

Precision tests of fundamental symmetries with η and η' decays

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Decays of the neutral and long-lived η and η' mesons provide a unique, flavor-conserving laboratory to test low-energy Quantum Chromodynamics and search for physics beyond the Standard Model. In these proceedings, we specifically concentrate on discrete symmetry tests. We point out how stringent constraints from electric dipole moment measurements strongly limit the realistic opportunities to observe P - and CP -odd mechanisms in η or η' decays. For the far less investigated C - and CP -odd processes, we introduce a dispersion-theoretical formalism to assess these from Dalitz plot asymmetries in a model-independent way.

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1. The η and η' mesons: properties, symmetries, quantum numbers

Decays of the lightest flavor-neutral mesons, the η and the η' , post ample opportunities of various symmetry tests. Apart from parity, they both share the quantum numbers of the vacuum, $I^G(J^{PC}) = 0^+(0^{-+})$. The η with its mass $M_\eta = 547.86$ MeV is largely a (pseudo-) Goldstone boson of spontaneously broken chiral symmetry. Its width, $\Gamma_\eta = 1.31$ keV, is tiny largely because many of its decay modes, strong or electromagnetic, are forbidden at leading order due to P , C , CP , isospin/ G -parity, or angular momentum conservation [1]. The much heavier η' , $M_{\eta'} = 957.78$ MeV, is no Goldstone boson due to the $U(1)_A$ anomaly, however its width, $\Gamma_{\eta'} = 196$ keV, still makes it much narrower than, say, the $\omega(782)$ or $\phi(1020)$ vector resonances of comparable mass.

Amongst stringent tests of Standard Model (SM) physics with η and η' decays, we mention various processes of odd intrinsic parity that are determined by the Wess–Zumino–Witten anomaly [2, 3]; the extraction of the light quark mass difference $m_u - m_d$ from $\eta \rightarrow 3\pi$ [4] (and references therein); theory input for the anomalous magnetic moment of the muon [5] via the $\eta^{(\prime)}$ transition form factors [6] (see also Refs. [7, 8] for recent developments); or scalar resonance dynamics of, e.g., the $f_0(500)$ or $a_0(980)$ resonances in decays such as $\eta^{(\prime)} \rightarrow \pi^0\gamma\gamma$, $\eta' \rightarrow \eta\gamma\gamma$ [9, 10]. These and several more topics are covered extensively in a recent review [11].

Possible probes of physics beyond the Standard Model (BSM) fall broadly into two categories. On the one hand, there are searches for weakly-interacting new light particles, such as dark photons, protophobic or leptophobic gauge bosons of new $U(1)$ symmetries, light Higgs-like scalars, or axion-like particles; we do not discuss these here, but refer to Ref. [11] and references therein. In these proceedings, we concentrate on tests of discrete symmetries, in particular searches for possible new sources of CP -violation. As the η and η' mesons are C and P eigenstates, their decays are a flavor-conserving laboratory for such symmetry tests, with little or no SM background. The different possible classes of violation and conservation of C , P , and T (always assuming CPT to be conserved) are enlisted in Table 1. While the Standard Model weak interactions are in class IV, violating all three discrete symmetries separately, they are in many circumstances close to class I, violating C and P maximally, with CP almost conserved. In the following, we concentrate on class II (Sec. 2), P - and T - violating, but C -conserving interactions, which are closely connected to the physics of electric dipole moments (EDMs); and on class III (Sec. 3), T - and C -odd, but P -even interactions, which have been much less explored to date.

class	violated	conserved	interaction
0		C, P, T, CP, CT, PT, CPT	strong, electromagnetic
I	C, P, CT, PT	T, CP, CPT	(weak, with no KM phase or flavor mixing)
II	P, T, CP, CT	C, PT, CPT	
III	C, T, PT, CP	P, CT, CPT	
IV	C, P, T, CP, CT, PT	CPT	weak

Table 1: Possible classes I–IV of interactions that violate discrete spacetime symmetries, assuming CPT invariance. Table taken from Ref. [11].

