

# PoS

# Symmetries of the Entropy Balance Condition for the Universe

## Merab Gogberashvili<sup>*a,b,\**</sup>

<sup>a</sup> Ivane Javakhishvili Tbilisi State University,
3 Chavchavadze Avenue, Tbilisi 0179, Georgia
<sup>b</sup> Elevter Andronikashvili Institute of Physics,
6 Tamarashvili Street, Tbilisi 0177, Georgia

E-mail: Merab.Gogberashvili@tsu.ge

We attempt to describe geometry in terms of informational quantities for the universe considered as a finite ensemble of correlated quantum particles. As the main dynamical principle, we use the conservation of the sum of all kinds of entropies: thermodynamic, quantum and informational. In this picture, relativity invariance is a consequence of symmetries of this entropy balance condition. The two postulates, which are enough to derive the whole theory of Special Relativity, are reformulated as the principles of information entropy universality and finiteness of information density.

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#### \*Speaker

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Conventional belief is that the laws of physics are fundamental and can be used as the framework for understanding information processing about the universe. There also exists the opposite view that the laws of physics are just applications of the rules for processing information. The idea that nature is built by information serves as a basis for so-called emergent theories [1-7]. In this approach, even space-time is not an a priori objective reality, but construction of an observer and can be explained in terms of thermodynamic-statistical notions [8-10].

The key ingredient in the statistical description of nature is entropy, which captures uncertainties of all kinds and thus allows us to model different aspects of a physical system using a universal mathematical framework. Entropy is a powerful tool in general relativity, thermodynamics, information theories, quantum physics, etc. (see the recent review [11]). The concept of entropy is far from being well understood and a more general definition than that introduced by Boltzmann, Shannon, or von Neumann, could exist [12]. In general, entropy of a physical system contains distinct constituents and there are deep reasons to expect relations between them adopting the principle of total entropy conservation [13].

In this paper, we want to model the main features of standard Special Relativity using the informational probabilistic approach [14–20]. Conceptually, it is hard to imagine spacetime as being emerged from some informational pre-geometric variables; we need to generalize the concepts of particles and locality. The model is based on the fact that locality, one of the assumed properties of classical reality, is incompatible with observation; we already know that quantum theory is nonlocal. In our scenario, the universe is considered as a finite ensemble of non-locally correlated quantum particles, where the kinematics of space-time emerges from the properties of the ensemble. Of course, this is an approximate view, fundamental objects are fields and not particles, we can describe fields using the concept of excitation quanta only in the perturbative regime.

When one studies the structure of a physical system and its connections with the universe, uncertainty and thus the entropy of the system should decrease. In the limiting case, when all interactions of the system are considered, uncertainties will disappear and the total entropy should be zero. So, in our opinion, the total entropy of a finite universe model is not only constant, but can be assumed to be zero, since there should be absent observers outside the universe and prior to the big bang as well. In any measurement process the three systems: an object, memory (or apparatus) and the observer are involved. Then, the zero total entropy of the universe can be symbolically written as the sum of information (I), thermodynamic (S) and quantum (H) components [13–15]:

$$S_{\text{tot}} = I + S + \mathcal{H} = 0, \qquad (1)$$

which can be understood as the conservation law for all kinds of information in the universe. Evolution of a thermodynamically isolated physical system (S = const) can be described by transitions into each other of information and entanglement entropies, which are observer dependent. To fulfill the condition (1) and at the same time obey the second law of thermodynamics, it seems that in standard definitions of entropies we need to include negative components as well [21–23].

The expression (1) seems to be contradictory, since the information and entanglement entropies usually are assumed to be dimensionless. However, the standard definition of thermodynamic entropy, S, contains Boltzmann's constant  $k_B$  and is considered to be a dimensional quantity. We note that in basic physical laws  $k_B$  always appears together with the absolute temperature, T, and

may be considered as its measure in kelvins [24–26]. So, it is natural to relate  $k_B$  to T and consider S in (1) as the Shannon type dimensionless information entropy.

According to Landauer's and Brillouin's principles, the abstract notions of information and entropy can be associated with some physical parameters, like energy or mass [27, 28]. However, there are no unique standards of energy and mass, which appear additive as electric charge. These quantities also have no polarity and usually are taken to be positive. In [14] we had suggested that the convenient physical parameter to measure information (or entropy) is the action *S*, which like standard entropies usually is considered to be additive and have unique discrete value,  $\hbar$ .

