

Flavour Physics Phenomenology with SuperIso

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We present an overview of SuperIso v4.1 which is a public program for the calculation of flavour physics observables. We give examples of using SuperIso to constrain new physics scenarios with kaon physics and present the implications of rare B -decays.

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1. Introduction

The Standard Model of particle physics has been very successful in explaining the phenomena within the energies of the Large Hadron Collider. However, there are theoretical and observational indications that there is physics beyond the Standard Model (BSM). In addition to direct searches for new physics (NP) signals at particle accelerators, indirect searches play an important and complementary role in investigating BSM physics and can in principle be sensitive to much shorter scales than direct searches probing energies not directly reachable. Flavour observables and precision measurements of rare decays are among the most powerful indirect probes of new physics scenarios.

SuperIso is a public C program [1–4] with the purpose of studying new physics by analysing indirect searches mainly via flavour observables. The SuperIso program can be downloaded at <http://superiso.in2p3.fr>. Details on the installation and the description of the various subroutines can be found in Ref. [4] or in the manual which can be downloaded from the SuperIso website <http://superiso.in2p3.fr/superiso4.1.pdf>.

In what follows we first list the observables included in SuperIso, in section 3 we briefly review the recently added kaon observables, in section 4 we show the implications of rare B -decays and we conclude in section 5.

2. Observables

SuperIso’s main focus is on indirect searches via flavour physics but it also includes other indirect observables such as electroweak precision tests via oblique parameters S, T, U , and ρ, Γ_z as well as the muon anomalous moment ($a_\mu - 2$). Furthermore, direct search limits from LEP and Tevatron can also be imposed. Moreover, dark matter relic density and direct and indirect detection rates can be included via a separate code, SuperIso Relic [5–7], which is an extension of SuperIso.

The flavour physics observables included in SuperIso can be categorized into two classes of flavour changing charged current (FCCC) and flavour changing neutral current (FCNC) processes. The former includes tree-level decays involving neutrinos in the final state and consists of the branching ratios of $B \rightarrow \ell\nu$, $B \rightarrow D^*\ell\nu$, $D_s \rightarrow \ell\nu$, $D \rightarrow \mu\nu$, and $K \rightarrow \mu\nu$. SuperIso includes most of the measured FCNC transitions where there are a plethora of observables mostly in B -physics and also in kaon physics making it possible to probe different types of NP contributions. The FCNC transitions in SuperIso consist of:

Radiative decays:

- Branching Ratio (BR) of the inclusive decays $B \rightarrow X_{s,d}\gamma$
- BR and isospin asymmetry of $B \rightarrow K^*\gamma$

Leptonic decays:

- BR of $B_{s,d} \rightarrow \ell^+\ell^-$
- BR of $K_{L,S} \rightarrow \ell^+\ell^-$

Semi-leptonic decays:

- $B \rightarrow X_s \ell^+ \ell^-$: BR and angular observables (A_{FB} , zero-crossing $q_0^2[A_{\text{FB}}]$)
- $B^{(+)} \rightarrow K^{*(+)} \ell^+ \ell^-$: BR and angular observables (A_{FB} , zero-crossing $q_0^2[A_{\text{FB}}]$, F_L , A_T^i , $P_i^{(\prime)}$, S_i , R_{K^*} , $R_{K^{**}}$, ...)
- $B_s \rightarrow \phi \ell^+ \ell^-$: BR and angular observables (F_L , S_i , R_ϕ)
- $B^{(+)} \rightarrow K^{(+)} \ell^+ \ell^-$: BR and angular observables A_{FB} , F_H , R_K , $R_{K_S^0}$
- $\Lambda_b^+ \rightarrow \Lambda \ell^+ \ell^-$: BR and angular observables A_{FB} , F_L
- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: BR
- $K_L \rightarrow \pi^0 \nu \bar{\nu}$: BR

Furthermore, $\text{BR}(K_L \rightarrow \pi^0 \ell \bar{\ell})$ as well as the $\Delta S = 2$ and $\Delta B = 2$ meson-mixing observables are included in the development version of SuperIso.

