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Higher order QCD corrections for neutral B-meson oscillations

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In the work present new contributions to the decay matrix element $\Gamma^{q}{}_{12}$ of the $\mathrm{B_q}$-$\mathrm{\overline{B}_q}$ mixing, where q = d or s. Our new results improves the prediction of the width difference $\Delta\Gamma_q$ between the two neutral-meson eigenstates.

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1. Introduction

Experiments in high energy physics today allow us to check with great accuracy the weak and the strong interaction theories and discover new physics beyond the standard model. In addition to the great accuracy experimental data from LHCb [1] and Belle II [2], more precise theoretical SM calculations are needed.

The mass differences ΔM_s = (17.757±0.021)ps−1 (ΔM_s= $M_{\mathrm{B_s}}$-$M_{\bar{\mathrm{B}}_s}$) and ΔM_d = (0.5064 ± 0.0019)ps−1 (ΔM_d= $M_{\mathrm{B_d}}$-$M_{\bar{\mathrm{B}}_d}$) [3, 4] have been determined very precisely by the CDF [5] and LHCb [6] experiments from the $\mathrm{B_q}$-$\bar{\mathrm{B}}_\mathrm{q}$ oscillation frequencies. The experimental values of the width differences ($\Delta\Gamma_q = \Gamma_L^q$-$\Gamma_H^q$) [4,5],

$\Delta\Gamma_s^{exp} = (8.9 \pm 0.6)*10^{-2}\mathrm{ps}^{-1}$ (1)

$\Delta\Gamma_d^{exp} = (+1.32 \pm 6.58)*10^{-3}\mathrm{ps}^{-1}$ (2)

are based on measurements by LHCb [5, 6], ATLAS [7], CMS [8], and CDF [9].

Clearly, $\Delta\Gamma_s$ is a precision observable, while the three other quantities are still far from giving precise information on fundamental parameters. For a $\Delta\Gamma_s$it is worthwhile to study the clean sample of B → $J/\psi K_s$ decays [10]. While new physics will primarily enter M_{12}^q, scenarios in which Γ_{12}^q is affected have been studied as well [11, 12], especially the doubly Cabibbo-suppressed Γ_{12}^d could play a role in new-physics studies.

The purpose of the present paper is to do the next step in the calculation of NNLO QCD corrections to Γ_{12}. We calculate the penguin contributions with full dependence on the charm quark mass. These terms constitute an improvement for the prediction of $\Delta\Gamma_q$ compared to Ref. [13].

Penguin contributions are small in the Standard Model, because the Wilson coefficients of the corresponding operators are small, of order 0.05 or smaller. However, this makes these coefficients sensitive to contributions of new physics, which can easily be of the same size [14] as the SM coefficients. Thus in order to study such effects beyond the SM a precise knowledge of the penguin contributions to Γ_{12}^q is desirable. Our results show an importance of charm quark mass in penguin sector for $\Delta\Gamma_s$.

2. Theoretical framework

The effective ΔB = 1 weak Hamiltonian, relevant for b → s transition, reads [15].

$$H_{eff}^{\Delta B=1} = -\frac{G_F}{\sqrt{2}}\{\lambda_t^s[\sum_{i=1}^{6} C_i O_i + C_8 O_8] - \lambda_u^s \sum_{i=1}^{2} C_i(O_i^u - O_i\}: \quad (3)$$

Where $\lambda_t^s = V_{ts}{}^* V_{tb}$, $\lambda_u^s = V_{us}{}^* V_{ub}$ comprises the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The dimension-six effective operators in Eq. (3) are

O_1^u=$(\bar{s}_i u_j)_{(V-A)}\,(\bar{u}_j b_i)_{(V-A)}$, $O_2^u = (\bar{s}_i u_i)_{(V-A)}(\bar{u}_j b_j)_{(V-A)}$,

$$O_1=(\bar{s}_i c_j)_{(V-A)}(\bar{c}_j b_i)_{(V-A)}, \qquad O_2=(\bar{s}_i c_i)_{(V-A)}(\bar{u}_j c_j)_{(V-A)},$$
$$O_3=(\bar{s}_i b_i)_{(V-A)}(\bar{q}_j q_j)_{(V-A)}, \qquad O_4=(\bar{s}_i b_j)_{(V-A)}(\bar{q}_j q_i)_{(V-A)}, \tag{4}$$
$$O_5=(\bar{s}_i b_i)_{(V-A)}(\bar{q}_j q_j)_{(V+A)}, \qquad O_6=(\bar{s}_i b_j)_{(V-A)}(\bar{q}_j q_i)_{(V+A)}),$$
$$O_8=\frac{g_s}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu}(1-\gamma_5) T^a_{ij} b_j G^a_{\mu\nu}$$

