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New ideas in γ/ϕ_3 measurements

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I review the possible measurements that can be used to better constrain the CKM unitarity triangle parameter γ (also known as ϕ_3). These include: the approach to perform unbinned model-independent Dalitz plot analysis of the $B \to DK$ transitions followed by $D \to K^0_S\pi^+\pi^-$ decay; double Dalitz plot analysis of $B \to DK\pi$, $D \to K^0_S\pi^+\pi^-$ decays; measurements with four-body D meson decays, and quantum-correlated analysis of $\chi_{c1}(3872) \to D\overline{D}$ decays.

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1. Introduction

The parameter γ (also known as ϕ_3) of the Unitarity Triangle is the complex phase of the Cabibbo-Kobayashi-Maskawa matrix [1] that is solely responsible for the CP violation in the Standard Model of electroweak interactions. Experimentally, γ is accessible in tree-level transitions where the weak diagrams involving $b \to c$ and $b \to u$ transitions interfere. The most theoretically clean observables sensitive to γ are CP-violating rates of decays of b hadrons to open-charm final states. All the hadronic unknowns in these decays can be obtained from the experiment, which results in extremely low theory uncertainties on γ measurement below 10^{-7} [2].

The value of γ is constrained in the combination of many measurements: time-integrated asymmetries of $B \to DX$ decays with different D meson final states, from the amplitude analyses of multibody D decays coming from $B \to DX$ transitions, as well as from time-dependent decay rates of the decays such as $B^0_s \to D_s K$. In the following I will only consider time-integrated measurements. In that case, the rate Γ_+ of the $B \to DX$, $D \to f$ decay chain and its CP-conjugate counterpart Γ_- can be written as

$$\Gamma_\pm \propto r_B^2 + r_D^2 + 2\kappa r_B r_D \cos(\delta_B - \delta_D \pm \gamma), \tag{1}$$

where r_B and δ_B are the ratio and the strong phase difference between the $b \to u$ and $b \to c$ amplitudes, r_D and δ_D are the ratio and strong phase difference between the $D^0 \to f$ and $\overline{D}^0 \to f$ amplitudes, and $0 \leq \kappa \leq 1$ is the coherence factor that takes into account the variations of the amplitudes in the case of multibody B or D decays ($\kappa \equiv 1$ for two-body decays).

The parameters $r_{B,D}$, $\delta_{B,D}$ and, for multibody final states, κ are in general unknown. It is important, however, that r_B and δ_B do not depend on the $D \to f$ decay, while r_D, δ_D are independent of the $B \to DX$ decay. Because of such factorisation, when multiple B and D transitions are considered, one has more observables ($\Gamma_\pm$ rates) than unknowns, and thus all the unknown parameters including γ can be obtained from the experiment. In the case of multibody decays, different kinematic regions can be treated as independent decay modes, which increases the number of observables. Careful choice of kinematic regions can also improve coherence (maximise the coherence factor κ) and, thus, the precision on γ.

Although the precision of γ measurement has improved significantly in recent years, there are still resources to improve our knowledge even with the data that is available today. Since the measurement is basically free from theory uncertainties, any experimental improvement immediately pays off. It is important to perform the analysis with potential sensitivity to γ in as many modes as possible for two reasons. First, since many of the hadronic uncertainties are shared between different decay modes, the precision of the combined measurement can be higher than the plain average of independent measurements of γ. Second, independent measurements allow for better control of systematic uncertainties, making the measurement more robust. In addition to adding new modes, it is interesting to consider the ways to improve current measurements by either applying statistically more optimal techniques, or better controlling the systematic uncertainties.

Here, I will review several approaches to improve our knowledge of γ that have not yet materialise in real experimental analyses.