3. Leptonic and semi-leptonic kaon decays

As an example of the observables included in SuperIso, we describe below with more detail the leptonic and semi-leptonic kaon decays which have been implemented more recently. To describe these decays, we employ an effective Hamiltonian similar to the one used for B -physics:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t^{sd} \frac{\alpha_e}{4\pi} \sum_k \left(C_k^{sd,\ell} O_k^{sd,\ell} + C_{Q_k}^{sd,\ell} Q_k^{sd,\ell} \right), \quad (1)$$

with $\lambda_t^{sd} \equiv V_{td} V_{ts}^*$ where the four-fermion operators are given by

$$O_9^{sd,\ell^{(\prime)}} = (\bar{s} \gamma_\mu P_{L(R)} d) (\bar{\ell} \gamma^\mu \ell), \quad O_{10}^{sd,\ell^{(\prime)}} = (\bar{s} \gamma_\mu P_{L(R)} d) (\bar{\ell} \gamma^\mu \gamma_5 \ell), \quad (2)$$

$$Q_1^{sd,\ell^{(\prime)}} = (\bar{s} P_{R(L)} d) (\bar{\ell} \ell), \quad Q_2^{sd,\ell^{(\prime)}} = (\bar{s} P_{R(L)} d) (\bar{\ell} \gamma_5 \ell), \quad (3)$$

$$O_{L(R)}^{sd,\ell} = (\bar{s} \gamma_\mu P_{L(R)} d) (\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu), \quad (4)$$

with ℓ referring to the lepton flavour. The corresponding Wilson coefficients $C_k^{sd,\ell^{(\prime)}}$ contain the short-distance effects from the SM as well as from new physics

$$C_k^{sd,\ell} = C_{k,\text{SM}}^{sd,\ell} + C_{k,\text{NP}}^{sd,\ell}. \quad (5)$$

3.1 Branching ratio of $K \rightarrow \pi \nu \bar{\nu}$

The branching fractions of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decays with a sum over all neutrino flavours are given by [8, 9]

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{\kappa_+(1 + \Delta_{\text{EM}})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\nu_\ell} \left[\text{Im}^2(\lambda_t C_L^{\nu_\ell}) + \text{Re}^2\left(-\lambda_c^{sd} \frac{\lambda^4 P_c(X)}{s_W^2} + \lambda_t^{sd} C_L^{\nu_\ell}\right) \right], \quad (6)$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\nu_\ell} \text{Im}^2[\lambda_t C_L^{\nu_\ell}], \quad (7)$$

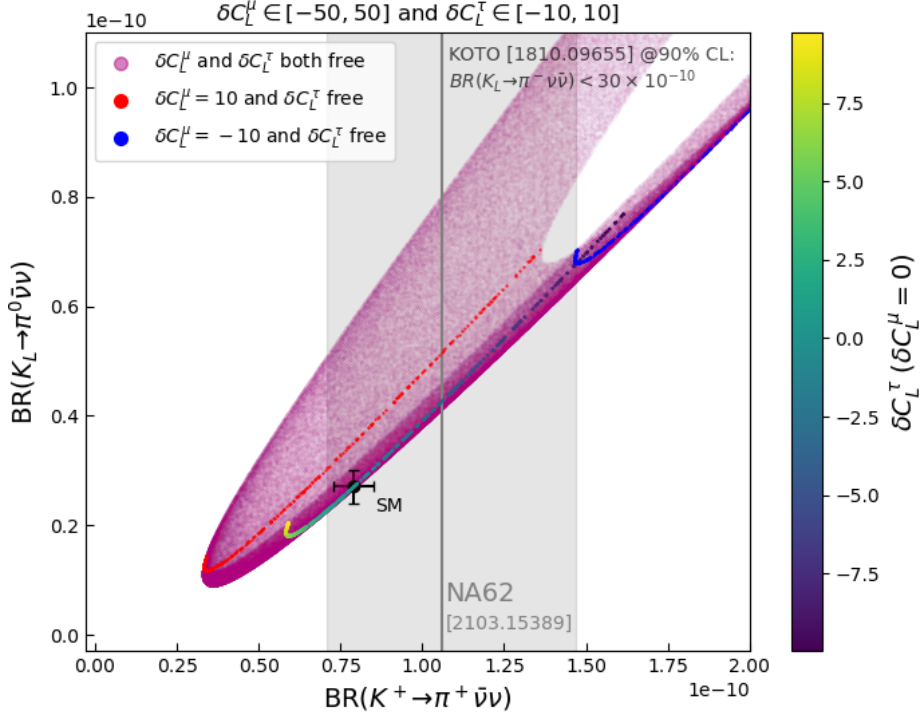


Figure 1: $\text{BR}(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$ vs $\text{BR}(K_L \rightarrow \pi^0 \bar{\nu} \nu)$ varying $\delta C_L^\mu \in [-10, 10]$ and $\delta C_L^\tau \in [-50, 50]$.