Here i, j are color indices and summation over q = u, d, s, c, b. $V\pm A=\gamma_\mu(1\pm\gamma_5)$.

$C_1,\ldots\ldots,C_6$, C_8 are the corresponding Wilson coefficients. G_F is the Fermi constant. Γ_{12} is calculated as follows. $\Gamma_{12}=\mathrm{Abs}\langle B_s|\int d^4x T H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0)|\bar{B}_s\rangle$ (5)

where 'Abs' denotes the absorptive part of the matrix element and T is the time ordering operator.

Γ_{12} we can present it in the following form[16]

$$\Gamma_{12}=-\lambda_t^2[\Gamma_{12}^{cc}+2\frac{\lambda_u}{\lambda_t}(\Gamma_{12}^{cc}-\Gamma_{12}^{uc})+\frac{\lambda_u^2}{\lambda_t^2}(\Gamma_{12}^{cc}+\Gamma_{12}^{uu}-2\Gamma_{12}^{uc}) \tag{6}$$

where the coefficients Γ_{12}^{ab} a, b = u, c are positive. The leading term (in powers of $\frac{\Lambda_{QCD}}{m_b}$) reads

$$\Gamma_{12}^{ab}=\frac{G_f^2 m_b^2}{24\pi M B_s}[G^{ab}\langle B_s|Q|\bar{B}_s\rangle - G_S^{ab}\langle B_s|Q_S|\bar{B}_s\rangle \tag{7}$$

Where $Q=(\bar{s}_i b_i)_{(V-A)}(\bar{s}_j b_j)_{(V-A)}$, $Q_S=(\bar{s}_i b_j)_{(S-P)}(\bar{s}_j b_i)_{(S-P)}$, $a_{(S-P)}=a(1-\gamma_5)$ (8)

3. Phenomenology

Here, in particular, we have observed diagrams in FIG.1. In general, quarks b, d, s, c, u can contribute to the loop. However, the contribution of the u, d, s quarks is the same, because the masses are very small. The diagrams shown in Fig.1 are counted in the Refs [16].

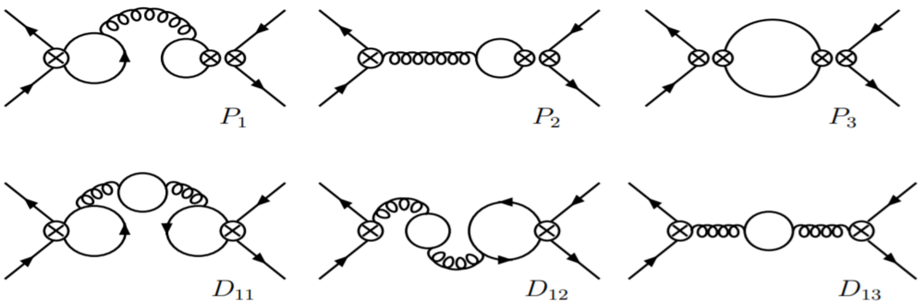

FIG. 1. Diagrams for the penguin contribution at $O(a_s{}^2 N_f)$. The small Wilson coefficients C_{3-6} are counted as $O(a_s)$. P_1, P_2 are diagrams with one insertion of a penguin operator O_3, ..., O_6, depicted as two circles with crosses, and one insertion of $O_2{}^{u,c}$or O_8, shown as a single circle with cross. P_3 denotes a one-loop diagram with two insertions of penguin operators O_3, ..., O_6. D_{11}, D_{12} and D_{13} are diagrams with insertions of operators $O_2{}^{u,c}$ or O_8. (The notation follows Ref. [13].)

