

## New results on inclusive semi-leptonic $B$ decays at the B-factories

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This proceeding summarises recent studies on inclusive semi-leptonic  $B$  decays, which provide key experimental inputs to ultimately increase the precision of inclusive  $|V_{ub}|$  and  $|V_{cb}|$  determinations using data-driven approaches. The presented analyses investigate the complete Belle data set of  $711 \text{ fb}^{-1}$  of integrated luminosity, produced at the KEKB accelerator complex with a centre-of-mass energy corresponding to the  $\Upsilon(4S)$  resonance of  $\sqrt{s} = 10.58 \text{ GeV}$ .

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## 1. $|V_{ub}|$ and $|V_{cb}|$

Precise determinations of the absolute value of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements provide a potent test of the Standard Model of Particle Physics (SM). A well-established strategy to determine the absolute values of  $V_{ub}$  and  $V_{cb}$  is to use measurements of semi-leptonic  $B$  meson decays with  $b \rightarrow u\ell\nu$  and  $b \rightarrow c\ell\nu$  transitions. Determinations of  $|V_{ub}|$  and  $|V_{cb}|$  are extracted by employing two complementary approaches: the exclusive approach focuses on the reconstruction of a specific decay mode, while the inclusive approach aims to measure the sum of all possible final states entailing the same quark-level transition. Current world averages of  $|V_{ub}|$  and  $|V_{cb}|$  from exclusive and inclusive determinations exhibit disagreements of approximately 3 standard deviations between both techniques [1]. This disagreement has posed a longstanding puzzle.

## 2. Reconstruction of inclusive semi-leptonic $B$ decays

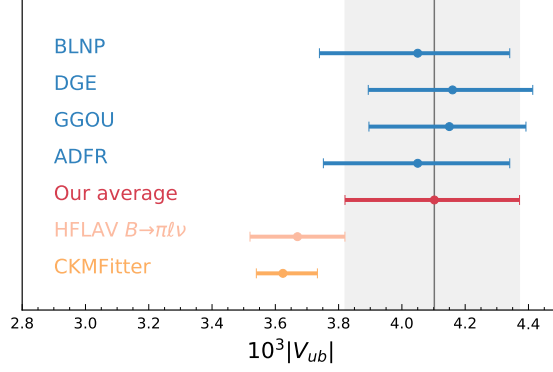
The analyses presented in this proceeding provide first measurements of key experimental input with the aim to ultimately increase the precision of inclusive  $|V_{ub}|$  and  $|V_{cb}|$  determinations. To this end, inclusive semi-leptonic  $B$  meson decays are studied using the complete Belle data set, comprising  $(772 \pm 10) \times 10^6$   $B$  meson pairs, that was produced at the KEKB accelerator complex with a centre-of-mass energy of  $\sqrt{s} = 10.58$  GeV. In order to identify and reconstruct kinematic variables describing inclusive semi-leptonic  $B$  meson decays, a multivariate algorithm based on a hierarchical approach, known as the Full Reconstruction [2], is employed to reconstruct one of the two  $B$  mesons through exclusive hadronic decay channels. The output classifier score of this neural network represents the estimated quality of the reconstructed candidates, denoted as the  $B_{\text{tag}}$ , and the best candidates for each event are selected. Continuum processes, where  $e^+e^- \rightarrow q\bar{q}$  ( $q = u, d, s, c$ ), are suppressed by requiring the  $B_{\text{tag}}$  candidates to have a beam-constrained mass of  $M_{bc} = \sqrt{(s/2)^2 - |\mathbf{p}_{\text{tag}}|^2} > 5.27$  GeV. A direct consequence of this reconstruction technique allows for the kinematics of the remaining  $B$  meson to be inferred using conservation laws. By using the momentum of the  $B_{\text{tag}}$  candidate together with the precisely known initial beam-momentum, the signal  $B$  rest frame is defined as  $p_{\text{sig}} = (\sqrt{s}, \mathbf{0}) - \left( \sqrt{m_B^2 + |\mathbf{p}_{\text{tag}}|^2}, \mathbf{p}_{\text{tag}} \right)$ . Since multiple leptons are likely to originate from a double semi-leptonic  $b \rightarrow c \rightarrow s$  cascade, exactly one signal lepton per event is selected. Additionally, the charge of the signal lepton is required to be opposite to the charge of the  $B_{\text{tag}}$  candidate. The four-momentum of the hadronic system,  $p_X$ , is determined from the sum of the remaining unassigned charged tracks and neutral energy clusters. Subsequently, the hadronic mass of the  $X$  system is calculated as  $M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$ . With the  $X$  system reconstructed, the four-momentum of the neutrino in the event is estimated as  $P_{\text{miss}} = (E_{\text{miss}}, \mathbf{p}_{\text{miss}}) = p_{\text{sig}} - p_X - p_\ell$ , while the four-momentum transfer squared is calculated as  $q^2 = (p_{\text{sig}} - p_X)^2$ .

