

Standard Model Predictions for Rare K and B Decays without $|V_{cb}|$ and $|V_{ub}|$ Uncertainties*

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The persistent tensions between inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ weaken the power of theoretically clean rare K and B decays in the search for new physics (NP). We demonstrate how this uncertainty can be practically removed by considering within the SM suitable ratios of various branching ratios. This includes the branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_S \rightarrow \mu^+ \mu^-$, $B_{s,d} \rightarrow \mu^+ \mu^-$ and $B \rightarrow K(K^*) \nu \bar{\nu}$. Also ϵ_K , ΔM_d , ΔM_s and the mixing induced CP-asymmetry $S_{\psi K_S}$, all measured already very precisely, play an important role in this analysis. The highlights of our analysis are 16 $|V_{cb}|$ and $|V_{ub}|$ independent ratios that often are independent of the CKM parameters or depend only on the angles β and γ in the Unitarity Triangle with β already precisely known and γ to be measured precisely in the coming years by the LHCb and Belle II collaborations. Once γ is measured precisely these 16 ratios taken together are expected to be a powerful tool in the search for new physics. On the other hand within the SM they imply that to obtain the SM predictions for rare Kaon decays only the experimental values of $|\epsilon_K|$ and $S_{\psi K_S}$ are needed. In particular we find: $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.60 \pm 0.42) \times 10^{-11}$ and $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = (2.94 \pm 0.15) \times 10^{-11}$. This are the most precise determinations to date. Similarly, to obtain the SM predictions for all rare B decay branching ratios only the knowledge of the experimental values of $\Delta M_{s,d}$ is required.

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1. Introduction

The rare K and B decays and the quark mixing being GIM suppressed in the Standard Model (SM) and simultaneously being often theoretically clean are very powerful tools for the search of New Physics (NP) [1]. Unfortunately the persistent tensions between inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ (see e.g. [2–6]) weaken this power significantly. As recently reemphasized by us [7] this is in particular the case of the branching ratios for K -meson decays and the parameter ε_K that exhibit stronger $|V_{cb}|$ dependences than rare B decay branching ratios and the $\Delta M_{s,d}$ mass differences.

To cope with this difficulty one can consider within the SM suitable ratios of two properly chosen observables so that the dependences on $|V_{cb}|$ and $|V_{ub}|$ are eliminated [7–9]. While in [8, 9] B physics observables were considered, our analysis in [7] was dominated by the K system and its correlation with rare B decays and $B_{s,d}^0 - \bar{B}_{s,d}^0$ mixing. In this manner we could construct 16 $|V_{cb}|$ independent ratios that were either independent of the CKM parameters or only dependent on the angles β and γ , that can be determined in tree-level processes. Having one day precise experimental values for the ratios in question and also precise values on β and γ will hopefully allow one to identify particular pattern of deviations from SM expectations independently of $|V_{cb}|$ and $|V_{ub}|$ pointing towards a particular extension of the SM.

This note summarizes the main results of [7] and is arranged as follows. In Section 2 we will simply list the 16 ratios in question and briefly discuss their first implications. In Section 3 we report on inconsistencies between the determinations of $|V_{cb}|$ from ε_K , ΔM_d and ΔM_s . In Section 4, combining these ratios with the assumption of no NP contributions to $|\varepsilon_K|$, ΔM_s , ΔM_d and $S_{\psi K_S}$, we will present SM predictions for all branching ratios considered by us that are most accurate to date. Our note closes with a short outlook in Section 5.

2. $|V_{cb}|$ and $|V_{ub}|$ Independent Ratios

As four basic CKM parameters we will use

$$\boxed{\lambda = |V_{us}|, \quad |V_{cb}|, \quad \beta, \quad \gamma} \quad (2.1)$$

with β and γ being two angles in the UT. Their determination from mixing induced CP-asymmetries in tree-level B decays and using other tree-level strategies is presently theoretically cleaner than the determination of $|V_{ub}|$. A recent review of such determinations of β and γ can be found in [1, 10, 11].

