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Neutral meson mixing in the $B^0_{(s)}$ sector from Lattice QCD

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We summarise the current status of lattice QCD computations of neutral meson parameters in the B^0_d and the B^0_s systems. We comment on recent results and anticipate further improvements on the uncertainties in the future.

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1. Introduction

The flavour eigenstates of neutral mesons oscillate. This yields an experimentally precisely measurable mass and width difference between the CP eigenstates of the $B^0_{(s)}$-$\overline{B}^0_{(s)}$-system. The most recent experimental measurements for the mass differences $\Delta m_d^{\mathrm{exp}}$ and $\Delta m_s^{\mathrm{exp}}$ for the $B_d - \overline{B}_d$ and the $B_s - \overline{B}_s$ systems, respectively are [1, 2]

$$\begin{aligned}\Delta m_d^{\mathrm{exp}} &= (0.5065 \pm 0.0019)\,\mathrm{ps}^{-1}\,,\\ \Delta m_s^{\mathrm{exp}} &= (17.7656 \pm 0.0057)\,\mathrm{ps}^{-1}\,.\end{aligned} \tag{1}$$

These oscillations are mediated through the quark flow box diagrams displayed in figure 1, where $q = d$ corresponds to the case of $B^0_d - \overline{B}^0_d$ and $q = s$ to $B^0_s - \overline{B}^0_s$ mixing. Due to the hierarchy of the CKM matrix and the large top-quark mass, the short-distance contribution is top and CKM enhanced, whilst the long-distance contribution is CKM suppressed, so that the short-distance contribution dominates. This feature makes lattice QCD computations of the corresponding matrix elements feasible (as opposed to e.g. D^0-$\overline{D}^0$ mixing, where lattice QCD predictions are limited to the sub-leading short distance contribution, see for example ref. [3]).

There are 5 independent, parity even, local, dimension 6 operators O_i in an effective Hamiltonian for $\Delta B = 2$. In the SM, only the operator O_1 contributes to the mass difference Δm_q, whilst the operators O_1, O_2 and O_3 contribute to the width difference. The operators O_4 and O_5 do not contribute in the Standard Model, but by virtue of being a loop mediated process, neutral meson mixing is a sensitive probe for New Physics. To account for possible beyond the Standard Model scenarios, a precise determination of the non-perturbative matrix elements of all five operators is desirable.

The mass differences Δm_q can be parameterised in terms of known functions, CKM factors and non-perturbative matrix elements which can be computed via lattice QCD computations. More precisely

$$\Delta m_q = \frac{G_F^2 m_W^2 m_{B_q}}{6\pi^2} \left|V_{tb}V_{tq}^*\right|^2 S_0(x_t)\eta_{2B} f_{B_q}^2 \hat{B}^{(1)}_{B_q}\,, \tag{2}$$

where $q = d$ or s, $x_t = m_t^2/M_W^2$, $S_0(x_t)$ is an Inami-Lim function [4] and η_{2B} captures short-distance QCD corrections. Furthermore f_{B_q} is the decay constant of the B_q meson and $\hat{B}^{(1)}_{B_q}$ is the renormalisation group independent bag parameter for the operator $O^{(1)}$. Precise non-perturbative predictions of $f^2_{B_{(q)}} \hat{B}^{(1)}_{B_q}$, attainable via lattice QCD simulations, enable the extraction of the combination of CKM matrix elements $\left|V_{tb}V_{tq}^*\right|$. Additionally, it is convenient to define the $SU(3)$ breaking ratio ξ [5], defined by

$$\xi^2 = \frac{f^2_{B_s}\hat{B}^{(1)}_{B_s}}{f^2_{B_d}\hat{B}^{(1)}_{B_d}}\,. \tag{3}$$

By combining ξ with the experimental measurements of Δm_d and Δm_s, one can extract the ratio $|V_{td}/V_{ts}|$ from

$$\left|\frac{V_{td}}{V_{ts}}\right|^2 = \left(\frac{\Delta m_d}{\Delta m_s}\right)_{\mathrm{exp}} \frac{m_{B_s}}{m_{B_d}} \left(\xi^2\right)_{\mathrm{lat}}\,. \tag{4}$$