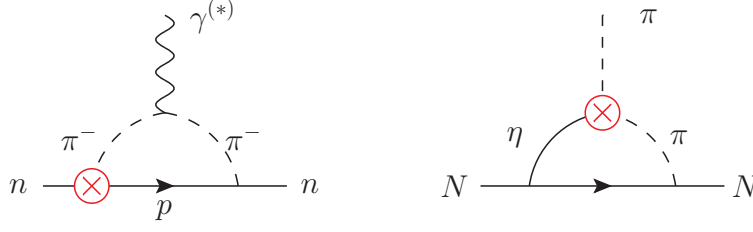


Figure 1: Pion loop contribution to the neutron EDM via a CP -violating pion–nucleon coupling (left), and loop-induced CP -violating pion–nucleon vertex function generated through an $\eta \rightarrow \pi\pi$ coupling (right). The red crosses indicated CP -violating vertices. Figures taken from Ref. [11].

2. P - and CP -violation

Permanent EDMs such as of neutron or proton are signs of P - and CP -violating interactions, or class II in the nomenclature of Table 1. As the weak contribution to these is vanishingly small, experimental limits in particular on the neutron EDM have been employed to constrain the (renormalizable) dimension-four P - and CP -odd operator known as the QCD θ -term,

$$\mathcal{L}_\theta = \frac{g_s^2 \theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}. \quad (1)$$

At the same time, the θ -term induces a coupling for the decay $\eta \rightarrow \pi\pi$ [12, 13]:

$$\mathcal{B}(\eta \rightarrow \pi^+\pi^-) = \frac{\bar{g}_{\eta\pi\pi}^2}{16\pi M_\eta \Gamma_\eta} \sqrt{1 - \frac{4M_{\pi^\pm}^2}{M_\eta^2}}, \quad \bar{g}_{\eta\pi\pi} = \frac{2\bar{\theta} M_\pi^2 m_u m_d}{\sqrt{3} F_\pi (m_u + m_d)^2}, \quad (2)$$

where $\bar{\theta} = \theta + \arg \det(\mathcal{M})$ is related to the Lagrangian parameter θ via the phase of the determinant of the quark mass matrix \mathcal{M} . The best experimental upper limit on this decay, $\mathcal{B}(\eta \rightarrow \pi^+\pi^-) < 4.4 \times 10^{-6}$ [14], implies $|\bar{\theta}| < 4 \times 10^{-4}$, which however is a much weaker limit than the one derived from the neutron EDM, $|\bar{\theta}| \lesssim 10^{-10}$ (see, e.g., Ref. [15] or references therein). However, given the suggested suppression of $\bar{\theta}$, various P - and CP -odd operators of dimension six may be competitive in size [16, 17], so one may wonder whether a tiny EDM may not still be reconciled with a signal in $\eta \rightarrow \pi\pi$. However, this possibility has been debunked frequently [11, 18–21]. Figure 1(left) shows how the leading nonanalytic contribution to the neutron EDM is, in fact, generated from the θ -term: it generates a CP -violating pion–nucleon coupling, and the photon coupling to the charged-pion loop induces a logarithmically (in M_π) enhanced contribution to the EDM. However, a CP -odd $\eta \rightarrow \pi\pi$ coupling in turn induces such a CP -odd pion–nucleon coupling at one loop, see Fig. 1(right), such that, at the two-loop level, any $\eta \rightarrow \pi\pi$ decay generates an EDM contribution. The estimated relation between neutron EDM d_n and coupling $\bar{g}_{\eta\pi\pi}$ reads [11]

$$d_n \approx 7 \times 10^{-16} \left(\frac{\bar{g}_{\eta\pi\pi}}{\text{GeV}} \right) e \text{ cm}, \quad (3)$$

which results in the limit $\mathcal{B}(\eta \rightarrow \pi^+\pi^-) < 2 \times 10^{-17}$, eleven orders of magnitude below the current measurement [14], and way beyond experimental reach in the foreseeable future.

