# PROCEEDINGS OF SCIENCE

# PoS

# Latest Measurements of the CKM angle $\phi_3$ at Belle

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Determination of the parameters of the Standard Model helps test the Cabibbo-Kobayashi-Maskawa (CKM) mechanism for *CP* violation asymmetry violation as well as search for new physics effects beyond the Standard Model (SM). The most recent results for the determination of  $\phi_3$  using the full Belle data set of 711 fb<sup>-1</sup> of integrated luminosity, produced at the KEKB accelerator complex with a centre-ofmass energy corresponding to the  $\Upsilon(4S)$  resonance of  $\sqrt{s} = 10.58$  GeV are summarized in this proceeding.

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#### 1. Introduction

The measurement of the parameters of the Standard Model is essential since it not only validates the Standard Model but also helps in the search for New Physics effects. The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] gives an excellent description of the current experimental measurements of charge-conjugation-parity (CP) violation. In the CKM matrix, out of the three angles of the CKM unitarity triangle (UT),  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$ ,  $\phi_3$  angle is the one that ihas the least precision [2]. It is the only CP-violating parameter that describes the UT that can be measured solely in tree-level processes. The determination of  $\phi_3$  will thus provide precision measurements of this UT angle. Currently, the experimental precision on  $\phi_3$  is of the order of 4° [3]. This proceeding summarizes some of the latest Belle results that use the full Belle data set corresponding to an intergrated luminosity of 711  $fb^{-1}$ , collected over a decade-long period of successul operation from 1999 to 2010.

#### 2. Latest Results from Belle

# **2.1** $\bar{B^0} \to D^+ h^- (h = K, \pi)$

The first analysis presented is the latest Belle measurement of the branching fraction of the Cabibbo favoured (CF)  $\bar{B^0} \to D^+\pi^-$  and the Cabibbo suppressed(CS)  $\bar{B^0} \to D^+K^-$  using the full Belle data set with data analysis performed using the Belle II software framework [4]. This measurement is very interesting due to two-fold reasons. Firstly, understanding these two-body heavy-light *B* decays yield a vital input for the ratio of the fragmentation fractions of  $B_s^0$  and  $B_d^0$  mesons,  $f_s/f_d$ , which in turn is a crucial input for the rare  $B_s \to \mu^+\mu^-$  decays. Secondly, the measurement of the branching fractions of these decays can address the reason behind the observed tension between the theoretical predictions and experimental observations of the branching fractions of these modes. The ratio of the branching fractions of the CS  $\bar{B^0} \to D^+K^-$  and the CF  $\bar{B^0} \to D^+\pi^-$  modes  $(R^D)$  is related to the physics parameters of interest, namely Cabibbo angle  $\theta_C$ , and meson decay constants,  $f_K$  and  $f_{\pi}$ :  $R^D = \frac{B(\bar{B^0} \to D^+K^-)}{B(\bar{B^0} \to D^+\pi^-)} \approx tan^2 \theta_C (f_K/f_{\pi})^2$ .

Although both these two-body hadronic decays have similar kinematic properties, since  $\bar{B^0} \rightarrow D^+\pi^-$  is a CF mode, it has a higher yield as compared to  $\bar{B^0} \rightarrow D^+K^-$ . Hence, this mode is chosen to finalise the event selection criteria. As is common in most Belle analyses, the kinematic variables used in this analysis are the beam-energy-constrained-mass,  $M_{bc} = \sqrt{(E_{beam}^2 - p_B^2)}$ , and the energy difference,  $\Delta E = E_B - E_{beam}$ , where  $p_B$  is the momentum of the B candidate,  $E_B$  is the energy of the B candidate and  $E_{beam}$  is the beam energy. The cross-feed from  $\bar{B^0} \rightarrow D^+\pi^-$  decays in the  $\bar{B^0} \rightarrow D^+K^-$  sample, where the pion is mis-identified as kaon is determined using a simultaneous fit to the samples enriched in prompt tracks that are identified as either pions or kaons.