## 3. Partial branching fractions of $B \rightarrow X_u \ell \nu$ and determination of $|V_{ub}|$

This analysis [3] measures partial branching fractions in three kinematic regions covering approximately 31% to 86% of the accessible  $B \rightarrow X_u \ell \nu$  phase space to extract inclusive  $|V_{ub}|$ . Signal  $B \rightarrow X_u \ell \nu$  decays are simulated with a ‘‘hybrid’’ approach [4, 5] as a mixture of specific exclusive

Fit variable	Phase space region	$10^3 \Delta \mathcal{B}$
$M_X$	$E_\ell^B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}$	$1.09 \pm 0.05 \pm 0.08$
$q^2$	$E_\ell^B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}, q^2 > 8 \text{ GeV}^2$	$0.67 \pm 0.07 \pm 0.10$
$E_\ell^B$	$E_\ell^B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}$	$1.11 \pm 0.06 \pm 0.14$
$E_\ell^B$	$E_\ell^B > 1 \text{ GeV}$	$1.69 \pm 0.09 \pm 0.26$
$M_X : q^2$	$E_\ell^B > 1 \text{ GeV}$	$1.59 \pm 0.07 \pm 0.16$

**Table 1:** Measured partial branching fractions for various phase space regions with the corresponding uncertainties due to statistics and systematics, respectively.

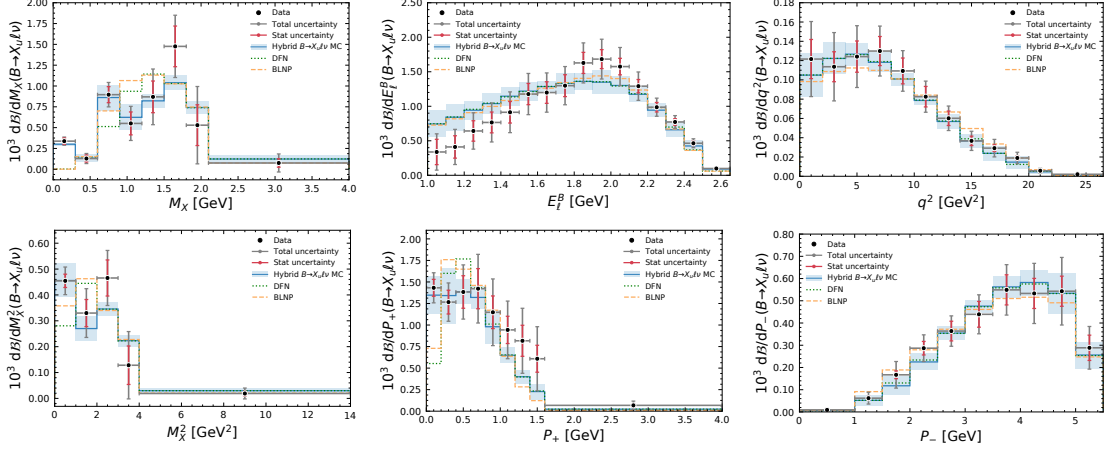


**Figure 1:** The obtained values of  $|V_{ub}|$  from the four calculations and the arithmetic average compared to the exclusive determination and the expectation from CKM unitarity.