We show the impact of non-zero charm quark mass, which enters via operator O_2 in the penguin sector.Using the NNLO $\Delta B = 1$ results of the Wilson Coefficients C_1,C_2[17,18], and the complete NLO expressions for C_{3-6}, C_8[13], values in Tab 1.

We obtain

$$\frac{\delta\Delta\Gamma^{1,p}{}_s(z)}{\Delta\Gamma^{NLO}{}_s(z)} = -14.5\% \text{ (pole)} \qquad \frac{\delta\Delta\Gamma^{1,p}{}_s(z)}{\Delta\Gamma^{NLO}{}_s(z)} = -11.2\% \ (\overline{MS}) \tag{9}$$

and the new $a_s^2 N_f$ corrections are

$$\frac{\delta\Delta\Gamma^{2,N_f\cdot p}{}_s(z)}{\Delta\Gamma^{NLO}{}_s(z)} = 2.4\% \text{ (pole)} \qquad \frac{\delta\Delta\Gamma^{2,N_f\cdot p}{}_s(z)}{\Delta\Gamma^{NLO}{}_s(z)} = 1.8\% \ (\overline{MS}) \tag{10}$$

where $\delta\Delta\Gamma^{1,p}{}_s(z)$ denotes the contribution to $\Delta\Gamma_s$ from the penguin sector at order αs and $\delta\Delta\Gamma^{2,N_f\cdot p}{}_s(z)$ is the corresponding contribution at order $a_s^2 N_f$

$\overline{m}_b\ (\overline{m}_b) = (4.18 \pm 0.03)$ GeV	[19]	$\overline{m}_c(\overline{m}_c) = (1.2982 \pm 0.0013_{stat} \pm 0.0120_{syst})$ GeV	[27,28,29]
$\overline{m}_s\ (\overline{m}_b) = (0.0786 \pm 0.0006)$ GeV	[20]	$\overline{m}_c(\overline{m}_c) = (165.26 \pm 0.11_{stat} \pm 0.30_{syst})$ GeV	[27]
$m_b^{pow} = 4.7$ GeV	[21]	$a_s(M_Z) = 0.1181(11)$	[22]
$M_{B_s} = 5366.88$ MeV	[22]	$M_{B_d} = 5279.64$ MeV	[22]
$B_{B_s} = 0.813 \pm 0.034$	[23]	$B_{B_d} = 0.806 \pm 0.041$	[23]
$\tilde{B}'_{S,B_s} = 1.31 \pm 0.09$	[23]	$\tilde{B}'_{S,B_d} = 1.20 \pm 0.09$	[23]
$B_{R_0}^s = 1.27 \pm 0.52$	[23]	$B_{R_0}^d = 1.02 \pm 0.55$	[23]
$B_{\tilde{R}_2}^s = 0.89 \pm 0.35$	[24]	$B_{\tilde{R}_2}^d = B_{\tilde{R}_2}^s$	
$B_{\tilde{R}_3}^s = 1.14 \pm 0.39$	[24]	$B_{\tilde{R}_3}^d =]B_{\tilde{R}_3}^s$	
$B_{R_2}^q = -B_{\tilde{R}_2}^q$	[25]	$B_{R_3}^q = \frac{5}{7} B_{\tilde{R}_3}^q + \frac{2}{7} B_{\tilde{R}_2}^q$	[25]
$f_{B_s} = (0.2307 \pm 0.0013)$ GeV	[26]	$f_{B_d} = (0.1905 \pm 0.0013)$ GeV	[26]
$\sin(2\beta) = 0.7083^{+0.0127}_{-0.0098}$	[27]	$R_t = 0.9124^{+0.0064}_{-0.0100}$	[27]
$\lvert V_{us}\rvert = 0.22483^{+0.00025}_{-0.00006}$	[27]		