Backgrounds are classified as continuum  $q\bar{q}$  (q = u, d, s, c) background, combinatorial  $B\bar{B}$  background and the cross-feed backgrounds from  $\bar{B^0} \to D^+h^-$ , ( $h = K, \pi$ ). An unbinned maximumlikelihood fit is performed to the  $\Delta E$  distribution simultaneously in pion and kaon enriched samples. The total  $D^+\pi^-$  yield from the simulatneous fit (Fig. 1) is used to determine the branching fraction of the  $\bar{B^0} \to D^+\pi^-$  decay.



**Figure 1:**  $\Delta E$  distributions for candidates obtained from the (left) pion-enriched  $\bar{B^0} \rightarrow D^+\pi^-$  and (right) kaon enriched  $\bar{B^0} \rightarrow D^+K^-$  data samples.

Belle measured the individual branching fractions of the modes are:  $BF(\bar{B^0} \rightarrow D^+\pi^-) = (2.48 \pm 0.01 \pm 0.09 \pm 0.04) \times 10^{-3} BF(\bar{B^0} \rightarrow D^+K^-) = (2.03 \pm 0.05 \pm 0.07 \pm 0.03) \times 10^{-4}$ , and the ratio of their branching fractions is  $R^D = (8.19 \pm 0.20 \pm 0.23) \times 10^{-2}$ . The recent result by Belle using the full data set of 711 fb<sup>-1</sup> shows that the individual branching fractions are lower than the theory predictions, however, the ratio agrees within uncertainties.

# **2.2** $\bar{B^0} \to D^{*+}h^-(h = K, \pi)$

The second analysis is based on two-body hadronic decays,  $\bar{B^0} \to D^{*+}h^-(h = K, \pi)$  that provide a unique test bed for QCD factorization test. Measurement of  $\bar{B^0} \to D^{*+}h^-$  along with the semi-leptonic decay rate  $d\Gamma(\bar{B^0} \to D^{*+}l^-\bar{\nu})/dq^2$  at a fixed lepton momentum transfer,  $q^2 = m_h^2$ helps in the determination of a physics parameter  $|a_1(q^2)| = |a_1(h)|$ , which is of prime importance in the hadronic B-decay studies. The determination of this parameter necessisated the measurement of both these modes by a single experiment. Using the full Belle dataset, the analysis results will soon be made public. This will be the first measurement of the branching fraction of  $BF(\bar{B^0} \to D^+K^-)$ and the branching fraction ratio,  $R^{K/\pi} = \frac{B(\bar{B^0} \to D^{*+}K^-)}{B(\bar{B^0} \to D^{*+}\pi^-)}$ .

**2.3** 
$$B^{\pm} \rightarrow D^0(K_s^0 \pi^+ \pi^-) K^{\pm}$$

The third analysis presented is the first measurement of  $\phi_3$  using a model-independent Dalitz plot method [5] at Belle using the full data set. In this analysis, measurements of the strong phase of the  $D^0 \rightarrow K_s^0 \pi^+ \pi^-$  amplitude is taken from the CLEO Collaboration. This measurement where Ddecays to a three-body final state dominates the current sensitivity of  $\phi_3$  measurement. The CKM angle  $\phi_3$  appears in the interference between  $b \rightarrow c\bar{u}s$  and  $b \rightarrow u\bar{c}s$  transitions. The difference in the Dalitz plots for the D mesons from  $B^+$  and  $B^-$  decays lead to the determination of  $\phi_3$ .

The amplitude of the  $B^{\pm} \to D^0(K_s^0 \pi^+ \pi^-) K^{\pm}$  is a superposition of  $B^+ \to \bar{D}^0 K^+$  and  $B^+ \to D^0 K^+$  amplitudes:  $A_B(m_+^2, m_-^2) = \bar{A} + r_B e^{i(\delta_B + \phi_3)} A$ . Here,  $m_+^2$  and  $m_-^2$  are the Dalitz plot variables, the squared invariant masses of  $K_s^0 \pi^+$  and  $K_s^0 \pi^-$  combinations respectively,  $A(\bar{A})$  is the amplitude of  $D^0 \to K_s^0 \pi^+ \pi^- (\bar{D}^0 \to K_s^0 \pi^+ \pi^-)$  decay,  $r_B$  is the ratio of the absolute values of the  $B^+ \to \bar{D}^0 K^+$  and  $B^- \to D^0 K^+$  amplitudes, and  $\delta_B$  is the strong-phase difference between them. In the case of CP conservation in the D decay, both A and  $\bar{A}$  are equal. The Dalitz plot density of the D decay



