

Axions in Baryon Chiral Perturbation Theory

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I give an overview of recent developments in the study of the coupling of the QCD axion to nucleons and other octet baryons. I demonstrate how axions can be included into heavy baryon chiral perturbation theory, and present recent numerical results for the several axion-baryon couplings in SU(2) and SU(3) chiral perturbation theory for the Kim–Shifman–Vainstein–Zakharov (KSVZ) and the Dine–Fischler–Srednicki–Zhitnitsky (DFSZ) axion model.

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1. From the QCD θ -vacuum to the QCD axion

Soon after the discovery of the instanton solution of non-Abelian gauge field theories [1], it became clear that the theory of the strong interaction, Quantum Chromodynamics (QCD), has a rich, complex vacuum structure [2, 3]. While this instanton solution was the starting point for the resolution of the $U(1)_A$ problem proposed by t’Hooft [4, 5], it at the same time lead to a new, still unresolved issue in QCD: the strong CP-problem. QCD with the θ -term is not CP-invariant as a non-vanishing θ angle leads to a contribution to the neutron electric dipole moment (nEDM) $\propto \bar{\theta} = \theta + \text{Arg det } \mathcal{M}$ [6] and affects meson and nucleon masses, as well as nuclear interactions and stellar and Big Bang nucleosynthesis [7, 8]. However, theoretical estimations of the nEDM, which roughly vary between $|d_n| \approx 10^{-16} \bar{\theta} e \text{ cm}$ and $|d_n| \approx 10^{-15} \bar{\theta} e \text{ cm}$ [9, 10], and recent measurements yielding $|d_n^{\text{exp}}| < 1.8 \times 10^{-26} e \text{ cm}$ (90% C.L.) [11], imply that $\bar{\theta} \lesssim 10^{-11}$. This is indeed a remarkable result and certainly in need of explanation: Why $\bar{\theta}$ is such a small quantity, whereas in principle it might take on any value between $[-\pi, \pi]$? It seems that there are at least no anthropic constraints that require θ to be of $O(10^{-11})$ [7, 12], not even $O(10^{-2})$ (though $\bar{\theta}$ of $O(1)$ has considerable impact on the universe [7]). Other alternatives, such as a vanishing quark mass seem to be improbable (see, e.g. [13], and references therein).

The Peccei–Quinn (PQ) mechanism [14, 15] coming with a new global chiral symmetry, now often labeled $U(1)_{\text{PQ}}$, is another solution to the strong-CP problem that is vividly discussed up to the present day. One of the reasons is that the pseudoscalar Nambu–Goldstone boson resulting from the spontaneous PQ-symmetry breakdown, called the axion [16, 17], became a reasonable dark matter candidate [18–24]. This in particular applies to the canonical “invisible” axion models, the KSVZ axion model [25, 26] and the DFSZ axion model [27, 28], but also in general to any kind of axion-like particles (ALPs).

In QCD/QED, the traditional axions couple to gluons, quarks, and photons, which means that on a less fundamental level, axions also interact with mesons, nucleons, and other baryons. Here I discuss recent developments in the study of the axion-nucleon coupling [29–33] in $SU(2)$ heavy baryon chiral perturbation theory (HBCHPT) and the general axion-baryon coupling in $SU(3)$ HBCHPT as developed in Ref. [34, 35]. This is motivated from the fact that the axion-nucleon coupling plays a crucial role in determining the traditional axion window [18, 19, 36]

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}, \quad (1)$$

where f_a is the axion decay constant, whose large value causes the axion’s weak coupling to Standard Model particles, but also the smallness of its mass. The process predominantly considered in the literature in the context of the axion-nucleon coupling is nuclear bremsstrahlung in massive stellar objects (see the overviews [9, 37–39]), which requires a precise knowledge of the strength and structure of the axion-nucleon coupling. This is also true for axion-nucleon scattering

$$aN \rightarrow \pi N \quad (2)$$

which might be of some relevance in protosupernova cores, as has been proposed recently [40, 41]. Moreover, it has been suggested that there might be a considerable amount of hyperons existing in the cores of neutron stars (see, e.g., [42] and references therein). Provided this is indeed the case, this would make it necessary to consider also the axion’s coupling to baryons other than the proton and neutron when determining the axion’s contribution to the cooling of neutron stars.