Alternative η decays to investigate P - and CP -violation that have been suggested in the past are $\eta \rightarrow \pi^+\pi^-\gamma^{(*)}$ [22, 23], where, e.g., an asymmetry in the angle between the $\pi^+\pi^-$ and $\ell^+\ell^-$

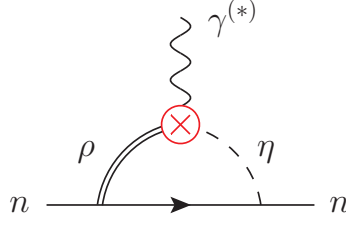


Figure 2: One-loop contribution to the neutron EDM, induced by a CP -odd $\eta \rightarrow \rho\gamma$ vertex. Figure taken from Ref. [11].

decay planes would be an experimental CP -odd signature [23]. Such an effect could be generated by the interference of magnetic (M) and electric (E) transitions,

$$\mathcal{A}(\eta \rightarrow \pi^+ \pi^- \gamma^{(*)}) = M \epsilon_{\mu\nu\alpha\beta} p_+^\mu p_-^\nu k^\alpha \epsilon^\beta + E ((\epsilon \cdot p_+)(k \cdot p_-) - (\epsilon \cdot p_-)(k \cdot p_+)). \quad (4)$$

In the SM, $M \propto e/F_\pi^3$ is given by the chiral anomaly at leading order [24–26], while $E = 0$. In Refs. [22, 23], nonvanishing contributions to E are constructed based on a four-quark operator of the form $(\bar{s} i\sigma_{\mu\nu} \gamma_5 (p - k)^\nu s) (\bar{q} \gamma^\mu q)$, where the light-quark vector current $(\bar{q} \gamma^\mu q)$ hadronizes into a P -wave $\pi\pi$ pair, while the strange-quark operator induces the $\eta \rightarrow \gamma^{(*)}$ transition. However, also this mechanism is heavily restricted through indirect EDM constraints: the P -wave $\pi\pi$ pair develops, via strong final-state interactions, resonant enhancement as the $\rho(770)$, such that, in a simplified manner of speaking, the E -term induces a CP -odd $\eta \rightarrow \rho\gamma$ vertex. Via the mechanism shown in Fig. 2, this once more generates a neutron EDM at one loop. As a result, the corresponding coupling can be estimated to be $E/M \lesssim 10^{-11}$ [11], which generates asymmetries way beyond any experimental accessibility at any time soon.

A loophole in this overall seemingly rather bleak picture of possible searches for CP -violation in $\eta^{(\prime)}$ decays has been identified in Ref. [27], where new symmetry tests in decays with dimuon pairs in the final state such as $\eta \rightarrow \mu^+ \mu^-$, $\eta \rightarrow \mu^+ \mu^- \gamma$, and $\eta \rightarrow \mu^+ \mu^- e^+ e^-$ have been investigated; the corresponding CP -odd observables in the first two of these necessarily involve muon polarization. These are induced by scalar–pseudoscalar quark–muon operators of the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2\nu^2} \text{Im} c_{\ell edq}^{2222} \left[(\bar{\mu} \mu) (\bar{s} i \gamma_5 s) - (\bar{\mu} i \gamma_5 \mu) (\bar{s} s) \right] + [u-, d\text{-quarks}]. \quad (5)$$

Their possible contribution to EDMs is suppressed to the two-loop level, as the muon (pseudo)scalar bilinears cannot directly couple to a single photon on account of vanishing VS and VP two-point functions in QCD+QED (due to C -parity). As a result, EDM constraints on such operators are significantly weaker, in particular for strange quarks, whose matrix elements in nucleons are suppressed. The resulting constraint on the coefficient was found to be $|\text{Im} c_{\ell edq}^{2222}| < 0.04$ [27], which results in CP -odd asymmetries in particular in $\eta \rightarrow \mu^+ \mu^-$ that may be testable at REDTOP [28].

The analysis of Ref. [27] has since been generalized to $\eta \rightarrow \pi^0 \mu^+ \mu^-$ [29] and is in the process of being tested in $\eta^{(\prime)} \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ [30], but so far, $\eta \rightarrow \mu^+ \mu^-$ seems to remain the most promising channel to observe these operators.