**Figure 2:** Results of the fit to the  $B^{\pm} \to DK^{\pm}$  sample (a) Signal yield in bins of the  $D \to K_s^0 \pi^+ \pi^-$  Dalitz plot: from  $B^- \to DK^-$  (red) and  $B^+ \to DK^+$  (blue) and flavor sample (histogram). (b) Difference of signal yields between the two decays.

from  $B^+ \to DK^+$  depends on the functions of the the cosine and sine of the strong-phase difference between the  $D^0 K_s^0 \pi^+ \pi^-$  and  $\overline{D^0} K_s^0 \pi^+ \pi^-$  amplitudes, which are denoted as  $c_i$  and  $s_i$  respectively.

In this model-independent approach, the Dalitz plot is divided into 2N bins symmetric under the exchange of the Dalitz plot variables. The bin index *i* ranges -N to +N (excluding zero) and the exchange of  $m_+^2$  and  $m_-^2$  corresponds to the exchange *i* to -i and vice-versa. CLEO measured the strong phase parameters using N = 8. Hence, this analysis uses 16 bins. The data fits for both are performed in two ways, one with separate fits for the number of events in bins and the other as a combined fit. In the former method, at first the fit to all events in the Dalitz plot is performed, followed by a 4-dimensional fit in each bin. Significantly different signal yields in the bins of  $B^+$ and  $B^-$  data is observed in the  $B \rightarrow DK$  sample (Fig. 2). Since probability to obtain this difference as a result of a statistical fluctuation is 0.42%, this value can be taken as the model-independent measure of the CP violation significance.

The analysis yields  $\phi_3 = 77.3^{+15.1}_{-14.9} \pm 4.1 \pm 4.3^\circ$ , and, the suppressed amplitude ratio  $r_B = 0.145 \pm 0.030 \pm 0.010 \pm 0.011$ . Here the first error is statistical, the second is the experimental systematic uncertainty, and the third is the error due to the precision of the strong-phase parameters obtained by CLEO. The latest Belle result using  $B \rightarrow D^*K$  using 605 fb<sup>-1</sup> data yields:  $\phi_3 = 78.4^{+10.8}_{-11.6} \pm 3.6 \pm 8.9^\circ$ . The combined Belle and Belle II analysis result using  $B^+ \rightarrow D^0(K_s^0 h^+ h^-)h^\pm$ , where  $h = K, \pi$  are presented recently [6].

## **2.4** $B^- \to D(K^+\pi^-\pi^0)K^-$

The fourth analysis was based on the determination of  $\phi_3$  by the Atwood–Dunietz–Soni (ADS) method in the suppressed decay  $B^- \to D(K^+\pi^-\pi^0)K^-$ , where *D* could be either  $D^0$  or  $\bar{D^0}$  using full Belle dataset of 711 fb<sup>-1</sup> [7]. Unlike previous ADS measurements that have used  $D \to K^+\pi^-$  subdecay mode, this analysis uses the  $D^0 \to K^+\pi^-\pi^0$  decay mode that has higher branching fraction. Inspite of having a reduced acceptance due to the presence of a  $\pi^0$  in the final state, this mode is more sensitive to the determination of  $\phi_3$ . This is the first analysis that uses  $B^- \to D^0K^-$ , where the favored  $B^- \to D^0K^-$  decay followed by the doubly Cabibbo-suppressed (DCS)  $D^0 \to K^+\pi^-\pi^0$ decay interferes with the suppressed  $B^- \to \bar{D^0}K^-$  decay followed by the Cabibbo-favored (CF)  $\bar{D^0} \to K^+\pi^-\pi^0$  decay. The physics observables obtained using this analysis are the direct CP asymmetry between the suppressed  $B^-$  and  $B^+$  decays,  $A_{DK}$ , the ratio of branching fractions of the above suppressed decay to the favored decay,  $R_{DK}$ , and the ratio of DCS and CF D decays,  $r_D$ .