2. QCD with axions

After a suitable chiral rotation, the axion-quark interaction part of the QCD Lagrangian below the PQ symmetry breaking scale can be written in matrix notation as

$$\mathcal{L}_{aq} = -(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}) + \bar{q} \gamma^\mu \gamma_5 \frac{\partial_\mu a}{2f_a} (\mathcal{X}_q - \mathcal{Q}_a) q, \quad (3)$$

where $q = (u, d, s, c, b, t)^\top$ collects the quark spinors, a is the axion field, and

$$\mathcal{M}_a = \exp\left(i \frac{a}{f_a} \mathcal{Q}_a\right) \mathcal{M}, \quad \mathcal{Q}_a = \frac{\mathcal{M}^{-1}}{\langle \mathcal{M}^{-1} \rangle} \approx \frac{1}{1+z+w} \text{diag}(1, z, w, 0, 0, 0). \quad (4)$$

Here, $\mathcal{M} = \text{diag}\{m_q\}$ is the 6×6 quark mass matrix, and $z = m_u/m_d$ and $w = m_u/m_s$ (this particular form of \mathcal{Q}_a has been chosen in order to avoid a mixing between the axion and the neutral Nambu–Goldstone bosons of chiral symmetry breaking). Furthermore, $\mathcal{X}_q = \text{diag}\{X_q\}$ is the 6×6 axion-quark coupling matrix. For the models under consideration, the KSVZ and the DFSZ, one has

$$X_q^{\text{KSVZ}} = 0, \quad X_{u,c,t}^{\text{DFSZ}} = \frac{1}{3} \sin^2 \beta, \quad X_{d,s,b}^{\text{DFSZ}} = \frac{1}{3} \cos^2 \beta = \frac{1}{3} - X_{u,c,t}^{\text{DFSZ}}, \quad (5)$$

where β is related to the vacuum expectation values of the two Higgs doublets in the DFSZ model.

From this Lagrangian one has to determine the external fields a_μ and $a_\mu^{(s)}$ that enter chiral perturbation theory. In SU(2) this can be achieved by separating the 2-dimensional flavor subspace of the two lightest quarks from the rest and by decomposing the matrix $(\mathcal{X}_q - \mathcal{Q}_a)$ into traceless parts and parts with non-vanishing trace, which results in

$$\begin{aligned} \mathcal{L}_{aq} = & -(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}) + \left(\bar{q} \gamma^\mu \gamma_5 \left(c_{u-d} \frac{\partial_\mu a}{2f_a} \tau_3 + c_{u+d} \frac{\partial_\mu a}{2f_a} \mathbb{1} \right) q \right)_{q=(u,d)^\top} \\ & + \sum_{q=\{s,c,b,t\}} \left(\bar{q} \gamma^\mu \gamma_5 c_q \frac{\partial_\mu a}{2f_a} q \right) \end{aligned} \quad (6)$$

with τ_3 being the third Pauli matrix and

$$c_{u\pm d} = \frac{1}{2} \left(X_u \pm X_d - \frac{1 \pm z}{1+z+w} \right), \quad c_s = X_s - \frac{w}{1+z+w}, \quad c_{c,b,t} = X_{c,b,t}. \quad (7)$$

Let c_i , $i = \{1, \dots, 5\}$, refer to the isoscalar couplings $\{u+d, s, c, b, t\}$, then one finds

$$a_\mu = c_{u-d} \frac{\partial_\mu a}{2f_a} \tau_3, \quad a_{\mu,i}^{(s)} = c_i \frac{\partial_\mu a}{2f_a} \mathbb{1} \quad \text{in SU(2)}. \quad (8)$$

In any of the following equations, a summation over repeated i is implied. Accordingly, the flavor subspace separation, this time with respect to the three lightest quarks, yields for the SU(3) case

$$\begin{aligned} \mathcal{L}_{aq} = & -(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}) + \left(\bar{q} \gamma^\mu \gamma_5 \frac{\partial_\mu a}{2f_a} \left(c^{(1)} \mathbb{1} + c^{(3)} \lambda_3 + c^{(8)} \lambda_8 \right) q \right)_{q=(u,d,s)^\top} \\ & + \sum_{q=\{c,b,t\}} \left(\bar{q} \gamma^\mu \gamma_5 \frac{\partial_\mu a}{2f_a} X_q q \right), \end{aligned} \quad (9)$$