In the world ensemble picture, the universe can be in a superposition of a finite number of different configurations. Not only there are configurations which occur more often than other ones, but there are no two configurations with the same weight. The observation of a configuration k, changes it and we can assign an ordering (time) in a natural way,

$$k \sim t , \qquad (2)$$

because any k'-configuration contains k-configuration if k' > k. In this picture, the relation of thermodynamic entropy with information entropies (1) is the origin of subjective times. So, thermodynamic entropy is related to the time of the event and can serve as the measure of the observer's information, i.e. the entropic arrow of time seems to be an emergent property associated with the transfer of information. By relating of the thermodynamic entropy with action,

$$S \sim \frac{S}{\hbar}$$
, (3)

we can develop a concept of energy to account for the amount of change in a system that we observe in time. Any measurement, even a thought experiment, is accompanied by the transfer of at least one quant of the action  $\hbar$  and leads to the growth of thermodynamic entropy. Maximal probability has the configuration with highest thermodynamic entropy (i.e. minimal information entropy *I*, as follows from (1)) corresponding to zero interactions, or equilibrium.

If we accept the thesis that action and information are related (3), the maximal entropy (minimal information) principle [29–31] will become equivalent to the familiar minimal action principle. However, the principle of least information is more general, especially for quantum descriptions, since in physical equations one can introduce the information terms (consciousness could also have quantum origins [32]) that correspond to measurement processes and to observers.

The condition of entropy neutrality (1) suggests information quantization [33], such as the discreteness of the charge might be thought as the consequence of Gauss' law for the electrically neutral universe – charge inside of even infinitesimally small volume should be canceled by outside charges. Analogously, there should exist a unit of elementary information – bit. For a system whose entanglement entropy vanishes, the simplified relation (1) obtains the form:

$$I\left(1+\frac{S}{I}\right) = 0. \qquad (\mathcal{H} \to 0) \tag{4}$$

The existence of the unit of information suggests that in the process of obtaining information, the related action exhibits discrete features in  $\hbar$ -s, i.e. upon measurements matter behaves as particles.

So, a bit of information is always associated with a particle and is accompanied by the transfer of elementary action.

From the introduction of the concept of time (2) for a finite world ensemble follows that there should be a limit to how fast information can move through the universe. Consider a physical system in the volume V that stores some information and the entropy S. If we neglect entropy supplied by internal sources, the time derivative of the entropy that is contained within this volume should be equal to the flux of the thermal entropy of matter  $S_m$  through the boundary A [34–36],

$$\frac{dS}{dt} = S_m v \frac{A}{V} \,. \tag{5}$$

In this equation it appears the quantity with the dimension of velocity v (information speed), which controls the rate of information growth in thermalizing states. From (5) follows that, in a simple model, the entropy of the system, initially contained within the volume V and entangled with outside matter, grows linearly in time (entanglement tsunami) [37],

$$S = S_m v t \frac{A}{V} \,. \tag{6}$$

To estimate the information speed v for the entire universe let us write the equation (5) for the Hubble sphere  $R_H = c/H$ . Using the black hole type entropy for the Hubble horizon [38],

$$S_H = \frac{A_H c^3}{4G\hbar} = \frac{\pi c^5}{G\hbar H^2} , \qquad (7)$$

and the Gibbs–Duhem relation,  $\rho + p/c^2 = k_B T_H S_m/V_H$ , where  $\rho$  stand for matter density, p represents pressure density and  $T_H = \hbar H/2\pi k_B c^2$  denotes the de Sitter temperature associated to the horizon [39], the relation (5) leads to the standard acceleration cosmological equation [14, 15], but only if

$$v = c . (8)$$

So, in thermodynamic formalism, the speed of light c can be understood as the world ensemble parameter that corresponds to the information velocity. This information speed measures the response time of the world ensemble on the transfer of one bit of information, i.e. exchange of one particle.

In an information probabilistic approach space is not an a priori objective reality and the metric represents the potential knowledge (or ignorance) of an observer. Geometry does not exist without matter, in practice we measure quantum particles whose actual locations have no meaning. An observer associates to each particle the entropy  $S(x^i)$  and the probability distribution,  $p(x^i) \sim e^{-S(x^i)}$ , labelled by some set of informational coordinates  $x^i$ . To find the dimension of the information space, or the number of independent coordinates  $x^i$ , consider the simplified model of the world ensemble consisting of N identical particles (generalization for many spices is obvious). The total action of the system, which we use in the definition of the total thermodynamic entropy (3), contains the factor  $N^3$ , and not the factor  $\sim N$ , as in a local model [18–20]. It is known that the maximal entropy of a random variable with n realizations is  $\ln n$ . The maximal entropy of the model universe appears to be  $\sim \ln N^3 = 3 \ln N$ . Then, in local measurements an observer obtains three copies of the entropy,  $\sim \ln N$ , which is used to define information distances. So, in the definition of

a particle distribution function,  $p(x^i)$ , the number of the information coordinates,  $x^i$ , three may be required (i = 1, 2, 3).

