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QCD corrections to B-meson mixing at two loops and beyond

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We report on the updated theoretical predictions for the width difference $\Delta\Gamma_s$ and the flavor-specific CP asymmetry a_{fs}^s in the $B_s - \bar{B}_s$ mixing. Working at leading power in the Heavy Quark Expansion we evaluated all previously unknown two-loop contributions to this observable, which marks an important step in the task of reducing uncomfortably large scale uncertainties stemming from uncalculated QCD corrections. Our new the Standard-Model prediction for the ratio of the width and mass differences in the $\overline{\text{MS}}$ scheme reads $\Delta\Gamma_s/\Delta M_s = (5.20 \pm 0.69) \cdot 10^{-3}$.

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1. Introduction

Modern flavor physics is a rich field for theoretical investigations that encompass not only model building and precision fits but also higher order perturbative calculations. The latter are especially important for observables that are well accessible to experimental measurements and feature small uncertainties due to large amounts of collected data and well understood systematic errors. One of the well known sources for such observables is the mixing of neutral B mesons. The $B_s - \bar{B}_s$ oscillations described via

$$i\frac{d}{dt}\begin{pmatrix}|B_s(t)\rangle \\ |\bar{B}_s(t)\rangle\end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B_s(t)\rangle \\ |\bar{B}_s(t)\rangle\end{pmatrix}, \quad \text{with} \quad \hat{M} = \begin{pmatrix}M_{11} & M_{12} \\ M_{12}^* & M_{22}\end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22}\end{pmatrix} \tag{1}$$

allow us to extract three physical quantities that can be calculated using quantum field theoretical methods. Diagonalizing the matrices $\hat{M}$ and $\hat{\Gamma}$ we can introduce the width and mass differences between mass eigenstates, $\Delta\Gamma_s$ and ΔM_s respectively, as well as the CP asymmetry in flavor-specific decays, a_{fs}^s. More explicitly, we have

$$\Delta M = M_H - M_L \approx 2|M_{12}|, \quad \Delta\Gamma = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos(\phi_{12}), \quad a_{\text{fs}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{12}, \tag{2}$$

where we neglected corrections of order $O(|\Gamma_{12}|^2/|M_{12}|^2)$ and used that

$$M_{12} = |M_{12}|e^{i\phi_M}, \quad \Gamma_{12} = |\Gamma_{12}|e^{i\phi_\Gamma}, \quad \cos(\phi_\Gamma - \phi_M) = -\cos(-\pi + \phi_\Gamma - \phi_M) \equiv -\cos(\phi_{12}). \tag{3}$$

Notice that while ϕ_Γ and ϕ_M depend on phase conventions (e. g. for the CKM matrix), ϕ_{12} is a physical CP-violating phase.

Precision flavor observables are very important for new physics searches due to their sensitivity to possible virtual Beyond Standard Model (BSM) particles appearing in the loops. In this respect $\Delta\Gamma_s$ and ΔM_s are of complementary nature: The former would deviate from its SM value in the case of light particles with masses below the electroweak scale, while the latter could help to find heavy (multi-TeV) degrees of freedom. Furthermore, one may also consider the ratio of both quantities $\Delta\Gamma_s/\Delta M_s$ which has the nice property of being independent of the CKM matrix element V_{ts}, while each observable by itself is proportional to $|V_{ts}|^2$. This allows us to circumvent existing issues related to the exclusive and inclusive determinations of $|V_{cb}|$, which is the main ingredient for obtaining $|V_{ts}|$. In addition to that, most hadronic uncertainties cancel from the ratio. Hence, better theoretical predictions for $\Delta\Gamma_s$ that is conventionally regarded as a SM precision probe can readily improve the limits on new physics from ΔM_s.

Experimental measurements for these two quantities resulted in

$$\Delta M_s^{\text{exp}} = (17.7656 \pm 0.0057)\ \text{ps}^{-1} \quad [1], \tag{4}$$
$$\Delta\Gamma_s^{\text{exp}} = (0.082 \pm 0.005)\ \text{ps}^{-1} \quad [2–8], \tag{5}$$

Theoretical predictions for $\Delta\Gamma_s$ [9–16] are still far away from the per cent level accuracy, which should motivate the theorists to improve their calculations. In the framework of the heavy-quark expansion (HQE), the calculation of $\Delta\Gamma_s$ can be understood as a double series in the strong coupling α_s and the ratio Λ_{QCD}/m_b.