The $|V_{cb}|$ independent ratios constructed by us have the general power-like structure

$$R_i = C_i [\sin \gamma]^{p_1} [\sin \beta]^{p_2}, \quad (2.2)$$

with the coefficients C_i either being constants or being very weakly dependent on β and γ . Whenever the ratios are not already in the form of (2.2), we derive an approximate power-law expression, with non-integer exponents. The latter are indeed fitted to describe as power-law functions of parameters some more complicated exact expressions, with the best possible accuracy, which is $\lesssim 2\%$. The dependence on $|V_{us}|$ is negligible, the angle β is already known from $S_{\psi K_S}$ asymmetry

with respectable precision and there is a significant progress by the LHCb collaboration on the determination of γ from tree-level strategies [12]:

$$\boxed{\beta = (22.2 \pm 0.7)^\circ, \quad \gamma = (65.4^{+3.8}_{-4.2})^\circ.} \quad (2.3)$$

Moreover, in the coming years the determination of γ by the LHCb and Belle II collaborations should be significantly improved so that precision tests of the SM using our strategy will be possible.

The 16 $|V_{cb}|$ independent ratios in question are given as follows

$$\boxed{R_0(\beta) = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})^{0.7}} = (2.03 \pm 0.08) \times 10^{-3} \left[\frac{\sin 22.2^\circ}{\sin \beta} \right]^{1.4} = (2.03 \pm 0.11) \times 10^{-3}.} \quad (2.4)$$

$$\boxed{R_{\text{SL}} = \frac{\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\text{SD}}}{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})} = 1.55 \times 10^{-2} \left[\frac{\lambda}{0.225} \right]^2 \left[\frac{Y(x_t)}{X(x_t)} \right]^2,} \quad (2.5)$$

where $X(x_t)$ and $Y(x_t)$ are well known one-loop functions with $x_t = m_t^2/M_W^2$ [1]. Then, we have

$$\boxed{R_1(\beta, \gamma) = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{[\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)]^{1.4}}, \quad R_2(\beta, \gamma) = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{[\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)]^{1.4}.} \quad (2.6)$$

In particular the ratio R_1 , implying the correlation in Fig. 2 should be of interest in the coming years due to the improved measurement of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ by NA62, of $B_s \rightarrow \mu^+ \mu^-$ by LHCb, CMS and ATLAS and of γ by LHCb and Belle II. Explicitly we have

$$R_1(\beta, \gamma) = (55.24 \pm 2.48) \left[\frac{\sin \gamma}{\sin 67^\circ} \right]^{1.39} \left[\frac{G(22.2^\circ, 67^\circ)}{G(\beta, \gamma)} \right]^{2.8} \left[\frac{230.3 \text{ MeV}}{F_{B_s}} \right]^{2.8}, \quad (2.7)$$

$$R_2(\beta, \gamma) = (8.29 \pm 0.40) \times 10^3 \left[\frac{\sin(67^\circ)}{\sin \gamma} \right]^{1.41} \left[\frac{190.0 \text{ MeV}}{F_{B_d}} \right]^{2.8}, \quad (2.8)$$

where F_{B_d} and F_{B_s} are weak decay constants and

$$G(\beta, \gamma) = 1 + \frac{\lambda^2}{2} (1 - 2 \sin \gamma \cos \beta). \quad (2.9)$$

Furthermore, there are also the following $|V_{cb}|$ independent ratios

$$\boxed{R_3(\beta, \gamma) = \frac{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{[\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)]^2}, \quad R_4(\beta, \gamma) = \frac{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{[\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)]^2},} \quad (2.10)$$

which are given by

$$R_3(\beta, \gamma) = (2.17 \pm 0.09) \times 10^6 \left[\frac{\sin \gamma \sin \beta}{\sin(67^\circ) \sin(22.2^\circ)} \right]^2 \left[\frac{G(22.2^\circ, 67^\circ)}{G(\beta, \gamma)} \right]^4 \left[\frac{230.3 \text{ MeV}}{F_{B_s}} \right]^4, \quad (2.11)$$

$$R_4(\beta, \gamma) = (2.79 \pm 0.13) \times 10^9 \left[\frac{\sin(67^\circ)}{\sin \gamma} \frac{\sin \beta}{\sin(22.2^\circ)} \right]^2 \left(\frac{190.0 \text{ MeV}}{F_{B_d}} \right)^4. \quad (2.12)$$