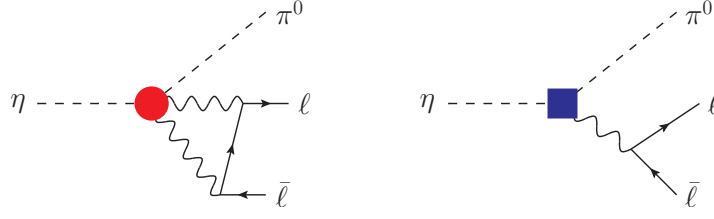


Figure 3: C -conserving two-photon (left) and C -violating single-photon (right) mechanism for the decay $\eta \rightarrow \pi^0 \ell^+ \ell^-$. Figure taken from Ref. [11].

3. C - and CP -violation

Class III interactions in the sense of Table 1 are harder to motivate theoretically from the point of view of a fundamental underlying theory. Effective operators on the quark level appear at mass dimension seven and involve both four-quark or fermion–gauge-boson operators such as [31–33]

$$\frac{1}{\Lambda^3} \bar{\psi}_f \gamma_5 D_\mu \psi_f \bar{\psi}_{f'} \gamma^\mu \gamma_5 \psi_{f'} + \text{h.c.}, \quad \frac{1}{\Lambda^3} \bar{\psi}_f \sigma_{\mu\nu} \lambda_a \psi_f G_a^{\mu\lambda} F_\lambda^\nu. \quad (6)$$

As these involve fermion helicity flips, they are actually of mass dimension eight when formulated in terms of fields invariant under the SM symmetry group (in other words, the prefactor $1/\Lambda^3$ would have to be interpreted as v/Λ_{BSM}^4 , where v is the Higgs vacuum expectation value). Electroweak radiative corrections mix classes II and III, such that indirect EDM constraints will also limit the prefactors of the operators (6).

As the flavor-neutral light pseudoscalars $\eta^{(\prime)}, \pi^0$ are charge-conjugation eigenstates with eigenvalues $C = +1$, any decay that involves only these mesons as well as an *odd* number of photons is directly a test of C -conservation. Examples of such C -forbidden decays are $\pi^0, \eta^{(\prime)} \rightarrow 3\gamma$ or $\eta^{(\prime)} \rightarrow 2\pi^0 \gamma^{(*)}$. This is more complicated for the seemingly simplest such decays, $\eta \rightarrow \pi^0 \gamma^{(*)}$ as well as analogous η' decays: as a consequence of gauge invariance (and angular momentum conservation), the corresponding matrix elements can be decomposed according to

$$\langle \pi^0(k) | j_\mu(0) | \eta(p) \rangle = [q^2(p+k)_\mu - (M_\eta^2 - M_\pi^2)q_\mu] F_{\eta\pi^0}(q^2) \quad (7)$$

(cf. the similar, although flavor-changing, decays $K \rightarrow \pi \ell^+ \ell^-$ [34, 35]), which vanish for real photons $q^2 = 0$. Potential nonvanishing, C -odd transitions can therefore only be assessed in the corresponding dilepton decays $\eta \rightarrow \pi^0 \ell^+ \ell^-$, for which, however, a C -conserving two-photon exchange, see Fig. 3, forms an irreducible SM background. The corresponding SM branching ratios have been recalculated recently [36], $\mathcal{B}(\eta \rightarrow \pi^0 e^+ e^-) = 2.1(5) \times 10^{-9}$ and $\mathcal{B}(\eta \rightarrow \pi^0 \mu^+ \mu^-) = 1.2(3) \times 10^{-9}$ (as well as similar orders of magnitude for $\eta' \rightarrow \pi^0 \ell^+ \ell^-$ and $\eta' \rightarrow \eta \ell^+ \ell^-$), based on a vector-meson-dominance model for the corresponding two-photon decays [9]. These are to be compared to the current experimental upper limits, $\mathcal{B}(\eta \rightarrow \pi^0 e^+ e^-) < 7.5 \times 10^{-6}$ [37] and $\mathcal{B}(\eta \rightarrow \pi^0 \mu^+ \mu^-) < 5 \times 10^{-6}$ [38]. Evidence for a C -odd signal can hence only be claimed if measured branching ratios significantly exceed the theoretical SM prediction, or in the nowadays rather unlikely case that interference effects between C -even and C -odd mechanisms allow us to observe Dalitz plot asymmetries in differential decay distributions.