**Figure 3:**  $\Delta E$  projections of the fits for the suppressed (two leftmost) and favored (two rightmost) *Dh* modes.

The neutral pion candidates are reconstructed from from photon pairs that have an invariant mass within a mass range that corresponds to approximately  $3.2\sigma$  in resolution around the nominal  $\pi^0$  mass. The  $\pi^0$  candidate along with a pair of oppositely charged tracks, with desired particle identification condition for kaon or pion are used to reconstruct a neutral *D* candidate. The invariant mass of the reconstructed *D* candidates is required to lie in the mass range that corresponds to approximately  $2.5\sigma$  in resolution around the nominal  $D^0$  mass with a mass-constrained fit. Finally, the *B* candidate is reconstructed using the *D* candidate and charged track, with appropriate selection. To suppress charm background, a cut on the mass difference between the  $D^*$  and *D* candidates ( $\Delta M$ ) is applied. A veto is applied to reduce the  $B^- \rightarrow D(K^-\pi^+\pi^0)K^-$  peaking background contribution. A neural network based variable  $(C'_{NB})$  is used to suppress continuum background using nine discriminating variables.

The signal yield is extracted using an unbinned extended maximum likelihood fit to  $\Delta E$  and the neural network output  $C'_{NB}$  distributions. Fig. 3 shows the projections of the fits in  $\Delta E$  variable for the suppressed and favored Dh,  $h = Kor\pi$  modes. The signal peaks for the suppressed DK, and  $D\pi$  are visible.

This analysis reports  $A_{DK} = 0.41 \pm 0.30 \pm 0.05$ , and  $R_{DK} = (1.98 \pm 0.62 \pm 0.24) \times 10^{-2}$ . This was the first evidence of the signal for this suppressed decay with a significance of 3.2  $\sigma$ . This analysis yielded the first measurements of  $A_{DK}$ ,  $R_{D\pi}$ , and  $A_{D\pi}$ . These results can be used to constrain the  $\phi_3$  measurement using the ADS method.

# **2.5** $B^0 \to D^0(K\pi)K^{*0}(K\pi)$

The fifth analysis is about the  $\phi_3$  measurement at Belle using the neutral B meson decay,  $B^0 \rightarrow D^0(K\pi)K^{*0}(K\pi)$ .  $\phi_3$  measurement has been done usually using the charged B meson decays. However, this analysis uses the neutral B meson decays. The analysis is done using full Belle dataset of 711  $fb^{-1}$  [8].

The analysis yields  $R_{DK^{*0}} = \Gamma(B^0 \to [K^-\pi^+]_D K^+\pi^-) / \Gamma(B^0 \to [K^+\pi^-]_D K^+\pi^-) = 4.5^{+5.6+2.8}_{-5.0-1.8} \times 10^{-2}$ . Since the value is not significant, credible upper limt of  $R_{DK^{*0}} < 0.16$  at 95% confidence level was set.  $R_{DK^{*0}}$  can be used to extract  $\phi_3$  by combining with other observables related to the dynamical parameters,  $r_B^{DK}$ , and,  $\delta_B^{DK}$ .

**2.6** 
$$B^0 \to D(K_s^0 \pi^+ \pi^-) K^{*0}$$

The sixth analysis  $B^0 \to D(K_s^0 \pi^+ \pi^-) K^{*0}$  analysis was the first model-independent Dalitz analysis of using full Belle dataset of 711 fb<sup>-1</sup> [9]. Here  $K^{*0}$  refers to  $K^{*0}(892)$  and D refers to either  $D^0$  or  $\overline{D^0}$  when the  $D^0$  flavor is untagged. The flavor of the B meson is identified by the kaon charge. Similar Dalitz analysis procedure as in Sec. 2.3 is followed. The physics observables of interest is r, which is defined as the ratio of the absolute values of the  $B^0 \rightarrow D^0 K^+ \pi^-$  and  $B^0 \rightarrow \overline{D^0} K^+$  amplitudes. If the  $B^0$  decay can be considered as a  $DK^{*0}$  two-body decay,  $r_s$  becomes unity. This analysis yields  $r_s^2 = \Gamma(B^0 \rightarrow D^0 K^+ \pi^-)/\Gamma(B^0 \rightarrow \overline{D^0} K^+ \pi^-)$  to be < 0.87 at 68% confidence level.