In view of future improved results from NA62 and Belle II, of particular interest are the ratios

$$R_5(\beta, \gamma) = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{[\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})]^{1.4}}, \quad R_6(\beta, \gamma) = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{[\mathcal{B}(B^0 \rightarrow K^{0*} \nu \bar{\nu})]^{1.4}}. \quad (2.13)$$

Explicitly

$$R_5(\beta, \gamma) = (2.69 \pm 0.51) \times 10^{-3} \left[\frac{\sin \gamma}{\sin 67^\circ} \right]^{1.39} \left[\frac{G(22.2^\circ, 67^\circ)}{G(\beta, \gamma)} \right]^{2.8}, \quad (2.14)$$

$$R_6(\beta, \gamma) = (9.07 \pm 1.23) \times 10^{-4} \left[\frac{\sin \gamma}{\sin 67^\circ} \right]^{1.39} \left[\frac{G(22.2^\circ, 67^\circ)}{G(\beta, \gamma)} \right]^{2.8}, \quad (2.15)$$

with $G(\beta, \gamma)$ defined in (2.9). Next, in view of future LHCb and Belle II measurements, of particular interest are the ratios

$$R_7 = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)}, \quad R_8 = \frac{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})}{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)} \quad (2.16)$$

with

$$(R_7)_{\text{SM}} = (1.20 \pm 0.17) \times 10^3 \left(\frac{230.3 \text{ MeV}}{F_{B_s}} \right)^2, \quad (2.17)$$

$$(R_8)_{\text{SM}} = (2.62 \pm 0.25) \times 10^3 \left(\frac{230.3 \text{ MeV}}{F_{B_s}} \right)^2. \quad (2.18)$$

Turning our attention to ε_K and $\Delta M_{s,d}$ we define two $|V_{cb}|$ independent ratios

$$R_9(\beta, \gamma) = \frac{|\varepsilon_K|}{(\Delta M_d)^{1.7}}, \quad R_{10}(\beta, \gamma) = \frac{|\varepsilon_K|}{(\Delta M_s)^{1.7}}. \quad (2.19)$$

The explicit expressions for them read

$$R_9(\beta, \gamma) = 6.405 \times 10^{-3} \text{ ps}^{1.7} \left(\frac{\sin 67^\circ}{\sin \gamma} \right)^{1.73} \left(\frac{\sin \beta}{\sin 22.2^\circ} \right)^{0.87} \bar{R}_d^\varepsilon, \quad (2.20)$$

$$R_{10}(\beta, \gamma) = 1.516 \times 10^{-5} \text{ ps}^{1.7} \left(\frac{\sin \gamma}{\sin 67^\circ} \right)^{1.67} \left(\frac{\sin \beta}{\sin 22.2^\circ} \right)^{0.87} \left(\frac{G(22.2^\circ, 67^\circ)}{G(\beta, \gamma)} \right)^{3.4} \bar{R}_s^\varepsilon. \quad (2.21)$$

Here

$$\bar{R}_d^\varepsilon = \left(\frac{214.0 \text{ MeV}}{\sqrt{\hat{B}_{B_d}} F_{B_d}} \right)^{3.4} \left(\frac{2.307}{S_0(x_t)} \right)^{1.7} \left(\frac{0.5521}{\eta_B} \right)^{1.7}, \quad (2.22)$$

$$\bar{R}_s^\varepsilon = \left(\frac{261.7 \text{ MeV}}{\sqrt{\hat{B}_{B_s}} F_{B_s}} \right)^{3.4} \left(\frac{2.307}{S_0(x_t)} \right)^{1.7} \left(\frac{0.5521}{\eta_B} \right)^{1.7}. \quad (2.23)$$