and in order to include the chirality flipped contributions, the Wilson coefficients should be replaced by $C_i \rightarrow C_i + C'_i$. In eq. 6 above, $P_c(X) = P_c^{\text{SD}}(X) + \delta P_{c,u}$ where $P_c^{\text{SD}}(X)$ corresponds to the short-distance charm contributions calculated at NNLO accuracy [10] and $\delta P_{c,u}$ refers to long-distance contributions [11]. The electromagnetic radiative correction due to photon exchanges is given by $\Delta_{\text{EM}} = -0.003$ for $E_{\text{max}}^\gamma \approx 20$ MeV, and the factors $\kappa_{+,L}$ are [12]

$$\kappa_L = (2.231 \pm 0.013) \cdot 10^{-10} \left[\frac{\lambda}{0.225} \right]^8, \quad (8)$$

$$\kappa_+ = (0.5173 \pm 0.0025) \cdot 10^{-10} \left[\frac{\lambda}{0.225} \right]^8. \quad (9)$$

The $K \rightarrow \pi \nu \bar{\nu}$ branching ratios are powerful probes of short-distance effects in the kaon sector as their SM predictions are very precise with $\lesssim 10\%$ uncertainty. The effect of generic lepton flavour violating new physics contributions in the muon and tau sectors can be seen in Fig. 1 where $\delta C_L^\mu \in [-10, 10]$ and $\delta C_L^\tau \in [-50, 50]$. On the experimental side, for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ there is currently only an upper bound from KOTO [13] which is two orders of magnitude larger than the SM prediction. The main constraint is from $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ measured by NA62 collaboration [14] in agreement with the SM prediction which is $(7.86 \pm 0.61) \times 10^{-11}$. The red (blue) curve corresponds to the case where δC_L^τ is varied while δC_L^μ is fixed to 10 (-10). The effect on the two branching ratios when having NP effects solely in the tau sector can be seen with the color map (putting $\delta C_L^\mu = 0$).

3.2 Branching ratio of $K_{L,S} \rightarrow \mu\mu$

The branching fractions of the $K_S \rightarrow \mu^+\mu^-$ and $K_L \rightarrow \mu^+\mu^-$ decays, are given by [11, 15]

$$\begin{aligned} \text{BR}(K_S^0 \rightarrow \mu^+\mu^-) = \tau_S \frac{f_K^2 m_K^3 \beta_{\mu,K}}{16\pi} \left(\frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \times & \left\{ \beta_{\mu,K}^2 \left| \frac{\sqrt{2}\pi}{G_F \alpha_e} N_S^{\text{LD}} - \frac{m_K}{m_s + m_d} \text{Re}(\lambda_t C_{Q_1}) \right|^2 \right. \\ & \left. + \left| \frac{2m_\mu}{m_K} \text{Im} \left(-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10} \right) + \frac{m_K}{m_s + m_d} \text{Im}(\lambda_t C_{Q_2}) \right|^2 \right\}, \end{aligned} \quad (10)$$

$$\begin{aligned} \text{BR}(K_L^0 \rightarrow \mu^+\mu^-) = \tau_L \frac{f_K^2 m_K^3 \beta_{\mu,K}}{16\pi} \left(\frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \times & \left\{ \beta_{\mu,K}^2 \left| \frac{m_K}{m_s + m_d} \text{Im}(\lambda_t C_{Q_1}) \right|^2 \right. \\ & \left. + \left| \frac{\sqrt{2}\pi}{G_F \alpha_e} N_L^{\text{LD}} - \frac{2m_\mu}{m_K} \text{Re} \left(-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10} \right) - \frac{m_K}{m_s + m_d} \text{Re}(\lambda_t C_{Q_2}) \right|^2 \right\}, \end{aligned} \quad (11)$$

with $\beta_{\mu,K} \equiv \sqrt{1 - 4m_\mu/M_K^2}$. In order to include chirality flipped contributions, the Wilson coefficients should be replaced by $C_i \rightarrow C_i - C'_i$. The short-distance SM contributions are given by C_{10}^{SM} and the charm contribution Y_c ($\equiv \lambda^4 P_c(Y)$), known with NNLO accuracy [16]. The long-distance contributions are denoted as $N_{S,L}^{\text{LD}}$ [15]

$$N_S^{\text{LD}} = (-2.65 + 1.14i) \times 10^{-11} (\text{GeV})^{-2}, \quad (12)$$

$$N_L^{\text{LD}} = \pm [0.54(77) - 3.95i] \times 10^{-11} (\text{GeV})^{-2}, \quad (13)$$

where in the latter equation the sign is unclear since theoretically as well as experimentally the sign of the intermediate 2γ contribution due to $\mathcal{A}(K_L \rightarrow (\pi^0)^* \rightarrow \gamma\gamma)$ remains unknown.