modes and non-resonant contributions using Monte Carlo (MC). In order to effectively suppress the dominant background process, namely  $B \rightarrow X_c \ell \nu$  decays, a multivariate selection based on Boosted Decision Trees (BDT) is implemented. The signal yield is then extracted from five separate template fits where systematic uncertainties are included as nuisance parameter constraints. The resulting branching fractions determined from each fit variable are summarised in Table 1, where the most inclusive measurements correspond to the region with lepton energies in the  $B$  rest frame of  $E_\ell^B > 1 \text{ GeV}$ . Using the most precise measurement determined from a two-dimensional fit of  $M_X$  and  $q^2$ ,  $|V_{ub}|$  is extracted from four theoretical predictions for the partial decay rate: BLNP [6], DGE [7, 8], GGOU [9], and ADFR [10, 11]. The arithmetic average of the four determinations is calculated to obtain  $|V_{ub}| = (4.10 \pm 0.09_{\text{stat}} \pm 0.22_{\text{syst}} \pm 0.15_{\text{theo}}) \times 10^{-3}$ . This value is compatible with the determination from exclusive semi-leptonic decays of  $|V_{ub}^{\text{excl.}}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$  [1] with 1.3 standard deviations, as compared in Fig. 1. Furthermore, the result is compatible with the value expected from CKM unitarity from a global fit [12] of  $|V_{ub}| = (3.62_{-0.08}^{+0.11}) \times 10^{-3}$  with 1.6 standard deviations.

#### 4. Differential branching fractions of $B \rightarrow X_u \ell \nu$

The first measurements of differential spectra of inclusive  $B \rightarrow X_u \ell \nu$  decays [13] are reported as a function of six kinematic variables in the  $E_\ell^B > 1 \text{ GeV}$  region of phase space: the lepton energy  $E_\ell^B$ , the four-momentum-transfer squared  $q^2$ , light-cone momenta  $P_\pm = (E_X^B \mp |\mathbf{p}_X^B|)$ , the

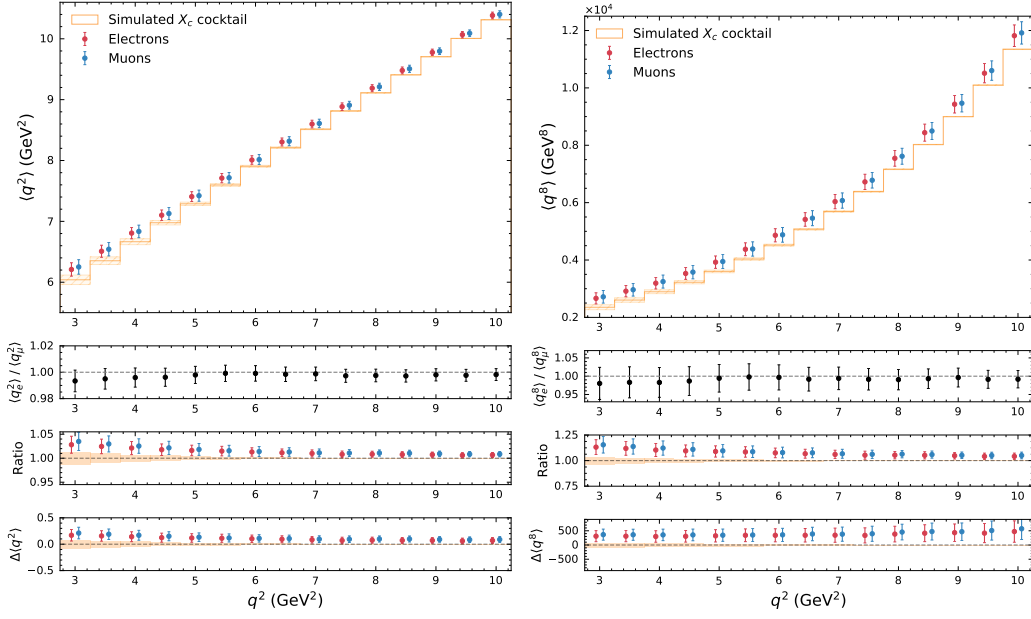


**Figure 2:** The measured differential  $B \rightarrow X_u \ell \nu$  branching fractions, together with the hybrid MC prediction and two inclusive calculations scaled to  $\Delta\mathcal{B} = 1.59 \times 10^{-3}$ .