Finally, there are also the following four ratios

$$R_{11}(\beta, \gamma) = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{|\varepsilon_K|^{0.82}} = (1.31 \pm 0.05) \times 10^{-8} \left(\frac{\sin \gamma}{\sin 67^\circ} \right)^{0.015} \left(\frac{\sin 22.2^\circ}{\sin \beta} \right)^{0.71}, \quad (2.24)$$

$$R_{12}(\beta, \gamma) = \frac{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{|\varepsilon_K|^{1.18}} = (3.87 \pm 0.06) \times 10^{-8} \left(\frac{\sin \gamma}{\sin 67^\circ} \right)^{0.03} \left(\frac{\sin \beta}{\sin 22.2^\circ} \right)^{0.98}, \quad (2.25)$$

$$R_q = \frac{\mathcal{B}(B_q \rightarrow \mu^+ \mu^-)}{\Delta M_q} = 4.291 \times 10^{-10} \frac{\tau_{B_q} (Y_0(x_t))^2}{\hat{B}_q S_0(x_t)}, \quad q = d, s. \quad (2.26)$$

The first two of these formulae express explicitly the fact that combining on the one hand $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and ε_K and on the other hand $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ε_K allows within the SM to determine to a very good approximation the angle β independently of the value of $|V_{cb}|$ and γ . This is one of the most interesting new results of our paper. The CKM independence of R_q has been pointed out already in [8].

There are two first interesting consequences of these $|V_{cb}|$ independent relations. First, combining the LHCb and Belle II results we find [7]

$$(R_7)_{\text{EXP}} = (3.86 \pm 1.48) \times 10^3. \quad (2.27)$$

The central value is by a factor of 3.2 larger than the SM prediction (2.17) but due to large error in the experimental $B^+ \rightarrow K^+ \nu \bar{\nu}$ branching ratio the tension is only at 1.8σ .

The second result is for R_s in (2.26) for which one finds [9]

$$(R_s)_{\text{SM}} = (2.042_{-0.058}^{+0.083}) \times 10^{-10} \text{ps}, \quad (R_s)_{\text{EXP}} = (1.61_{-0.17}^{+0.19}) \times 10^{-10} \text{ps}, \quad (2.28)$$

that is a 2.1σ tension.

While the ratios presented here are independent of $|V_{cb}|$ and $|V_{ub}|$ they depend on the hadronic parameters which enter in particular the ratios involving $|\varepsilon_K|$, ΔM_d and ΔM_s . This brings us to the next important topic.

3. $|V_{cb}|(\beta, \gamma)$ from $|\varepsilon_K|$, ΔM_d and ΔM_s

When one day the angle γ will be measured precisely we will be able to answer the question whether all these SM predictions for the ratios in question can be satisfied by the data simultaneously. We have just seen two possible tensions. But in fact, we can do more already today. Namely, we can ask the question, whether the SM can describe the very precise data for ε_K , ΔM_s and ΔM_d with the same values of $|V_{cb}|$, β and γ . Also, theory in this three cases is in a good shape due to progress from LQCD groups over many years and in the case of ε_K due to the progress in [13].

To answer this question we considered in our paper the following hadronic parameters

$$F_{B_d} \sqrt{\hat{B}_{B_d}} = 214.0(39) \text{ MeV}, \quad F_{B_s} \sqrt{\hat{B}_{B_s}} = 261.7(38) \text{ MeV} \quad (3.1)$$

in the case of $\Delta M_{d,s}$ and $\hat{B}_K = 0.7625(97)$ in the case of ε_K . The latter one is the PDG average, while the values in (3.1) are the averages of $2 + 1$ and $2 + 1 + 1$ LQCD results quoted by PDG obtained in [9]. The result of this exercise is shown in Fig. 1.