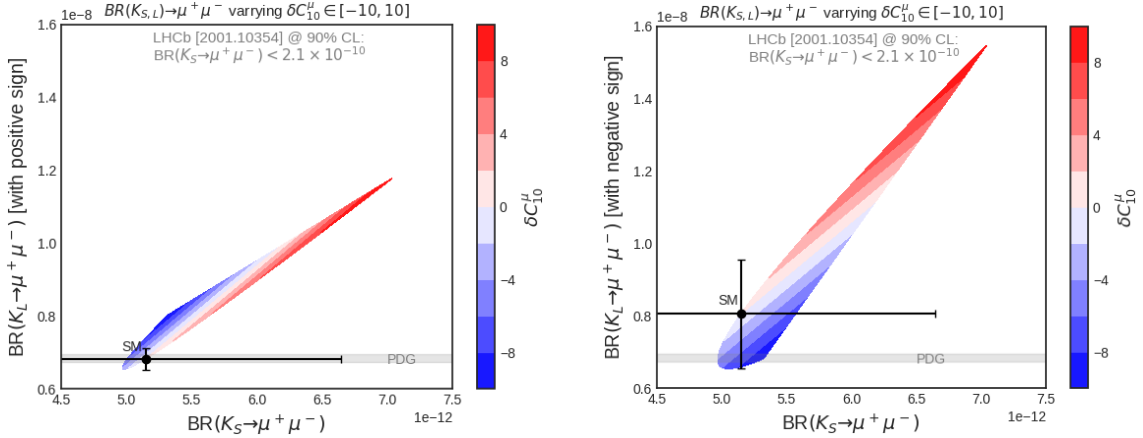


Figure 2: $\text{BR}(K_S \rightarrow \bar{\mu}\mu)$ vs $\text{BR}(K_L \rightarrow \bar{\mu}\mu)$ with positive (negative) sign for the long-distance contribution to $K_L \rightarrow \bar{\mu}\mu$ on the left (right) panel.

The effect of NP contributions from $\delta C_{10} \in [-10, 10]$ on $\text{BR}(K_{L,S} \rightarrow \mu^+\mu^-)$ can be seen in Fig. 2. On the experimental side, $\text{BR}(K_L \rightarrow \mu^+\mu^-)$ is very precisely known [17] with an uncertainty of less than 2%, however, considering the theoretical uncertainty of long-distance contributions and the sign ambiguity, there is room for NP effects. On the other hand, for the $K_S \rightarrow \mu^+\mu^-$ which

is a much rarer decay, with three orders of magnitude suppression compared to the $K_L \rightarrow \mu^+ \mu^-$ in the SM, there is currently only an upper bound from LHCb [18] which is about two orders of magnitude larger than the SM prediction.

For a detailed analysis of NP phenomenology with rare kaon decays, especially for scenarios involving lepton flavour universality violation (LFUV) see Ref. [19] where in addition to the above decay modes, the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ decays as well as LFUV via $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ have been considered. Furthermore, in Ref. [19] the overall NP fit to the rare kaon decays has been presented.

4. Global fit to rare B -decays

In recent years, the most promising signs of new physics have been in rare B -decays where several observables have been measured to be in tension with their corresponding SM predictions. However, there are numerous B -physics observables, inter-dependent via one or more Wilson coefficients and in order to get a consistent global picture of the implications B -physics measurements,