A sizable reduction of theoretical uncertainties requires two ingredients. On the one hand, at leading order (LO) in the Λ_{QCD}/m_b-expansion one needs to evaluate all the relevant QCD corrections at next-next-to-leading order (NNLO) in α_s. On the other hand, it is also necessary to consider next-to-leading order (NLO) α_s corrections at NLO in the Λ_{QCD}/m_b-expansion. The main goal of our project is to address the former class of corrections i. e. to perform relevant two- and three-loop calculations at leading power.

2. Calculation

The essence of our $\Delta\Gamma_s$ calculation is the matching between two effective theories that describe the $\Delta B = 1$ and $\Delta B = 2$ processes. In the basis of [17] our effective Hamiltonian $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ reads

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-\lambda_t^s \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - \lambda_u^s \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}, \tag{6}$$

where we refer to [17] for the definitions of the current-current operators Q_{1-2}, four-quark penguin operators Q_{3-6} and the chromomagnetic penguin operator Q_8. The $Q_{1-2}^{u,cu,uc}$ versions of Q_{1-2} are obtained by replacing the two c quarks inside the operator by two u quarks or a combination of a c and a u quark respectively. Explicit expression for these operators can be found e. g. in [18]. Other quantities appearing in the effective Hamiltonian comprise the Fermi constant G_F, CKM matrix elements V_{ij} and $\lambda_a^s = V_{as}^* V_{ab}$. Then, C_i denote the Wilson coefficients of the $|\Delta B| = 1$ effective theory calculated from the matching to SM, where all degrees of freedom heavier than the b quark mass m_b have been integrated out. Eq. (6) contains renormalized physical operators. In the renormalization and regularization procedure one further encounters unphysical operators such as evanescent operators $E[Q_i]$ [19, 20]. Higher loop calculations involving evanescent operators become highly nontrivial when both ultraviolet and infrared poles are regularized dimensionally using the same ε from the spacetime dimension $d = 4 - 2\varepsilon$. Fortunately, the prescription for NLO calculations explained in [21] can be straightforwardly generalized to NNLO. For further details on the topic we refer to [16].

Let us now turn to the other side of our matching calculation. To calculate Γ_{12} we need to consider two insertions of $|\Delta B| = 1$ effective Hamiltonians with proper time-ordering and extract the absorptive part thereof. This bilocal matrix element can be simplified using Heavy Quark Expansion (HQE) [22–31], which yields [12]

$$\Gamma_{12} = -(\lambda_c^s)^2 \Gamma_{12}^{cc} - 2\lambda_c^s \lambda_u^s \Gamma_{12}^{uc} - (\lambda_u^s)^2 \Gamma_{12}^{uu}, \tag{7}$$

with

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \Big[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \Big] + O(\Lambda_{\text{QCD}}/m_b), \tag{8}$$

where $z \equiv m_c^2/m_b^2$ and

$$Q = \bar{s}_i \gamma^\mu (1-\gamma^5) b_i \, \bar{s}_j \gamma_\mu (1-\gamma^5) b_j, \qquad \widetilde{Q}_S = \bar{s}_i (1+\gamma^5) b_j \, \bar{s}_j (1+\gamma^5) b_i, \tag{9}$$

having labels i, j that denote the quark field color indices. The matching coefficients $H^{ab}(z)$ and $\widetilde{H}_S^{ab}(z)$ constitute the main goal of our perturbative calculations. Insertions of different operators on the $|\Delta B| = 1$ side of the matching and especially QCD corrections to the corresponding Feynman diagrams generate new contributions to $H^{ab}(z)$ and $\widetilde{H}_S^{ab}(z)$. Their knowledge is crucial to the task of improving the theory prediction for $\Delta\Gamma_s$.

