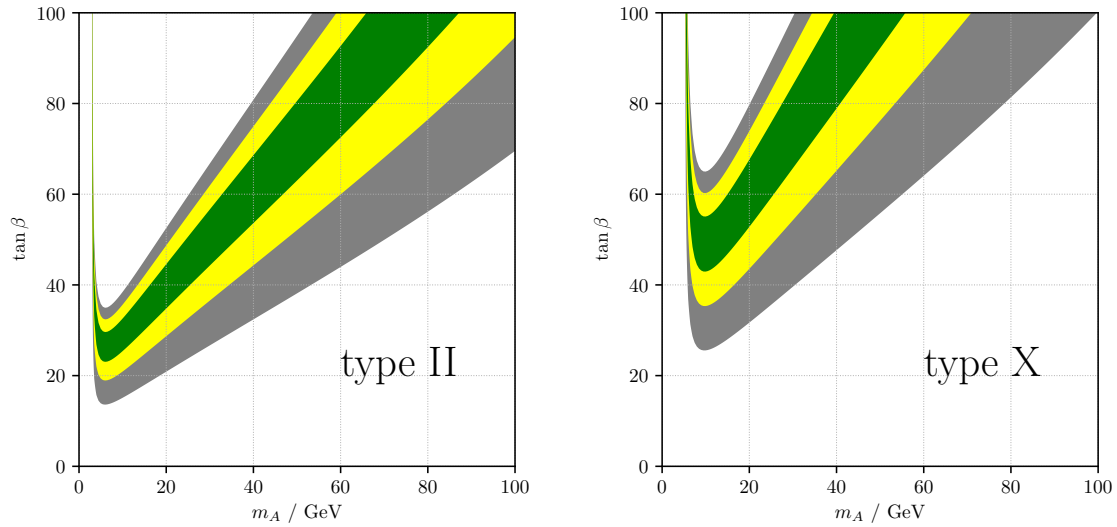
In the next example we compare the size of the bosonic and fermionic two-loop contributions in the type-II and FA2HDM. This example can be run in Mathematica using Mathlink, and the below source code shows the type-II part of the example

```

1 | Install["bin/gm2calc.mx"];
2 |
3 | CalcAmu2[mA_] :=
4 | {amu2LF, amu2LB} /. GM2CalcAmuTHDMMassBasis[

```





**Figure 5:** Prediction of the two-loop contributions to  $a_\mu^{\text{BSM}}$  from the types-II and X 2HDM shown in the left plots and right plot respectively. The contributions are a function of  $\tan(\beta)$  and  $m_A$  with  $m_h = 126$  GeV,  $m_H = m_{H^\pm} = 200$  GeV,  $\sin(\beta - \alpha) = 1$ ,  $\lambda_6 = \lambda_7 = 0$ ,  $m_{12}^2 = m_H^2/\tan(\beta) + (m_h^2 - \lambda_1 v^2)/\tan(\beta)^3$  (see Eq. (14) in [80]) and  $\lambda_1 = \sqrt{4\pi}$ . The green, yellow, and grey regions show point which can explain the  $a_\mu$  anomaly in Eq. (3) within 1, 2, and  $3\sigma$  respectively.

```

5 yukawaType      -> 2,
6 Mhh             -> { 125, 400 },
7 MAh            -> mA,
8 MHp            -> 440,
9 sinBetaMinusAlpha -> 0.999,
10 lambda6        -> 0,
11 lambda7        -> 0,
12 TB             -> 3,
13 m122           -> 200^2
14 ];
15
16 (* mA values in [130,500] GeV *)
17 mAValues = Subdivide[130, 500, 200];
18
19 (* calculation w/ running couplings *)
20 GM2CalcSetFlags[runningCouplings -> True];
21 type2values = CalcAmu2 /@ mAValues;
22
23 data2 = { mAValues, type2values[[All,1]], type2values[[All,2]] };
24
25 Export["type2.txt", N @ Transpose @ data2, "Table"];

```

In the above code line 1 loads the MathLink executable which is built by GM2Calc in its build step. Lines 3-14 constructs the function CalcAmu2 which takes  $m_A$  as input, and returns the individual one-loop, two-loop fermionic and two-loop bosonic contributions. The parameter choices of  $m_H = 400$  GeV,  $m_{H^\pm} = 440$  GeV,  $\tan(\beta) = 3$ ,  $\sin(\beta - \alpha) = 0.999$ , and  $m_{12}^2 = 40000$  GeV<sup>2</sup> are

taken from Ref. [81]. The range of  $m_A$  is specified on line 17. We choose to use running masses for the SM fermions, and then calculate the contributions to  $a_\mu^{\text{BSM}}$  on lines 20-21. Finally in lines 23-25 the data is assembled and outputted. The results of this scan are shown in the left plot of Fig. 6.