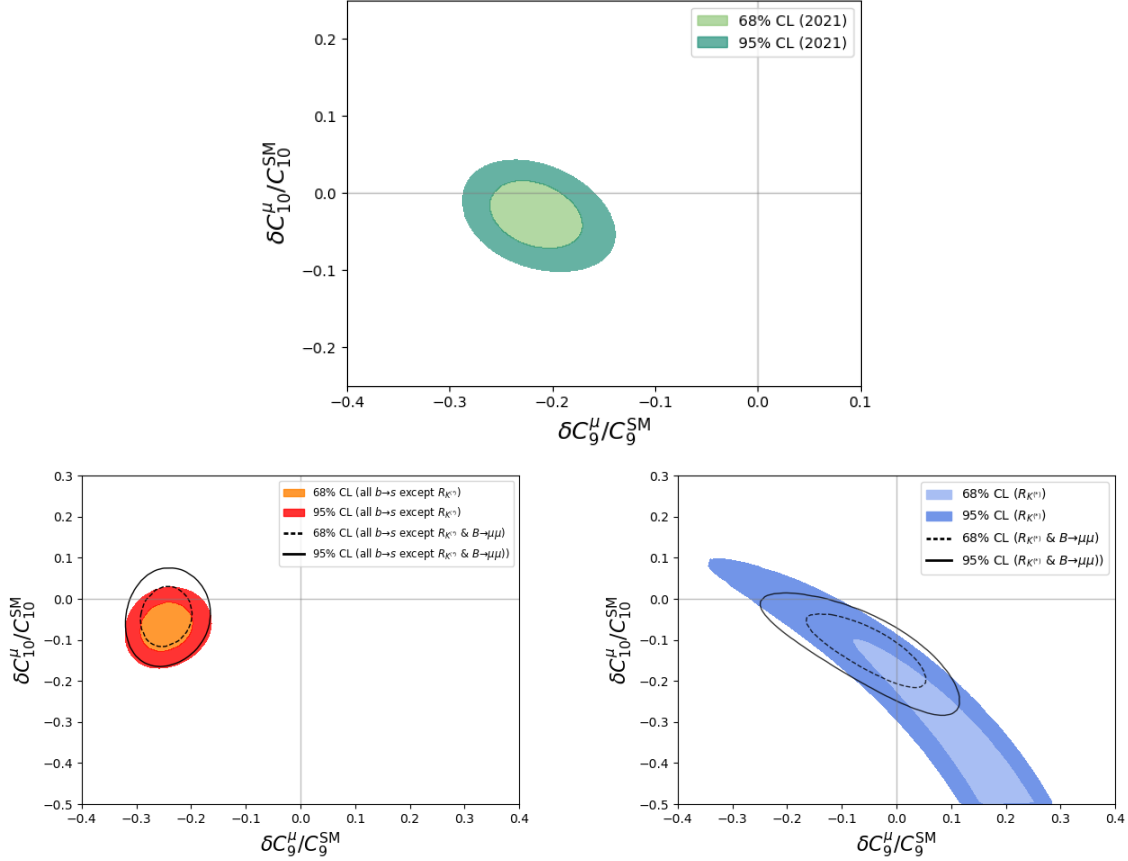


Figure 3: Two operator fits to (C_9^H, C_{10}^H) within 68 and 95% confidence level. The upper plot corresponds to the fit to all observables (assuming 10% power corrections). In the lower plots, on the left we have considered all observables except R_K and R_{K^*} (with the assumption of 10% power corrections) while on the right we have only used the data on R_K, R_{K^*} . Further details can be found in Ref. [20].

it is suitable to perform a global fit to all observables which are described via common Wilson coefficients. This can be done by doing a χ^2 fit to the relevant Wilson coefficients of the B -physics observables. In SuperIso, there are dedicated subroutines in order to calculate

$$\chi^2 = \sum_{i,j=1}^N \left(O_i^{\text{th}} - O_i^{\text{exp}} \right) C_{ij}^{-1} \left(O_j^{\text{th}} - O_j^{\text{exp}} \right), \quad (14)$$

where O_i^{exp} and O_i^{th} denote the experimental measurement and theoretical predictions of the i^{th} observable, respectively. $C_{i,j}$ describes the covariance matrix which is the sum of the theoretical and experimental covariance matrices. In SuperIso $C_{i,j}$ is calculated automatically for the SM and then used for all NP points. It is also possible to calculate $C_{i,j}$ for each NP point, however, computationally this would be very demanding and as shown in Ref. [21], the assumption that $C_{i,j}$ calculated for the SM can be used also for new physics points is a justified approximation.

In Fig. 3, two-dimensional fits to Wilson coefficients (C_9^μ, C_{10}^μ) are shown where in the first row all the relevant $b \rightarrow s$ data have been used while in the second row separate fits to the lepton flavour universality violating/conserving observables have been considered, showing the coherence of the two sets of data with the global fit.

5. Conclusions

Flavour observables offer powerful probes of physics beyond the Standard Model and can explore energy scales not reachable via direct detection. SuperIso calculates most of the constraining flavour observables and it also includes many other observables such as muon anomalous magnetic moment and the dark matter constraints. Here we presented only a few examples in model-independent new physics scenarios, but SuperIso also automatically calculates the Wilson coefficients of several model-specific scenarios such as two Higgs doublet models (2HDM), minimal supersymmetric Standard Model (MSSM) and next-to-minimal supersymmetric Standard Model (NMSSM). In the next versions, SuperIso is going to be extended to include further flavour observables as well as the automatic calculation of Wilson coefficients of non-minimal flavour violating MSSM scenarios [22]. Furthermore, there is also going to be a direct interface to MARTY [23] which is a C++ public program that calculates Wilson coefficients for generic BSM scenarios at one-loop level.

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