Obviously, the idea of testing C -odd interactions via certain asymmetries (instead of via rates of decays that are strictly possible only if C is violated) is even more appealing for decays

with much larger branching ratios than these dilepton decays with their heavily suppressed SM rates. Indeed, with precisely this argument, it has been suggested to search for C -violation in Dalitz plot asymmetries in $\eta \rightarrow \pi^+\pi^-\pi^0$ [39], one of the dominant η decay modes, precisely for the reasons that such an asymmetry scales *linearly* with (supposedly very small) BSM physics parameters, while decay rates for, e.g., $\eta \rightarrow 3\gamma$ scale with such parameters *squared*.¹ Subsequently, analogous suggestions have also been made for similar η' decays, $\eta' \rightarrow \eta\pi^+\pi^-$ and $\eta' \rightarrow \pi^+\pi^-\pi^0$, with the rationale that these are partly sensitive to underlying short-distance operators of different isospin [41].

The decay $\eta \rightarrow \pi^+\pi^-\pi^0$ manifestly breaks G -parity. Within the SM, charge conjugation is preserved and hence isospin has to be broken; with electromagnetic effects strongly suppressed [42–44], this makes $\eta \rightarrow 3\pi$ an ideal process to extract the light quark mass difference $m_u - m_d$ [4]. Allowing for BSM effects, two additional amplitudes that break C -invariance can be added, either conserving or breaking isospin, such that the full decay amplitude is written as

$$\mathcal{M}_C(s, t, u) = \mathcal{M}_0^C(s, t, u) + \xi \mathcal{M}_1^C(s, t, u) + \mathcal{M}_2^C(s, t, u), \quad \xi = \frac{(M_{K^+}^2 - M_{K^0}^2)_{\text{QCD}}}{3\sqrt{3}F_\pi^2}, \quad (8)$$

where the subscript denotes the total (three-pion) isospin of the final state, and ξ parameterizes the isospin breaking in the SM amplitude. Each of these amplitudes can subsequently be analyzed in the so-called Khuri–Treiman framework [45], which allows us to take all iterated pion–pion rescattering effects into account consistently. For this purpose, the amplitudes are decomposed into single-variable amplitudes (SVAs) according to reconstruction theorems [39, 46, 47] up to (and including) P -waves:

$$\begin{aligned} \mathcal{M}_1^C(s, t, u) &= \mathcal{F}_0(s) + (s-u)\mathcal{F}_1(t) + (s-t)\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s), \\ \mathcal{M}_0^C(s, t, u) &= (t-u)\mathcal{G}_1(s) + (u-s)\mathcal{G}_1(t) + (s-t)\mathcal{G}_1(u), \\ \mathcal{M}_2^C(s, t, u) &= 2(u-t)\mathcal{H}_1(s) + (u-s)\mathcal{H}_1(t) + (s-t)\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u), \end{aligned} \quad (9)$$

where the subscripts at the SVAs denote isospin (which in turn identifies the angular momentum uniquely via Bose symmetry). Obviously, $\mathcal{M}_{0,2}^C$ are odd under the exchange $t \leftrightarrow u$. For the SVAs, inhomogeneous Omnès solutions can be constructed,

$$\mathcal{A}_I(s) = \Omega_I(s) \left(P_{n-1}(s) + \frac{s^n}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dx}{x^n} \frac{\sin \delta_I(x) \hat{\mathcal{A}}_I(x)}{|\Omega_I(x)| (x-s)} \right), \quad (10)$$