According to Bekenstein's area-law entropy bound (7), the maximal entropy of a region of the size R, independently of its matter content, is  $S \sim R^2$ . Thus, an observer naturally associates S with the notion of a spherical volume of the radius,

$$R^{2} = g_{ij}x^{i}x^{j} \sim \mathcal{S}(x^{i}) , \qquad (i, j = 1, 2, 3)$$
(9)

in the center of which the probability distribution  $p(x^i)$  attains the maximum. In continuous measurements it is difficult to distinguish quantum particles, the information distance between two neighboring probability distributions,  $p(x^i)$  and  $p(x^i + dx^i)$ , is given by the variance [8–10],

$$dl^2 = g_{ij} dx^i dx^j . \qquad (i, j = 1, 2, 3)$$
<sup>(10)</sup>

Consider the entropy  $S(x^i, x'^i)$  of one distribution  $p(x^i)$  relative to another distribution  $p(x'^i)$ , which attains an absolute maximum at  $x^i = x'^i$ . In terms of  $S(x^i, x'^i)$  the information metric  $g_{ij}$  in (10) can be defined as

$$g_{nm} = -\frac{\partial \mathcal{S}(x^i, x^{\prime h})}{\partial x^{\prime m} \partial x^{\prime m}}, \qquad (i, n, m = 1, 2, 3)$$
(11)

so that  $S(x^i + dx^i, x^i) = -dl^2/2$  [8–10]. A small value of dl means that particles at the points  $x^i$  and  $x^i + dx^i$  are difficult to distinguish. We can also invert the logic and assert that the two points  $x^i$  and  $x^i + dx^i$  must be very close together because they are difficult to distinguish.

The metric (10) is known as the Fisher-Rao metric, or the information metric, which can be used to calculate the informational difference between measurements. The informational coordinates  $x^i$  are arbitrary, one can relabel distributions of particles. It is then easy to check that  $g_{ij}$  are the components of a tensor and that the distance  $dl^2$  is an invariant, a scalar under coordinate transformations. It is important that the metric tensor  $g_{ij}$  on the manifold of probability distributions is unique: there is only one metric (up to rescaling) that is invariant under sufficient statistics, i.e. another observer with the same thermodynamic entropy has no additional information [8–10].

The information distance (10) is dimensionless,  $g_{ij}$  measures distinguishability in units of the local uncertainty implied by the distribution  $p(x^i)$ . So, information geometry allows us to describe the conformal geometry of space - the local shapes but not absolute local sizes. The scale of distance turns out to be a property of the models we employ to describe it. One possible choice of gauge would be to choose the unit of length so that the evolving three-dimensional information manifold of distributions generates a four-dimensional space-time. The conditions for such a space-time gauge have been proposed, for example, in the context of Machian relational dynamics [40].

The Riemannian space-like metrics of information geometry (10) do not reproduce the lightcone structure of space-time; some additional ingredient is needed. According to (1), changes in information and entanglement entropies for a thermodynamically isolated physical system (coordinate transformations), leads to deformations in thermodynamic entropy associated with time evolution. This relation of thermodynamic and information entropies is analogous to the relativistic transformations. Quantum theory does not treat space and time on the same footing. Using the dimensional parameter of information speed (8), the information space (10) can be considered as a three-dimensional spacelike 'surface' embedded in four-dimensional space-time and thus c introduces the scale for the information distance (10). Having decided on a measure of information distance, dl, we can now also measure angles, areas, volumes and all sorts of other geometrical quantities. The introduction of notions of space-time and the relation of thermodynamic entropy to action integral, moves information theory from abstract mathematics to physics.

Now we are ready to try to translate relativity theory in information terms. In the Special Theory of Relativity the law of inertia has no known origin [41]. This theory also says nothing about why an inertial system exists, which is required to describe relative motions and inertial observers. In the world ensemble picture, all entangled particles are employed in the definition of the fundamental frame. Also, homogeneity and isotropy of the universe (with a huge number of particles) is obvious for a small system. In spite of the introduction of the preferred cosmological frame, the world ensemble model can imitate basic features of relativity theory, such as the relativity principle, the local Lorentz invariance and the so-called signal locality.