hadronic mass  $M_X$ , and the hadronic mass squared  $M_X^2$ . These measurements follow directly from the previously discussed analysis, however additional selection criteria of  $|\mathbf{E}_{\text{miss}} - \mathbf{p}_{\text{miss}}| < 0.1$  GeV and  $M_X < 2.4$  GeV are imposed to improve the resolution of key variables and further suppress background processes. The background subtracted spectra are obtained by estimating the remaining expected number of background events from a template fit to the  $M_X$  distribution, after which the singular-value-decomposition (SVD) method [14] is used to correct for detector resolution and acceptance effects. As a final step the unfolded yields are further corrected for efficiency and acceptance effects arising from the partial phase space defined by the  $E_\ell^B > 1$  GeV requirement. Additionally, uncertainties and correlations stemming from the background subtraction strategy and the modelling of the detector response are consistently propagated throughout the analysis procedure. The measured differential branching fractions are shown in Fig. 2 together with the  $B \rightarrow X_u \ell \nu$  hybrid MC and the fully inclusive DFN [15] and BLNP predictions. Overall fair agreement between the measured and predicted distributions is observed, in particular the hybrid MC better describes the  $B \rightarrow X_u \ell \nu$  in the resonance region. These distributions will not only allow for model-independent determinations of the shape function governing the non-perturbative dynamics of the  $b \rightarrow u$  transition, but also  $|V_{ub}|$  [16, 17].

## 5. $q^2$ moments of $B \rightarrow X_c \ell \nu$

Inclusive determinations of  $|V_{cb}|$  rely on predictions of the total rate and the spectral moments as a double series of non-perturbative matrix elements in  $\Lambda_{\text{QCD}}/m_b$  and  $\alpha_s$  using the Heavy Quark Expansion (HQE). The proliferation of these matrix elements at order  $1/m_b^4$  and higher complicates their extraction, thus limiting the precision of  $|V_{cb}|$ . A novel approach [18] exploits reparametrization invariance (RPI) to drastically reduce the number of parameters to a set of eight for the total rate. However, new measurements are required to determine the reduced set of parameters, since the  $q^2$  spectrum is found to be the unique observable that satisfies the key prerequisite giving rise to RPI for  $B \rightarrow X_c \ell \nu$  decays. The first systematic study [19] of the first to fourth moments of the  $q^2$  spectrum is performed as a function of progressively increasing threshold



**Figure 3:** The first and fourth order measured  $q^2$  moments for electrons and muons. In the ratio between moments, many of the associated systematic uncertainties cancel and all reported moments are compatible with the expectation of lepton flavour universality (bottom top). In addition, the measured and generated-level moments for all the threshold selections on  $q^2$  are compared as a ratio (bottom middle) and difference (bottom lower) for both electrons and muons. Note that the individual electron and muon moments are highly correlated.

selections, where  $q^2 \in [3.0, 10.0]$   $\text{GeV}^2$ . To not only ensure the selection of correctly reconstructed  $B \rightarrow X_c \ell \nu$  decays, but also greatly improve the resolution of the  $q^2$  distribution, this analysis requires  $|E_{\text{miss}} - |\mathbf{p}_{\text{miss}}|| < 0.5 \text{ GeV}$ . In addition, the total observed charge of the event is required to be  $|Q_{\text{tot}}| = |Q_{B_{\text{tag}}} + Q_X + Q_\ell| \leq 1$ , allowing for charge imbalance in events with low-momentum tracks. To extract the  $q^2$  moments, the background contributions are subtracted by assigning an event-by-event signal probability. The signal probability weights are determined using a two step procedure: first, a binned likelihood fit to the  $M_X$  distribution is performed to estimate the total number of background events. The signal and background templates for the fit are determined from MC samples, while systematic uncertainties are incorporated via nuisance parameter constraints. Next, binned signal probabilities, calculated as the difference between the measured  $q^2$  spectrum and the expected background and normalized to the measured distribution, are fitted with a polynomial function to extract continuous event-wise weights. To ensure an unbiased result, the measured  $q^2$  spectrum is corrected for detector resolution and selection effects. The measured moments are reported separately for electron and muon final states, allowing for a test of lepton flavour universality, as shown in Fig. 3.

## 6. Summary

Several exciting measurements of inclusive semi-leptonic  $B$  decays have recently been reported by the Belle Collaboration. These results provide crucial experimental inputs to ultimately increase

the precision of inclusive  $|V_{ub}|$  and  $|V_{cb}|$  determinations using novel, data-driven approaches. Furthermore, the accumulated knowledge of improved analysis techniques and MC modelling, together with the increasing size of the Belle II data set, will uncover new insights concerning the longstanding tensions between exclusive and inclusive determinations, while enhancing our understanding of the theory of semi-leptonic  $B$  meson decays.

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