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2. Unbinned model-independent measurements with $D \to K^0_S \pi^+\pi^-$ decays

The analysis of $B \to DK$ decays with $D \to K^0_S \pi^+\pi^-$ [3] is one of the methods that dominate γ sensitivity and allows for determination of γ without ambiguities. The value of γ is obtained from the analysis of three-body $D \to K^0_S \pi^+\pi^-$ amplitude which, as a function of two Dalitz plot variables $m^2_+ \equiv m^2(K^0_S\pi^+)$ and $m^2_- \equiv m^2(K^0_S\pi^-)$, equals

$$A_B(m^2_+, m^2_-) = A_D(m^2_+, m^2_-) + r_B e^{i\delta_B + i\gamma} A_D(m^2_-, m^2_+), \tag{2}$$

where $A_D(m^2_+, m^2_-)$ is the amplitude of the flavour-specific $D^0 \to K^0_S \pi^+\pi^-$ decay. Experiment is only sensitive to the magnitudes $|A_{B,D}|^2$ of the amplitudes, however, in order to obtain γ from Eq. 2 one needs to also know the phase difference between $A_D(m^2_+, m^2_-)$ and $A_D(m^2_-, m^2_+)$. The fit of the isobar model to the $D^0 \to K^0_S \pi^+\pi^-$ data can provide this information, but it results in the model uncertainty which is hard to evaluate. Therefore, recent analyses use model-independent approach with binned D decay phase space.

In the binned approach, the measurement of γ is performed by solving a system of equations that relates the yields in $D \to K^0_S \pi^+\pi^-$ bins from B decay and from flavour-tagged D^0 decays. The strong phase variations are encoded in the coefficients c_i and s_i (cosine and sine of the phase difference between D^0 and $\overline{D}^0$ decays averaged over the i-th bin area) which are obtained from the decays of quantum-correlated pairs of D^0 mesons produced in e^+e^- collisions near threshold [4]. Binned approach, however, is limiting the statistical precision of the measurement. Variations of the A_D amplitude over the kinematic space of the bin will reduce coherence between the amplitudes. Although the procedure to choose the optimal binning that maximises the statistical precision has been proposed [5], it has only 80–90% statistical power compared to the unbinned technique.

It is possible, however, to generalise the binned approach to make it more statistically efficient [6]. Instead of applying the binning to the magnitudes $|A_{B,D}|^2$, *i.e.* splitting them into independent kinematic regions and integrating over them, one can instead perform *weighted integrals* of the decay density with the series of weight functions $w_i(m^2_+, m^2_-)$:

$$a_i = \int_{\mathcal{D}} |A(m^2_+, m^2_-)|^2 \, w_i(m^2_+, m^2_-) \, dm^2_+ \, dm^2_-, \tag{3}$$

where A is the decay amplitude of the D meson in either a flavour eigenstate or from the B decay, and $\mathcal{D}$ is the full kinematic phase space of the decay. The coefficients a_i can then be connected by a similar system of equations as the one that connects the yields in bins for the binned model-independent approach. In the case of scattered data, the integral in Eq. 3 is replaced by a sum of weights over the events in the sample.

The flexibility in the choice of the weighting functions w_i allows one to reach better statistical sensitivity of the model-independent measurement. It was shown in Ref. [6] that the series of Fourier harmonics $w_{2n} = \sin(n\,\Delta\delta_D(m^2_+, m^2_-))$, $w_{2n+1} = \cos(n\,\Delta\delta_D(m^2_+, m^2_-))$ (where $n = 0, 1, \ldots$ and $\Delta\delta_D = \arg A_D(m^2_+, m^2_-) - \arg A_D(m^-_2, m^2_+))$ with already three terms $w_{1,2,3}$ provides the sensitivity comparable to the binned approach with the optimal binning. Further splitting of the kinematic space into two regions, where $|A_D(m^2_+, m^2_-)|$ is grater or less than $|A_D(m^2_+, m^2_-)|$, gives the sensitivity better than the binned approach. However, it is still worse than the unbinned model-dependent measurement.