TABLE I. Input parameters used in Sec. IV. $\overline{m}_s(\overline{m}_b)$ is calculated from $\overline{m}_s$ (2 GeV) = 0.09344 ± 0.00068 GeV [20]. The listed values for B_{B_q} and $\tilde{B}'_{S,B_q}$ are found by rescaling the numbers in Table V of Ref. [23] by 8/3 and 3, respectively (see Eq. (16)). m_b^{pow} is a redundant parameter calibrating the overall size of the hadronic parameters B_{R_i} which quantify the matrix elements at order Λ_{QCD}/m_b. $B_{R_0}^q$ is calculated from $\langle B_s|R_0|\bar{B}_s\rangle = -(0.66 \pm 0.27)\ GeV^4$ and $\langle B_d|R_0|\bar{B}_d\rangle = -(0.36 \pm 0.20)\ GeV^4$ [23] (with R_0 defined as in Ref. [21, 25]) with the central values of f_{B_s} and the quark and meson masses listed above, so that the error quoted for $B_{R_0}^q$ correctly reflects the error of only the matrix element (and not the uncertainty of the artificial conversion factor from matrix elements to bag parameters). In the same way $B_{\tilde{R}_{2,3}}^s$ is calculated from $\langle B_s|\tilde{R}_2|\bar{B}_s\rangle = (0.28 \pm 0.11)\ GeV^4$ and $\langle B_s|\tilde{R}_3|\bar{B}_s\rangle = (0.44 \pm 0.15)\ GeV^4$ [24]. The expressions for $B_{R_2}^q$ and $B_{R_3}^q$ hold up to Λ_{QCD}/m_b corrections. $B_{\tilde{R}_1}^q = 1.5$ and $B_{\tilde{R}_1}^s = 1.2$ [24] are phenomenologically irrelevant. The charm and bottom masses imply z = $m_c^2(m_c)/m_b^2(m_b) = 0.096$ leading to $\bar{z} = m_c^2(m_b)/m_b^2(m_b) = 0.052 \pm 0.002$ at NLO and we use the same value at NNLO.

Until the full NNLO calculation is available, we recommend to use the following updated NLO SM values for $\Delta\Gamma_q/\Delta M_q$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (4.33 \pm 0.83_{\text{scale}} \pm 0.11_{B,\widetilde{B_s}} \pm 0.94_{\Lambda_{QCD}/m_b}) \times 10^{-3} (\text{pole})$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (4.97 \pm 0.62_{\text{scale}} \pm 0.13_{B,\widetilde{B_s}} \pm 0.80_{\Lambda_{QCD}/m_b}) \times 10^{-3} (\overline{MS}) \quad (11)$$

$$\frac{\Delta\Gamma_d}{\Delta M_d} = (4.48 \pm 0.82_{\text{scale}} \pm 0.12_{B,\widetilde{B_s}} \pm 0.86_{\Lambda_{QCD}/m_b}) \times 10^{-3} (\text{pole})$$

$$\frac{\Delta\Gamma_d}{\Delta M_d} = (5.07 \pm 0.61_{\text{scale}} \pm 0.14_{B,\widetilde{B_s}} \pm 0.73_{\Lambda_{QCD}/m_b}) \times 10^{-3} (\overline{MS}) \quad (12)$$

4. Conclusion

In response to the recent progress in the lattice calculations of the non-perturbative matrix elements [23,24] we have further presented updated NLO values for $\Delta\Gamma_q$.

We have calculated the penguin contributions of order $a_s^2 N_f$ to the width difference $\Delta\Gamma_q$. $\Delta\Gamma_q$is a fundamental quantity characterizing the B_q-$\bar{B}_q$ mixing complex. $\Delta\Gamma_q$ terms have signs opposite to the NLO corrections. The calculated partial NNLO corrections are smaller than the corresponding NLO terms by factors of roughly 6 for $\Delta\Gamma_q$ respectively, indicating a good convergence of the perturbative series.

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References

[1] LHCb collaboration, A. A. Alves Jr. et al., *The LHCb detector at the LHC,* JINST 3 *(2008) S08005A. Cerri et al., Report from Working Group 4 : Opportunities in Flavour Physics at the HL-LHC and HE- LHC*, CERN Yellow Rep. Monogr. 7 (2019) 867 [`arXiv:1812.07638`].

[2] E. Kou et al. [Belle-II Collaboration et al.], *The Belle II Physics Book*, PTEP 2019, no.12, 123C01
(2019), Erratum: [PTEP 2020 (2020) no.2, 029201] [arXiv:1808.10567] .