In the actual calculation we express all Dirac and color structures encountered on the $|\Delta B| = 1$ side via tree-level matrix elements of $|\Delta B| = 2$ operators. This requires us to introduce additional operators

$$\widetilde{Q} = \bar{s}_i \gamma^\mu (1-\gamma^5) b_j \, \bar{s}_j \gamma_\mu (1-\gamma^5) b_i \,, \qquad Q_S = \bar{s}_i (1+\gamma^5) b_i \, \bar{s}_j (1+\gamma^5) b_j \,, \tag{10}$$

which equally enter the definitions of the evanescent $|\Delta B| = 2$ operators (cf. [15, 16]).

An important subtlety that we have to address is the treatment of the $1/m_b$-suppressed operator R_0 [9, 12] defined as

$$R_0 = Q_S + \alpha_1 \widetilde{Q}_S + \frac{1}{2}\alpha_2 Q. \tag{11}$$

Using Fierz identities and equations of motion on the level of the $|\Delta B| = 2$ effective Hamiltonian one can show that in 4 dimensions

$$\langle Q_S \rangle^{(0)} + \langle \widetilde{Q}_S \rangle^{(0)} + \frac{1}{2} \langle Q \rangle^{(0)} = O(1/m_b), \tag{12}$$

where $\langle \cdot \rangle^{(0)}$ denotes the tree-level matrix element of an operator. An important consequence of this relation is the fact that a $|\Delta B| = 2$ operator basis made of Q, Q_S and $\widetilde{Q_S}$ leads to ambiguous results since in the matching one can always use Eq. (12) to shift arbitrary pieces of Wilson coefficients between the three operators. Choosing to work in a two-operator basis (e. g. Q and $\widetilde{Q}_S$) seemingly leads to correct matching coefficients, but only when IR divergences of the amplitudes are regularized using a fictitious gluon mass m_g and not dimensionally. To obtain correct results when using dimensional regularization with $\varepsilon = \varepsilon_{\mathrm{UV}} = \varepsilon_{\mathrm{IR}}$ it is indeed necessary to include R_0 as defined in Eq. (11) to the operator basis. The point is that beyond tree-level the $1/m_b$-suppression of R_0 occurs only when IR divergences are regularized using m_g. In the case of dimensional IR regularization the unphysical evanescent piece of R_0, the so-called E_{R_0} is not suppressed anymore [9, 16], which may potentially lead to incorrect matching coefficients. The practical recipe to circumvent this issue without writing down the explicit form of E_{R_0} is to include the QCD correction factors $\alpha_{1,2}$ from Eq. (11) to the calculation when evaluating the matrix elements of R_0. Their task is to ensure that R_0 remains $1/m_b$-suppressed at each order in perturbation theory and thus to prevent a pollution of $H^{ab}(z)$ and $\widetilde{H}_S^{ab}(z)$ with unwanted contributions from R_0. Notice that the results for $\alpha_{1,2}$ given in [9] are sufficient only for matching calculations at $O(\alpha_s)$, if one uses dimensional regularization to treat IR divergences. The fermionic (i. e. proportional to the number of quark flavors n_f) contributions to $O(\alpha_s^2)$ were published in [13], while in our work we extended this result to include also nonfermionic pieces.

We refer to [15, 16] for further technical details regarding our matching calculation. It should be noted, however, that our results were obtained as an expansion in $z = m_c^2/m_b^2$ up to $O(z)$. This can be regarded as sufficient for phenomenological purposes, given the good convergence of the z-expansion. In short, we evaluated all possible insertions of two $|\Delta B| = 1$ operators (i. e. $Q_{1,2}$,

Q_{3-6} and Q_8) up to two loops and tackled the insertions of two current-current operators $Q_{1,2}$ at three loops. In most cases the fermionic parts of these results were already known in the literature, which provided us with useful cross checks. The computationally challenging nonfermionic pieces that constitute a genuine result of our work are now publicly available in a computer-readable form for all the one- and two-loop matching coefficients. For the sake of clarity we summarize the current status quo in Table 1.