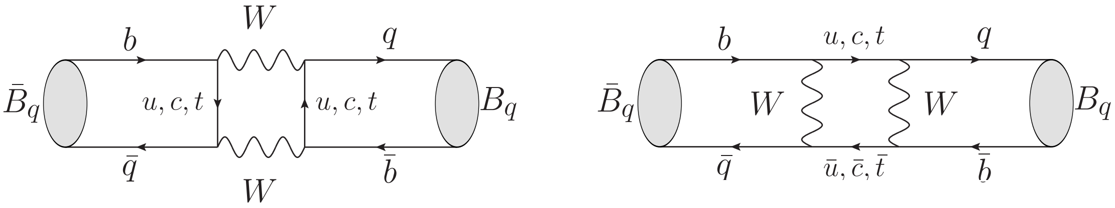

Figure 1: Quark flow diagrams of the box diagrams mediating neutral meson mixing for the case of $B^0_{(s)} - \overline{B}^0_{(s)}$ mesons.

This is favourable since various systematic and part of the statistical uncertainties cancel in such a ratio. The results for $|V_{td}|$, $|V_{ts}|$, and their ratio provide constraints which enter the global fits produced by the UT fit [6] and CKM-fitter [7] groups.

2. Neutral meson mixing results from lattice QCD

The large bottom quark mass poses significant challenges for the direct simulation of heavy-light systems. Currently all lattice simulations employ one of the following two approaches. In the first approach the heavy quark is simulated via an effective action such as static quarks, the non-relativistic QCD action [8, 9] or the Fermilab method [10, 11]. In the second approach simulations take place at unphysically light heavy quark masses. The results are then extrapolated to the physical b-quark mass [12, 13]. In addition, all current lattice QCD results for neutral meson mixing include heavier-than-physical pion mass ensembles. One leading uncertainty in current lattice QCD predictions stems from the chiral-continuum extrapolation. This extrapolation depends strongly on the available gauge field ensembles at or near the physical pion mass and gauge field ensembles with fine lattice spacings.

Figure 2 presents an overview over the lattice spacing and pion mass properties of the the gauge field ensembles that are used in the most recent computations [13–16]. For the discussion of older results [12, 17, 18], we refer the reader to the recent update of the flavour lattice averaging group (FLAG) [19].

The Fermilab/MILC result [14] has been discussed in previous iterations of this conference, so we only briefly summarise the computation. The light quarks are discretised using the asqtad action [20], the b-quark is discretised using the Fermilab method [10, 11]. The authors compute the product $f_{B_q}\sqrt{\hat{B}^{(i)}_{B_q}}$ for $q = d, s$ and for $i = 1, ..., 5$. The bag parameters $\hat{B}^{(i)}_{B_q}$ are then found by taking external input for f_{B_q}. The relative uncertainties for $f^2_{B_q}\hat{B}^{(1)}_{B_q}$ are approximately 8 and 6% for $q = d, s$, respectively. The relative uncertainty of ξ is approximately 1.5%.

The recent HPQCD result [15] uses the HISQ action [21] for the light quarks and the NRQCD action for the b-quark [8, 9]. The calculation provides values for the bag parameters $\hat{B}^{(i)}_{B_q}$ for $i = 1, ..., 5$ and $q = d, s$. The decay constants are then taken as an external input to convert this to $f_{B_q}\sqrt{\hat{B}^{(i)}_{B_q}}$. The inclusion of two ensembles at the physical pion mass significantly reduces uncertainties associated to the chiral extrapolation. The dominant uncertainty in this work arises from matching terms of the order α_s^2 and $\alpha_s\Lambda_{\text{QCD}}/m_b$. The quoted uncertainties for $\hat{B}^{(1)}_{B_q}$ are 5.0 and