[3] *Heavy Flavor Averaging Group (HFLAV)*,
`http://www.slac.stanford.edu/xorg/hfag/osc/summer 2017.`

[4] Y. Amhis et al. [HFLAV], Eur. Phys. J. C 77, no.12, 895 (2017) [arXiv:1612.07233].

[5] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 114 (2015) no.4, 041801 [arXiv:1411.3104].

[6] R. Aaij et al. [LHCb Collaboration], Phys. Lett. B 762 (2016) 253 [arXiv:1608.04855].

[7] G. Aad et al. [ATLAS Collaboration], JHEP 1608 (2016) 147 [arXiv:1601.03297].

[8] V. Khachatryan et al. [CMS Collaboration], Phys. Lett. B 757 (2016) 97 [arXiv:1507.07527].

[9] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 109 (2012) 171802 [arXiv:1208.2967].

[10] U. Nierste, *CP asymmetry in flavor-specific B decays, Proceeding of 39th Rencontres de Moriond on Electroweak Interactions and Unified Theories*, 21-28 March 2004, La Thuile, Aosta Valley, Italy [hep-ph/0406300].

[11] S. Jäger, M. Kirk, A. Lenz and K. Leslie, Phys. Rev. D 97 (2018) no.1, 015021 [arXiv:1701.09183]. S. Jäger, M. Kirk, A. Lenz and K. Leslie, JHEP 2003 (2020) 122 [arXiv:1910.12924].

[12] C. Bobeth, U. Haisch, A. Lenz, B. Pecjak and G. Tetlalmatzi-Xolocotzi, JHEP 1406 (2014) 040 [arXiv:1404.2531].

[13] H. Asatrian, A. Hovhannisyan, U. Nierste and A. Yeghiazaryan, JHEP 10, 191 (2017) [arXiv:1709.02160].

[14] Y. Y. Keum and U. Nierste, Phys. Rev. D 57 (1998) 4282 [hep-ph/9710512].

[15] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [hep-ph/9512380].

[16] H. Asatrian, Hrachya H. Asatryan, A. Hovhannisyan, U. Nierste, S. Tumasyan, A. Yeghiazaryan, Phys. Rev. D 102, 033007 (2020) [https://arxiv.org/abs/2006.13227].

[17] A. J. Buras, M. Gorbahn, U. Haisch and U. Nierste, JHEP 0611, 002 (2006) [JHEP 1211, 167 (2012)] [hep-ph/0603079].

[18] M. Gorbahn and U. Haisch, Nucl. Phys. B 713, 291 (2005) [hep-ph/0411071].

[19] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016) .

[20] S. Aoki et al. [Flavour Lattice Averaging Group], Eur. Phys. J. C 80 (2020) no.2, 113 [arXiv:1902.08191].

[21] A. Lenz and U. Nierste, JHEP 0706 (2007) 072 [hepph/0612167].

[22] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no.3, 030001 (2018).

[23] R. Dowdall, C. Davies, R. Horgan, G. Lepage, C. Monahan, J. Shigemitsu and M. Wingate, Phys. Rev. D 100, no.9, 094508 (2019) [arXiv:1907.01025].

[24] C. T. Davies et al. [HPQCD], Phys. Rev. Lett. 124, no.8, 082001 (2020) [arXiv:1910.00970].

[25] M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D 54 (1996), 4419-4431 [hep-ph/9605259].

[26] A. Bazavov, C. Bernard, N. Brown, C. Detar, A. El- Khadra, E. Gámiz, S. Gottlieb, U. Heller, J. Komijani, A. Kronfeld, J. Laiho, P. Mackenzie, E. Neil, J. Simone, R. Sugar, D. Toussaint and R. Van De Water, Phys. Rev. D 98 (2018) no.7, 074512 [arXiv:1712.09262].

[27] J. Charles et al. [CKMfitter Group Collaboration], Eur. Phys. J. C 41, 1 (2005) [hep-ph/0406184]. We use updated numbers from December 2019 at http://ckmfitter.in2p3.fr.

[28] J. H. Kühn, M. Steinhauser and C. Sturm, Nucl. Phys. B 778 (2007) 192 [hep-ph/0702103].

[29] I. Allison et al. [HPQCD Collaboration], Phys. Rev. D 78 (2008) 054513 [arXiv:0805.2999].