$\mathcal{A} \in \{\mathcal{F}, \mathcal{G}, \mathcal{H}\}$, where the Omnès functions $\Omega_I(s)$ are given in terms of known pion–pion phase shift input, and the subtraction polynomials $P_{n-1}(s)$ yield the free parameters of the dispersive representation. A minimal subtraction scheme for \mathcal{M}_1^C depends on three (real) constants only; it allows us to fit data for $\eta \rightarrow \pi^+\pi^-\pi^0$ [48] and $\eta \rightarrow 3\pi^0$ [49] very well, as well as to fulfill constraints from chiral perturbation theory at $\mathcal{O}(p^4)$ [4, 50]. Employing the same minimal assumptions, it can be shown [41] that $\mathcal{M}_{0,2}^C$ both depend on one single (complex) subtraction constant each, which can be matched unambiguously onto leading Taylor invariants:

$$\mathcal{M}_0^C(s, t, u) = g_0 (s-t)(u-s)(t-u) + \mathcal{O}(p^8), \quad \mathcal{M}_2^C(s, t, u) = g_2 (t-u) + \mathcal{O}(p^4). \quad (11)$$

¹The same problem obviously also affects the P - and CP -violating decays discussed in the previous section: decay rates for, e.g., $\eta^{(\prime)} \rightarrow \pi\pi$ or $\eta \rightarrow 4\pi^0$ [40] are quadratically suppressed in the small couplings.

The effective coupling constants $g_{0,2}$ can be constrained from the most recent KLOE Dalitz plot data [48], which overall restricts C -violation to the permille level. While the kinematical dependence of the two C -odd amplitudes and their different interference patterns can clearly not be resolved (all C/CP -violating signals vanish within $1-2\sigma$), the small phase space severely reduces the sensitivity to the isoscalar amplitude \mathcal{M}_0^C [39, 41].

In principle, this kinematic suppression could be overcome in the decay $\eta' \rightarrow \pi^+\pi^-\pi^0$, in which one would expect C -violation to be caused by essentially the same fundamental operators; and indeed, the relative theoretical sensitivity to g_0 would be enhanced by about two orders of magnitude compared to $\eta \rightarrow \pi^+\pi^-\pi^0$ [41]. However, as this is a comparably rare decay ($\mathcal{B}(\eta' \rightarrow \pi^+\pi^-\pi^0) \approx 3.6 \times 10^{-3}$), an analysis on C -violation based on the existing data by BESIII [51] is not too promising right now.

This is somewhat different for $\eta' \rightarrow \eta\pi^+\pi^-$ [52], for which a Khuri–Treiman-type dispersive analysis of the SM amplitude has also been performed already [53]. Here, the SM decay conserves isospin, while a C -odd contribution would change the isospin by 1. Charge asymmetries in the $\eta' \rightarrow \eta\pi^+\pi^-$ Dalitz plot therefore test a BSM operator of different isospin, which is independent of the ones constrained in $\eta \rightarrow \pi^+\pi^-\pi^0$. A Khuri–Treiman analysis of the corresponding amplitudes has also been performed in Ref. [41], and while the resulting constraints are not quite as strong yet, C -violation is restricted to the percent level in this channel. We also refer to that reference for graphical representations of the various Dalitz plot asymmetries.

4. Summary

Decays of η and η' mesons allow for a vast range of physics investigations, both within the Standard Model and beyond. Here we have concentrated in particular on possible new patterns of discrete symmetry violation. Electric dipole moments set rigorous limits on P - and CP -violation, putting most candidate decays way beyond foreseeable experimental reach; the exception being certain strange-quark–muon operators of dimension six that may be testable in particular in $\eta \rightarrow \mu^+\mu^-$. Very little theory work has been done on P -conserving, but C - and CP -violating mechanisms so far, however, a dispersion-theoretical framework has now been established to investigate charge asymmetries in $\eta^{(\prime)} \rightarrow \pi^+\pi^-\pi^0$ and $\eta' \rightarrow \eta\pi^+\pi^-$ Dalitz plots. New experimental results from forthcoming facilities such as the JLab Eta Factory [54] or REDTOP [28] are eagerly awaited.

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