It should be possible, however, to construct the model-independent fit procedure using the weighed integrals in such a way as to ensure the statistical sensitivity equivalent to the unbinned model-dependent fit. To achieve that, one has to choose the system of weighting functions w_i such that it spans completely the decay density $p_B = |A_B(m_+^2, m_-^2)|^2$ for any values of the unknowns r_B, γ and δ_B. Since, as follows from Eq. 2,

$$p_B \propto p_D + r_B^2 \bar{p}_D + 2(xC + yS), \tag{4}$$

the system of weight functions w_i can be obtained by the result of orthogonalisation of four functions, $p_D = |A_D(m_+^2, m_-^2)|^2$, $\bar{p}_D = |A_D(m_-^2, m_+^2)|^2$, $C = \sqrt{p_D \bar{p}_D} \cos \Delta\delta_D$, $S = \sqrt{p_D \bar{p}_D} \sin \Delta\delta_D$.

As in the case of the binned technique, the approach with weighting functions offers the optimal sensitivity if the amplitude A_D used to define the weights w_i matches the true one. If that is not the case, the sensitivity will deteriorate, while the measurement will still be unbiased. In the weighted formalism one can further improve this by choosing the set of more than four functions corresponding to alternative amplitude models; in that case the optimal precision can be reached if the true model is either of the ones used to define the weights, or is their linear combination.

The details of the γ measurements procedure with weight functions are still under development and are worked on as a part of inter-collaboration effort between BES-III, LHCb and Belle II collaborations.

3. Double Dalitz plot analysis

Open charm decays of neutral B mesons are promising from the point of view of γ measurement since the interference term for them is expected to be larger, with $r_B \simeq 0.3$, since both interfering amplitudes are colour-suppressed. Three-body $B^0 \to DK\pi$ decays have been proposed as a tool to measure γ via the simultaneous Dalitz plot analysis of decays with neutral D mesons reconstructed in (quasi) flavour-specific decay like $K^-\pi^+$, and in the CP-eigenstate such as K^+K^- or $\pi^+\pi^-$ [7].

To make this measurement in a model-independent fashion, the procedure that adds three-body $D \to K_S^0\pi^+\pi^-$ decays to the technique mentioned above was proposed [8]. In that case, the decay densities over the three-body $B \to DK\pi$ and $D \to K_S^0\pi^+\pi^-$ decays become correlated, with the amplitude of such decay being

$$A_{\text{dblDlz}} = \overline{A}_B \overline{A}_D + e^{i\gamma} A_B A_D, \tag{5}$$

where $A_B(\overline{A}_B)$ are the decay amplitudes of the $B^0 \to D^0 K^+\pi^-$ ($B \to \overline{D}^0 K^+\pi^-$), and $A_D(\overline{A}_D)$ are the amplitudes of $D^0 \to K_S^0\pi^+\pi^-$ ($\overline{D}^0 \to K_S^0\pi^+\pi^-$) decays.

After binning is applied to both the B and D Dalitz plots, one arrives to the system of equations relating the yields in bins similar to the one for the $B \to DK$ decays [5]. The coefficients containing the unknown phase difference between the $B \to D^0 K\pi$ and $B \to \overline{D}^0 K\pi$ amplitudes are treated as the free parameters. The binning of both the D and B decay Dalitz plots can be optimised based on the models of the decay amplitudes to obtain the best statistical power; the $D \to K_S^0\pi^+\pi^-$ binning in that case is the same as for the $B \to DK$ decays.

In the more recent study [9] this conceptual idea was further investigated using realistic amplitude models of the B decays and background contributions constrained by the available

LHCb data [10]. In particular, it was shown that the analysis can be performed using only three categories of events: $B^0 \to DK^+\pi^-$ with $D \to K^+\pi^-$, $D \to K^+K^-(\pi^+\pi^-)$ and $D \to K^0_S\pi^+\pi^-$ (plus corresponding CP-conjugate modes), and without the use of the suppressed $D \to K^-\pi^+$ decays. The latter, in the LHCb case, are strongly affected by the large $B^0_s \to D^*K\pi$ background and will be practically unusable. The study shows that even in the presence of backgrounds, the measurement of γ is possible with the precision of $(5-12)^\circ$ with the currently available LHCb dataset (Runs 1 and 2) and $(1.5-2.5)^\circ$ with the dataset expected after the first upgrade corresponding to 50 fb^{-1}.