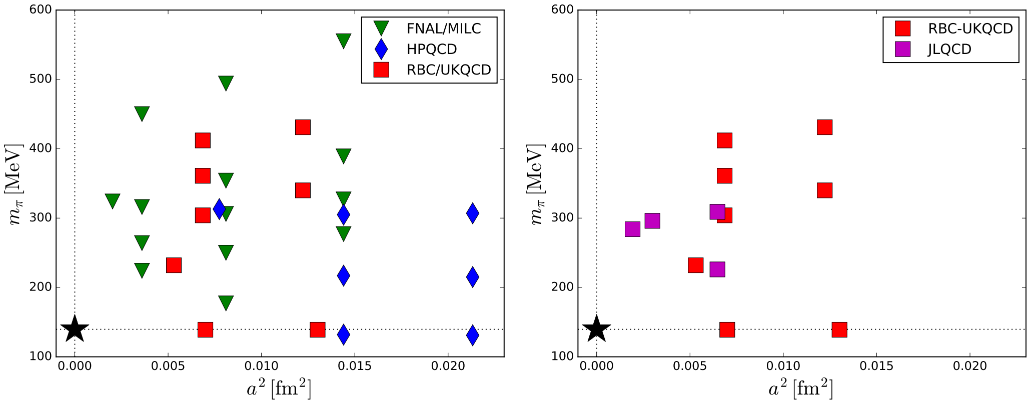

Figure 2: Gauge field ensembles as a function of the lattice spacing squared (a^2) and the pion mass squared m_π^2 used in recent computations of neutral meson mixing parameters. The black star corresponds to the physical point to which results need to be extrapolated. *Left*: The ensembles entering the FNAL/MILC analysis [14], the HPQCD analysis [15] and the RBC/UKQCD analysis [13] are shown as green triangles, blue diamond and red squares, respectively. *Right*: In Ref. [16] the RBC/UKQCD dataset (red squares) is supplemented with a recently generated JLQCD dataset (magenta squares). The gauge fields ensembles used in refs [13, 14, 16] include the dynamical effects of two degenerate light quarks and the strange quark ($N_f = 2+1$), whilst the ensembles used in ref. [15] also includes dynamical charm quark effects ($N_f = 2+1+1$).

4.3% for $q = d, s$, respectively, whilst the uncertainty of their ratio is 2.5%. For ξ an uncertainty of 1.3% is quoted.

The recent work by RBC/UKQCD [13] follows a different approach in the treatment of the heavy quark. Here the (to a good approximation) chirally symmetric domain wall fermion action [22–25] is used for light and heavy quarks, albeit with different choices for the domain wall parameters for the light and the heavy quarks [26]. The heavy quarks are simulated in the region from below the physical charm quark mass up to approximately half the bottom quark mass. Ref. [13] provides results for the ratios $\hat{B}_{B_s}/\hat{B}_{B_d}$, f_{B_s}/f_{B_d} and ξ. Because these ratios have a favourable behaviour as a function of the heavy quark mass, they allow a controlled extrapolation to the physical b-quark mass. As in the case of the HPQCD result, the inclusion of two ensembles with physical pion masses removes most uncertainties associated to the chiral extrapolation. The dominant uncertainty in this work is the extrapolation to the physical b-quark mass. However, by addition of ensembles with smaller lattice spacings, this uncertainty can be systematically improved upon. The total uncertainty for the quantity ξ is quoted at the percent level.

3. Summary and future prospects

The three computations [13–15] highlighted above are based on completely disjoint gauge field ensembles and are therefore independent predictions. These results are highly complementary as all the discretisation of the light and the heavy quark action differ between all three works. When comparing the numerical values obtained for the bag parameters $B^{(i)}_{B_q}$, the product $f^2_{B_q}\hat{B}^{(1)}_{B_q}$, and ξ, agreement between computations from the different groups can be seen. Combining the lattice results with the experimentally measured mass differences allows the extraction of $|V_{td}|$,

$|V_{ts}|$ and $|V_{td}/V_{ts}|$ at the 2.6%, 2.2% and 1.3%-level. However, in the $|V_{td}| - |V_{ts}|$ plane, an approximately 2 sigma deviation remains between the determinations from the different lattice results (see for example figure 7 of ref. [27]). Whilst the recent results are a significant improvement, the uncertainties on the CKM matrix elements are still dominated by the theory inputs, so additional work is required to improve upon their accuracy.

Recently, work in progress was reported for a joint effort between the RBC/UKQCD and the JLQCD collaborations [16]. This supplements the existing dataset from ref. [13] with additional ensembles with finer lattice spacings provided by the JLQCD collaboration. The bound of only simulating up to approximately half the physical b-quark mass was set due to the lattice spacing on the finest ensemble [28]. This can be significantly extended by including the JLQCD ensembles, enabling simulations close to the physical b-quark mass. The choice of the chirally symmetric all-domain-wall fermion set-up significantly simplifies the renormalisation structure and allows to draw on the expertise of the similar neutral kaon mixing programme [29, 30]. Using this the authors anticipate results for the full operator basis for the B_d^0 and the B_s^0 system in addition to the $SU(3)$-breaking ratios presented in ref. [16].

This review focusses on the determination of the dimension 6 operators from lattice QCD. Beyond this, in a recent computation [31] the HPQCD collaboration presented the first lattice QCD computation of the matrix elements of the dimension 7 operators which contribute to the width difference at next to leading order. Finally since the last iteration a new computation of the dimension 6 operators from sum rules has appeared [32].

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