We find that with the chosen hadronic parameters it is not possible to obtain simultaneous agreement with the data on ε_K , ΔM_d , ΔM_s and $S_{\psi K_S}$, used to determine β , within the SM independently of the value of $|V_{cb}|$ and γ . While these tensions are still moderate they could hint for

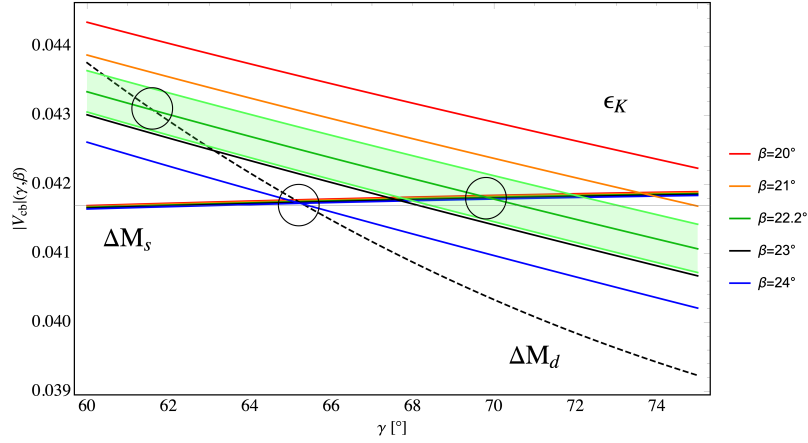


Figure 1: The values of $|V_{cb}|$ extracted from ϵ_K , ΔM_d and ΔM_s as functions of γ for different values of β . $|V_{cb}|$ extracted from ΔM_d is independent of β .

some NP at work. In this context a precise measurement of γ and the improvements on hadronic parameters will be important. The consequences of these findings are discussed in detail in [7]. But the message is clear. The ratios involving ϵ_K , ΔM_d , ΔM_s , that is

$$R_9, \quad R_{10}, \quad R_{11}, \quad R_{12}, \quad R_d, \quad R_s, \quad (3.2)$$

are not expected to agree simultaneously with the future data for the set of hadronic parameters in (3.1). This lead us to propose a different strategy for SM predictions for rare K and B decays summarized in the rest of this writing.

4. Improved SM Predictions for rare K and B Decays

While the ratios listed above are useful in the context of the testing of the SM and in the search for NP, they are not as interesting as the observables themselves. Therefore, assuming in addition no NP in ϵ_K , ΔM_d and ΔM_s and in the mixing induced CP-asymmetry $S_{\psi K_S}$, these ratios allowed to obtain $|V_{cb}|$ independent SM predictions for a number of branching ratios [7]. As these four quark mixing observables are very precisely measured and theoretically rather clean, the resulting predictions obtained in this manner turned out to be the most precise to date. We report on these predictions in Table 1.

However, in view of the inconsistencies in different determinations of $|V_{cb}|$ seen in Fig. 1 to obtain these results we did not perform on purpose the usual global fit of observables which would require in addition the input on $|V_{cb}|$ from tree-level decays in contradiction with the main strategy of our paper. We concluded therefore that it would be a bad idea to assume, as done in global fits, that NP is absent simultaneously in ϵ_K , ΔM_d , ΔM_s and $S_{\psi K_S}$. Therefore, to obtain SM predictions for rare Kaon decays we only assumed the absence of NP in ϵ_K and $S_{\psi K_S}$. To obtain predictions for $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \mu^+ \mu^-$ we assumed, following [8], the absence of NP in ΔM_s and in ΔM_d , respectively but not simultaneously. In our view this strategy for finding SM predictions is presently more powerful than any global fit which would include decays like $B \rightarrow K \mu^+ \mu^-$, $B \rightarrow K^* \mu^+ \mu^-$ and $B_s \rightarrow \phi \mu^+ \mu^-$ that seem to exhibit significant contributions from NP. Moreover, these *local*

Decay	Branching Ratio	Decay	Branching Ratio
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$B_s \rightarrow \mu^+ \mu^-$	$(3.62_{-0.10}^{+0.15}) \times 10^{-9}$
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$B_d \rightarrow \mu^+ \mu^-$	$(0.99_{-0.03}^{+0.05}) \times 10^{-10}$
$K_S \rightarrow \mu^+ \mu^-$	$(1.85 \pm 0.10) \times 10^{-13}$	$B^+ \rightarrow K^+ \nu \bar{\nu}$	$(4.45 \pm 0.62) \times 10^{-6}$
		$B^0 \rightarrow K^{0*} \nu \bar{\nu}$	$(9.70 \pm 0.92) \times 10^{-6}$

Table 1: Present most accurate $|V_{cb}|$ independent SM estimates of the branching ratios considered in the paper. The γ dependence is either very small or absent. See [7] for details.