Contribution	Literature result	This work
$Q_{1,2} \times Q_{3-6}$	2 loops, z-exact, n_f-part only [14]	2 loops, $O(z)$, full
$Q_{1,2} \times Q_8$	2 loops, z-exact, n_f-part only [14]	2 loops, $O(z)$, full
$Q_{3-6} \times Q_{3-6}$	1 loop, z-exact, full [32]	2 loops, $O(z)$, full
$Q_{3-6} \times Q_8$	1 loop, z-exact, n_f-part only [14]	2 loops, $O(z)$, full
$Q_8 \times Q_8$	1 loop, z-exact, n_f-part only [14]	2 loops, $O(z)$, full
$Q_{1,2} \times Q_{1,2}$	3 loops, $O(\sqrt{z})$, n_f-part only [13]	3 loops, $O(z)$, full

Table 1: Overview of the existing and new results required for the NNLO theory prediction of $\Delta\Gamma_s$ that were considered in this work. With "n_f-part only" we signify that the corresponding literature result provides only fermionic contributions, while "full" means that both fermionic and nonfermionic pieces are included.

3. Results

In [16] we published new theoretical predictions for the B_s-meson mixing observables that encompass our analytic two-loop results. While the numerical impact of the three-loop contribution is still in preparation, let us provide a brief overview of the numbers from [16].

Our starting point are the matching coefficients $H^{ab}(z)$ and $\tilde{H}_S^{ab}(z)$ with bottom and charm masses in the on-shell scheme, i. e. $z = (m_c^{\text{pole}}/m_b^{\text{pole}})^2$. From there we switch to the $\overline{\text{MS}}$ scheme for z by introducing $\bar{z} = (m_c(m_b)/m_b(m_b))^2$. The choice of the renormalization scale in the running masses $\mu_c = \mu_b = m_b(m_b)$ is motivated by the vanishing of potentially large $z \log z$-terms [33] that occurs only for $\mu_c = \mu_b$. Having arrived at $H^{ab}(\bar{z})$ and $\tilde{H}_S^{ab}(\bar{z})$ that depend only on $\overline{\text{MS}}$ parameters, we still need to make a scheme choice for the m_b^2 prefactor in Eq. (8). Leaving it in the on-shell scheme defines our "pole" scheme [13, 14], while converting the prefactor to the $\overline{\text{MS}}$ mass yields our "$\overline{\text{MS}}$" scheme. As far as the nonperturbative matrix elements are concerned, we use the parametrization

$$\langle B_s | Q(\mu_2) | \overline{B}_s \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B_{B_s}(\mu_2), \quad \langle B_s | \tilde{Q}_S(\mu_2) | \overline{B}_s \rangle = \frac{1}{3} M_{B_s}^2 f_{B_s}^2 \tilde{B}'_{S,B_s}(\mu_2). \tag{13}$$

Notice that although we employ lattice QCD results for the bag parameters B_{B_s} and $\tilde{B}'_{S,B_s}$, recent determinations [34] based on QCD/HQET sum rules [35–41], would have been an equally valid choice. Our numerical values for the input parameters are listed in Table 2. We set the renormalization scale μ_1 of the $\Delta B = 1$ theory to $m_b(m_b)$ or m_b^{pole} in the $\overline{\text{MS}}$ and pole schemes respectively. The $\Delta B = 2$ renormalization scale μ_2 is, however, always fixed to m_b^{pole}. The evolution of the $\Delta B = 1$ matching coefficients C_i from the high scale $\mu_0 = 165$ GeV $\approx m_t(m_t)$ to μ_1 is done using a private

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Input parameter	Numerical value
$\alpha_s(M_Z)$	0.1179 ± 0.001 [42]
$m_c(3\text{ GeV})$	0.993 ± 0.008 GeV [43]
$m_b(m_b)$	4.163 ± 0.016 GeV [43]
m_t^{pole}	172.9 ± 0.4 GeV [42]
M_{B_s}	5366.88 MeV [42]
B_{B_s}	0.813 ± 0.034 [44]
$\tilde{B}'_{S,B_s}$	1.31 ± 0.09 [44]
f_{B_s}	0.2307 ± 0.0013 GeV [45]
λ_u^s/λ_t^s	$-(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i$ [14]

Table 2: Numerical input parameters. We have $\bar{z} = 0.04974 \pm 0.00092$, while m_t^{pole} is obtained from $m_t(m_t) = (163.1 \pm 0.4)$ GeV in the $\overline{\text{MS}}$ scheme. The bag parameters B_{B_s} and $\tilde{B}'_{S,B_s}$ are understood to be evaluated at the scale $\mu_2 = m_b^{\text{pole}} = 4.56$ GeV.

code based on the results published in [46, 47]. We also include the effect of LO $1/m_b$-corrections to $\Delta\Gamma_s$ from [32].