4. Measurements with four-body D meson decays

Multibody (4-body and more) decay modes constitute a large fraction of the D^0 decay rate, and thus can add significantly to the precision of γ measurement. Such analyses can be made in the phase-space integrated way, by introducing a coherence factor (see Sec. 1) to account for amplitude variations as a function of kinematic degrees of freedom. However, a more optimal way would be to take these variations into account in the binned model-independent analysis. The complication here is that the phase space for four-body decay has five dimensions, and binning optimisation in the 5D phase space requires a complex amplitude model.

There are a few analyses where the quantum-correlated measurement from CLEO-c experiment are used to constrain the values of strong-phase coefficients c_i and s_i in four-body decays. One example is the analysis of $D^0 \to K^0_S\pi^+\pi^-\pi^0$ decays [11]. This analysis does not use an amplitude model for binning optimisation (with bins being defined as regions around the known resonances). The analysis provides the expected sensitivity with Belle II data sample (50 ab^{-1}) which is estimated to be $\sigma(\gamma) = 4.4^\circ$. Another kind of decay is $D^0 \to 4\pi$ which, being fully charged final state, is more promising for LHCb [12].

5. Quantum correlations in $\chi_{c1}(3872) \to D^0\overline{D}^0$ decays

As discussed in Sec. 2, quantum-correlated pairs of neutral D mesons produced at threshold are essential to constrain the strong phase difference between the D^0 and $\overline{D}^0$ amplitudes. Presently these measurements are done at e^+e^- machines operated at the $\psi(3770)$ resonance [4, 13] decaying into a pair of D mesons without additional particles. Since the initial state has charge parity $C = -1$, the wave function of the two D mesons is antisymmetric. This measurement, however, cannot be done in hadronic environment because the broad $\psi(3770)$ state is difficult to extract in presence of combinatoric background. Recently, the state $\chi_{c1}(3872)$ (also known as $X(3872)$) was suggested as a promising source of quantum-correlated D meson pairs [14].

The mass of $\chi_{c1}(3872)$ equals to the sum of D^0 and D^{*0} masses within the experimental precision, and the decay to $D^0\overline{D}^{*0}$ has the largest branching fraction of all its decay modes. The $\chi_{c1}(3872)$ state is thus clearly visible in the $D^0\overline{D}^0$ invariant mass spectrum in LHCb data at the very threshold, even without explicit reconstruction of the γ or π^0 from the D^{*0} decay [15].

The quantum numbers of $\chi_{c1}(3872)$ have been established to be $J^{PC} = 1^{++}$ [16]. The form of the wave function of the $D^0\overline{D}^0$ pair in the $\chi_{c1}(3872)$ decay is thus dependent on the D^{*0} decay mode: it is symmetric ($C = +1$) for the $D^0\overline{D}^0\pi^0$ decay, and antisymmetric ($C = -1$) for the $D^0\overline{D}^0\gamma$ decay. Ref. [14] shows that it is possible to statistically separate the two decay modes without reconstructing the soft neutral particle (which is difficult at LHCb) and suggests the kinematic variables to do this, such as the norm of the difference of 4-momenta of the two D mesons. With the data from the upgraded phase of LHCb this technique can offer a considerable statistics to add to the precision of D^0 strong phase determination.

6. Conclusion

I reviewed a few new techniques that could help to improve the experimental precision of the determination of the CKM phase γ. This is not an exhaustive list: a few other measurements sensitive to γ have been reviewed in other CKM2021 presentations, such as the open-charm decays of beauty baryons, or various time-dependent measurements of neutral B mesons.

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