In our model relativity emerges due to the existence of the class of 'inertial' observers with the same informational, but different thermodynamic and entanglement entropies. From the assumption (1) it follows that for observers with the constant information entropy about the universe, I = const, the changes in thermodynamic and entanglement entropies should cancel out, dS = -dH. So, relativity invariance is a consequence of symmetries of the entropy balance condition (1). Indeed, for two observers with zero overall entropies and the same information entropy, the sums of thermodynamic and entanglement entropies are equal. For the case of constant but different thermodynamic and entanglement ingredients we have an 'inertial' observer with the same information speed (8) and the same information entropy about the universe. Thus, for any physical system that can be treated as isolated, there exists a special state (inertial frame) for which the rest of the universe looks similar. Thus, the local relativistic invariance is a fictitious symmetry that has been artificially imposed in physical models to account the observer dependence of geometry. This fictitious symmetry disappears for large, cosmological scale systems, i.e. the relativity principle is subjective and local. Entropy approach leads to a new dimension of inertial frames of reference and the transformation between them. It also removes the concepts 'rest frame' and 'moving frame' and emphasizes the importance of observer and observation.

Let us demonstrate subjective properties of the relativity principle using our model universe of N identical quantum particles in the equilibrium. Consider an elementary fluctuation with action that is written as the product of two quantities, S = Et, one of which, the energy E, does not vary during transition from equilibrium to excited state and thus can be attributed to the fluctuation itself. The quantity E is subjective and is minimal from an observer's point of view (in the observer's frame). The quantity t (time) describes the change of fluctuation (when the model universe bounces back to equilibrium) and can be used as the kinematical parameter describing the evolution of the ensemble. In the linear case (6), for a multi-particle fluctuation, an observer can introducing some fictitious energy E' and the same time parameter t,  $Et + S_{\text{extra}} = E't$ . The extra action,  $S_{\text{extra}}$ , alters total thermodynamic entropy of the observer and leads to the variance of the distribution functions of particles of the ensemble in the expression of the emerged distance (10). Thus it can be interpreted as describing motions,  $S_{\text{extra}} = p_i x^i$ , and consider the relativity principle in 'space-time'.

Now let us try to re-formulate the Special Relativity axioms in information terms (see also [42]). Consider the following two axioms:

- 1. Principle of informational entropy universality: *The laws of physics have the same form for the observers with the same information, but different thermodynamic and entanglement entropies;*
- 2. Principle of finiteness of information density (Bekenstein bound [43]): A finite volume of space can only contain a finite amount of information.

From 2. (that replaces the requirement of invariance of c) one infers that only a finite amount of information can be transmitted in a finite time, otherwise this would require to 'move' an infinite volume of space. Hence, the speed of propagation of information (8) necessarily needs to be finite too. In the finite universe there should exist maximal speed, corresponding to the transfer of all information of the universe in its lifetime. Then, from 1. (analog of the principle or relativity), this velocity must be the same for all 'inertial' observers. The former considerations are enough to derive the whole theory of Special Relativity, with the additional assumptions of homogeneity of space and time and isotropy of the world ensemble.

According to (6), thermodynamic entropy of a moving particle increases as  $dS \sim d(vt)$ . Then, for a system whose entanglement entropy vanishes,  $dI \approx -dS$  ( $\mathcal{H} \rightarrow 0$ ). So, Lorentz contractions of the induced space-time coordinates appear to be the direct consequence of the relation (4). It is known that, if we define inertial observers with classical notions of space-time coordinates, the standard Lorentz transformations can be derived using only the relativity principle, supplemented by the assumptions of homogeneity, isotropy and smoothness [44, 45]. The existence of a universal speed also follows as a consequence of the relativity principle.

Another physical consequence of the principle of finiteness of information density 2. is that, in general, physical quantities do not have perfectly determine values at every time (or, alternatively, that the truth value of certain empirical statements, such as a particle location at a certain instant, is indeterminate). Hence, upholding the principle of finiteness of information density gives us a hint that physics should be at the same time indeterministic and relativistic. These two views are compatible if one regards (in)determinacy itself as relative.

To conclude, in this paper we attempted to describe emerged geometry in terms of informational quantities. The universe was considered as a finite ensemble of non-locally correlated quantum particles. As the main dynamical principle, the assumption of conservation of the sum of all kinds of entropies (thermodynamic, quantum and informational) was used. This entropy balance condition allows symmetry between different components of entropy that in constant cases can be understood as the manifestation of relativity invariance. Instead of mass or energy, we relate thermodynamic entropy to action that moves information theory from abstract mathematics to physics. The fundamental constant of speed is interpreted as the information velocity for the finite world ensemble of all quantum particles. The two postulates, which are enough to derive the whole theory of Special Relativity, are re-formulated as the principles of information entropy universality and finiteness of information density.

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