Our results for the ratio $\Delta\Gamma_s/\Delta M_s$ using NLO analytic results for M_{12}^s from [48] read

$$\begin{aligned}
\frac{\Delta\Gamma_s}{\Delta M_s} &= (4.70^{+0.32}_{-0.70}{}_{\text{scale}} \pm 0.12_{B\tilde{B}_S} \pm 0.80_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\text{pole})\,, \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= (5.20^{+0.01}_{-0.16}{}_{\text{scale}} \pm 0.12_{B\tilde{B}_S} \pm 0.67_{1/m_b} \pm 0.06_{\text{input}}) \times 10^{-3} \quad (\overline{\text{MS}})\,,
\end{aligned} \tag{14}$$

where the uncertainties are related to the variations of μ_1("scale"), bag parameters ("$B\tilde{B}_S$"), matrix elements of the power-suppressed corrections ("$1/m_b$") as well as $\alpha_s(m_Z)$, $m_b(m_b)$, $m_c(3\text{ GeV})$, m_t^{pole} and the CKM parameters ("input"). As one can see, the $1/m_b$ corrections currently constitute the largest source of uncertainties, which necessitates the calculation of the corresponding contributions at NLO in α_s. The scale uncertainties are equally nonnegligible, yet the final word on their importance can be said only upon incorporating the three-loop current-current contributions into our theoretical predictions.

As far as a_{fs}^s is concerned, we find

$$\begin{aligned}
a_{\text{fs}}^s &= (2.07^{+0.10}_{-0.11}{}_{\text{scale}} \pm 0.01_{B\tilde{B}_S} \pm 0.06_{1/m_b} \pm 0.06_{\text{input}}) \times 10^{-5} \quad (\text{pole})\,, \\
a_{\text{fs}}^s &= (2.02^{+0.15}_{-0.17}{}_{\text{scale}} \pm 0.01_{B\tilde{B}_S} \pm 0.05_{1/m_b} \pm 0.06_{\text{input}}) \times 10^{-5} \quad (\overline{\text{MS}})\,.
\end{aligned} \tag{15}$$

Last but not least, using the experimental value for ΔM_s [42],

$$\Delta M_s^{\text{exp}} = 17.7656 \pm 0.0057\ \text{ps}^{-1}\,, \tag{16}$$

we can finally arrive at the predictions for $\Delta\Gamma_s$

$$\begin{aligned}
\Delta\Gamma_s^{\text{pole}} &= (0.083^{+0.005}_{-0.012}{}_{\text{scale}} \pm 0.002_{B\tilde{B}_S} \pm 0.014_{1/m_b} \pm 0.001_{\text{input}})\ \text{ps}^{-1}\,, \\
\Delta\Gamma_s^{\overline{\text{MS}}} &= (0.092^{+0.0002}_{-0.003}{}_{\text{scale}} \pm 0.002_{B\tilde{B}_S} \pm 0.012_{1/m_b} \pm 0.001_{\text{input}})\ \text{ps}^{-1}\,,
\end{aligned} \tag{17}$$

which agree with the experimental values within the (still large) uncertainties.

Due to space limitations and the fact that the above results will be soon superseded by our upcoming NNLO predictions, we would like to abstain from discussing these numbers in further details. The interested reader is referred to Section 5 of [16] which offers a detailed analysis of the subject.

4. Summary

We reported on the theoretical status of QCD corrections to the width difference $\Delta\Gamma_s$ in $B_s - \bar{B}_s$ oscillations, where our main effort was concentrated on the reduction of scale uncertainties at leading order in the $\Lambda_{\mathrm{QCD}}/m_b$ expansion. The presented numerical results stem from our recent work [15, 16] where we computed all relevant matching coefficients at two-loop accuracy at $O(z)$ and provided analytic results in form of computer-readable expressions. The last missing ingredient to reach the NNLO accuracy for $\Delta\Gamma_s$ is the three-loop contribution from current-current operator insertions. We are currently in the progress of updating our numerical analysis to include this piece (again at $O(z)$) and expect to present preliminary numbers within the next months.

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