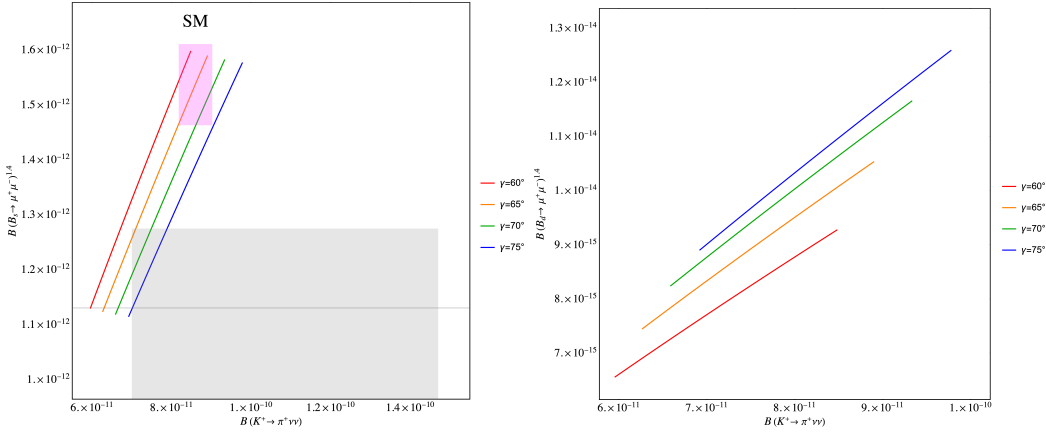


Figure 2: The correlations of $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ with $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)^{1.4}$ (left panel) and with $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)^{1.4}$ (right panel) as given in (2.7) and (2.8), for different values of γ within the SM. The ranges of branching ratios correspond to $38 \leq |V_{cb}| \times 10^3 \leq 43$ and $20^\circ \leq \beta \leq 24^\circ$. The SM area corresponds to the one in Table 1. The gray area represents the present experimental situation.

assumptions of the absence of NP, with the goal to obtain SM predictions, are weaker than made in *global* fits. But one should be aware of the fact that the SM predictions in Table 1 that are based on hadronic parameters in (3.1) are likely not be true simultaneously in view of the results in Fig. 1. But this is precisely what we are after!

5. Summary and Outlook

We have summarized the main results of our recent paper [7], where following and extending significantly the strategies of [8, 9, 14–19], we have proposed to search for NP in rare Kaon and B -meson decays without the necessity to choose the values of the CKM elements $|V_{cb}|$ and $|V_{ub}|$, that introduce presently large parametric uncertainties in the otherwise theoretically clean decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_S \rightarrow \mu^+ \mu^-$, $B_{s,d} \rightarrow \mu^+ \mu^-$, $B \rightarrow K(K^*) \nu \bar{\nu}$, in the parameter ε_K and $\Delta M_{s,d}$. The 16 $|V_{cb}|$ independent ratios of branching ratios and mixing observables, depending only on the angle β and γ in the UT, will certainly play an important role in the search for NP as soon as the branching ratios in question and the angle γ will be precisely measured. Several

plots of these ratios and also of correlations between various branching ratios can be found in our paper. We present here only two correlations in Fig. 2 which could turn out in the coming years to be the smoking guns of NP. For the set of hadronic parameters in (3.1) we have also identified inconsistencies between the determinations of $|V_{cb}|$ from ϵ_K , ΔM_d and ΔM_s . This led us to propose a novel strategy to obtain SM predictions for rare K and B decay branching ratios which is based on *locality* rather than *globality* as explained in Section 3. In this context we have found that within the SM to obtain the SM predictions for rare Kaon decays only the experimental values of $|\epsilon_K|$ and $S_{\psi K_S}$ are needed. Similarly, to obtain the SM predictions for all rare B decay branching ratios only the knowledge of the experimental values of $\Delta M_{s,d}$ is required.

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