**2.7**  $B^{\pm} \rightarrow D^0 (K^0_{\rm S} \pi^+ \pi^- \pi^0) K^{\pm}$ 

The last measurement presented is the first  $\phi_3$  result using  $B^{\pm} \to D^0 (K_S^0 \pi^+ \pi^- \pi^0) K^{\pm}$  decay with full Belle dataset of 711 fb<sup>-1</sup> [10]. The inputs for the measurements of the strong-phase difference of the  $D^0 \to K_S^0 \pi^+ \pi^- \pi^0$  decays  $(c_i, s_i)$  are taken from the CLEO-c [11], that uses 0.82 fb<sup>-1</sup> data collected at the  $\psi(3770)$  resonance. Several resonance substructures are seen. The binning of the phase space around these resonances in the absence of an amplitude model is done. The analysis yields the following results:  $\phi_3 = 5.7^{+10.2}_{-8.8} \pm 3.5 \pm 5.7^\circ$ ,  $r_B = 0.323 \pm 0.147 \pm 0.023 \pm 0.051$ , and,  $\delta_B = 83.4^{+18.3}_{-16.6} \pm 3.1 \pm 4.0^\circ$ . The 95% confidence level on  $\phi_3$  interval (-29.7, 109.5)° is consistent with the current world average.

This measurement can be improved upon once a suitable amplitude model for is available to provide guidance in choosing a more sensitive binning. Precise inputs for  $c_i$ ,  $s_i$  from BESIII will also help reduce the systematic uncertainty. Single-mode uncertainty on  $\phi_3$  is achievable with a 50 ab<sup>-1</sup> sample of data at Belle II experiment. Using pseudo experiments with  $c_i$ ,  $s_i$  as inputs with each experiment consisting of about 60,000 events and the input values of  $\phi_3$  and the hadronic parameters  $r_B$  and  $\delta_B$  using Ref. [12], the estimated uncertainty on  $\phi_3$  is found to be about 4.4°. The measurement precision can improve to about  $1 - 2^\circ$  with 50 ab<sup>-1</sup> data, that can be collected at Belle II.

## 3. Summary and Outlook

Precision measurement of  $\phi_3$  is important to establish CP violation in Standard Model. Latest measurements by Belle have strong impact in improving  $\phi_3$  precision. Several results are already available with the full Belle dataset. Belle II and LHCb will be major players in further improving  $\phi_3$  precision in future. Measurement precision of  $\phi_3$  will improve greatly with higher data statistics that can be collected at Belle II.

### References

- N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
- [2] J. Brod and J. Zupan, J. High. Energ. Phys. 051, 1401 (2014).
- [3] Y. Amhis et al, Heavy Flavor Averaging Group (HFLAV), Eur. Phys. J. C (2021) 81: 226, arXiv:1909.12524.
- [4] E. Waheed et al. (Belle Collaboration), Phys. Rev. D 105, 012003 (2021).

- [5] H. Aihara et al. (Belle Collaboration), Phys. Rev. D 85, 112014 (2012).
- [6] F. Abudinén, et. al (Belle and Belle II Collaboration), JHEP02, 063 (2022).
- [7] M. Nayak et al. (Belle Collaboration), Phys. Rev. D 88, 091104(R) (2013).
- [8] K. Negishi et al. (Belle Collaboration), Phys. Rev. D 86, 011101(R) (2012).
- [9] K. Negishi et al. (Belle Collaboration), PTEP, 043C01 (2016).
- [10] P. K. Resmi, J. Libby, K. Trabelsi et al. (Belle Collaboration), JHEP10, 178 (2019).
- [11] P.K. Resmi, J. Libby, S. Maldeb and G. Wilkins, JHEP01, 82 (2018).
- [12] CKMfitter group webpage, http://ckmfitter.in2p3.fr.