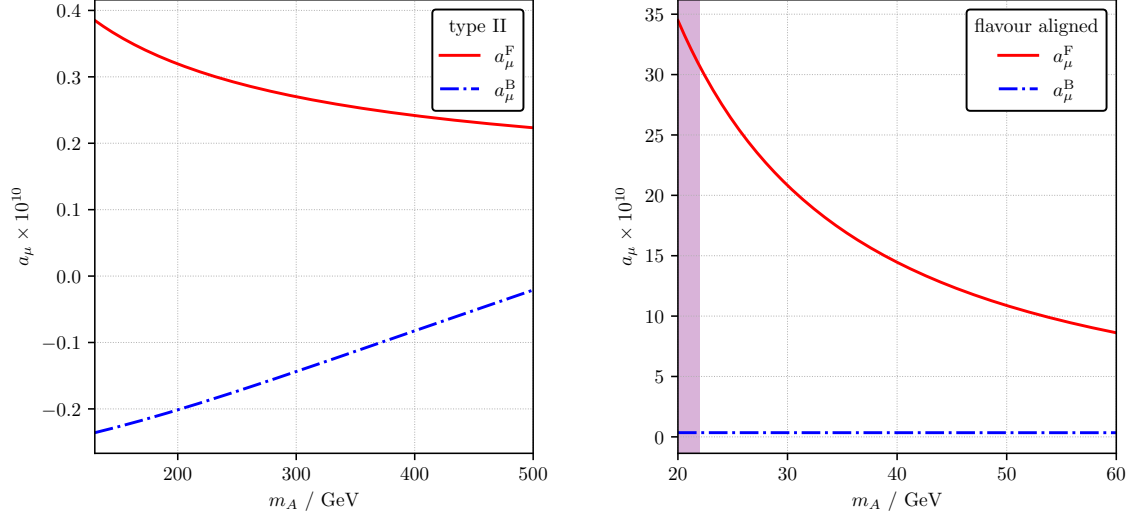
The source code for the flavour-aligned part of the example is shown below

```

1  Install["bin/gm2calc.mx"];
2
3  CalcAmuFA[mA_] :=
4  {amu1L, amu2LF, amu2LB} /. GM2CalcAmuTHDMMassBasis[
5  yukawaType      -> 5,
6  Mhh             -> { 125, 150 },
7  MAh             -> mA,
8  MHP             -> 150,
9  sinBetaMinusAlpha -> 0.999,
10 lambda6        -> 0,
11 lambda7        -> 0,
12 TB             -> 2,
13 m122           -> 3187.3 + mA*(3.27803 + 0.0165557*mA),
14 zetau          -> -0.1,
15 zetad         -> -0.1,
16 zetal         -> 50
17 ];
18
19 (* mA values in [20,60] GeV *)
20 mAValues = Subdivide[20, 60, 200];
21
22 (* calculation w/ running couplings *)
23 GM2CalcSetFlags[runningCouplings -> True];
24 favalues = CalcAmuFA /@ mAValues;
25
26 datafa = { mAValues, favalues[[All,1]], favalues[[All,2]], \
27           favalues[[All,3]] };
28 Export["flavouraligned.txt", N @ Transpose @ datafa, "Table"];

```

The parameter choices for the above example are taken from Ref. [82], and are chosen to show a region of the 2HDM's parameter space where it is possible to explain the value of  $a_\mu^{\text{BSM}}$  in Eq. (3). In the function CalcAmuFA on lines 3-17, we set the parameters to  $m_H = m_{H^\pm} = 150$  GeV,  $\zeta_u = \zeta_d = -0.1$ ,  $\zeta_l = 50$ ,  $\tan(\beta) = 2$ ,  $\sin(\beta - \alpha) = 0.999$ . We also fix  $m_{12}^2 = 3187.3 + m_A * (3.27803 + 0.0165557 * m_A)$  (in  $\text{GeV}^2$ ) so there are no signals from  $h \rightarrow AA$  decays, using Eq. (12) from [82]. The scan for this scenario is over  $m_A \in [20, 60]$  GeV, and the rest of the script goes similarly to the previous example. The results of this scan can be seen in the right panel of Fig. 6. The shaded purple right region on the left indicate that for low values of  $m_A \sim 20$  GeV we can explain the  $a_\mu$  anomaly in the FA2HDM.



**Figure 6:** Comparison of the bosonic and fermionic two-loop contributions to  $a_\mu^{\text{BSM}}$  in the 2HDM, which correspond to the red and blue dashed lines respectively. The left panel shows a scenario from the type-II 2HDM from Ref. [81], with the parameters  $m_H = 400$  GeV,  $m_{H^\pm} = 440$  GeV,  $\tan(\beta) = 3$ , and  $m_{12}^2 = 40000$  GeV<sup>2</sup>. The right panel shows a scenario in the FA2HDM from Figure 10 in Ref. [82], which uses the parameters  $m_H = m_{H^\pm} = 150$  GeV,  $\zeta_u = \zeta_d = -0.1$ ,  $\zeta_l = 50$ ,  $\tan(\beta) = 2$  and  $\sin(\beta - \alpha) = 0.999$ , and fixes  $m_{12}^2$  to avoid  $h \rightarrow AA$  decays. Also shown in the right panel is a shaded purple region which indicates when the combine one-loop, two-loop fermionic and two-loop bosonic contributions can explain the value of the  $a_\mu$  anomaly in Eq. (3).

#### 4. Conclusions

The 2HDM has been shown to be adept at explaining many of the outstanding anomalies of particle physics, including the  $a_\mu$  anomaly. GM2Calc2 provides an easy-to-use and powerful tool which can calculate the contributions to  $a_\mu^{\text{BSM}}$  in the types-I, II, X, Y, flavour-aligned, and general 2HDM. GM2Calc2 includes contributions at up to the two-loop level, features CKM mixing, lepton flavour violation, and complex Yukawa couplings, utilizing and building on the calculations done by [75]. We have provided an overview of the code, and given examples of its usage, based on the scenarios from Refs. [80–82]. Further details can be found in the original work [1] and on the GM2Calc [GitHub page](#).

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