where λ_3 and λ_8 are the third and eighth Gell-Mann matrices. Now

$$\begin{aligned} c^{(1)} &= \frac{1}{3} (X_u + X_d + X_s - 1), & c^{(3)} &= \frac{1}{2} \left(X_u - X_d - \frac{1-z}{1+z+w} \right), \\ c^{(8)} &= \frac{1}{2\sqrt{3}} \left(X_u + X_d - 2X_s - \frac{1+z-2w}{1+z+w} \right) \end{aligned} \quad (10)$$

and

$$a_\mu = \frac{\partial_\mu a}{2f_a} \left(c^{(3)} \lambda_3 + c^{(8)} \lambda_8 \right), \quad a_{\mu,i}^{(s)} = c_i \frac{\partial_\mu a}{2f_a} \mathbb{1} \quad \text{in SU(3),} \quad (11)$$

where this time $c_i = \{c^{(1)}, c_c, c_b, c_t\}$.

3. Heavy baryon chiral perturbation theory with axions

3.1 Basic definitions

The baryon fields B and meson fields Φ are collected in

$$B = N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \quad (12)$$

in the case of SU(2) HBCHPT, whereas

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma_3 + \frac{1}{\sqrt{6}}\Lambda_8 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma_3 + \frac{1}{\sqrt{6}}\Lambda_8 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda_8 \end{pmatrix}, \quad \Phi = \begin{pmatrix} \pi_3 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_3 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix} \quad (13)$$

in SU(3), where in any case Φ appears in the unitary matrix

$$u = \sqrt{U} = \exp\left(i \frac{\Phi}{2F_p}\right). \quad (14)$$

Note that the physical Σ^0 , Λ , π^0 , and η are mixed states parameterized by the mixing angle ϵ constrained from the relation

$$\tan 2\epsilon = \frac{\langle \lambda_3 \mathcal{M}_q \rangle}{\langle \lambda_8 \mathcal{M}_q \rangle}. \quad (15)$$

Here, the heavy baryon limit is applied meaning that $\mathcal{L}_{\Phi B}$ contains an expansion in the inverse baryon mass m_B (= nucleon mass in the SU(2) case). Furthermore, any Dirac bilinear is entirely expressed by means of the baryon four-velocity v_μ and the spin operator $S_\mu = \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu$. This means that the fields B are actually velocity dependent fields, which more correctly would be denoted B_v .

The axion enters HBCHPT in the following basic building blocks:

$$\begin{aligned} u_\mu &= i \left[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger - iu^\dagger a_\mu u - iua_\mu u^\dagger \right], & u_{\mu,i} &= 2a_{\mu,i}^{(s)}, \\ \Gamma_\mu &= \frac{1}{2} \left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - iu^\dagger a_\mu u + iua_\mu u^\dagger \right], \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, & \text{with } \chi &= 2B_0 \mathcal{M}_a. \end{aligned} \quad (16)$$

These objects are 2×2 matrices in the SU(2) case and 3×3 matrices in SU(3). The chiral connection is needed for the chiral covariant derivative

$$\mathcal{D}_\mu N = \partial_\mu N + \Gamma_\mu N \quad \text{in SU(2),} \quad (17)$$

$$[\mathcal{D}_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \quad \text{in SU(3).} \quad (18)$$

3.2 General form of the axion-baryon coupling

As chiral Lagrangians are organized with respect to chiral orders

$$\mathcal{L}_{\Phi B} = \mathcal{L}_{\Phi B}^{(1)} + \mathcal{L}_{\Phi B}^{(2)} + \mathcal{L}_{\Phi B}^{(3)} + \dots + \mathcal{L}_{\Phi}^{(2)} + \mathcal{L}_{\Phi}^{(4)} + \dots \quad (19)$$

it is clear from Eqs. (4), (8), and (11) that any axion-baryon coupling can be organized in terms of inverse powers of the expectedly large parameter f_a , because

$$a_\mu, a_{\mu,i}^{(s)} = \mathcal{O}(1/f_a), \quad \mathcal{M}_a = \exp\left(i \frac{a}{f_a} Q_a\right) \mathcal{M}_q = \mathcal{M}_q + i \frac{a}{f_a} \frac{1}{\langle \mathcal{M}_q^{-1} \rangle} + \mathcal{O}(1/f_a^2). \quad (20)$$

certainly only leading terms $\propto 1/f_a$ can contribute significantly, so the general axion-baryon coupling can be expressed as

$$\begin{array}{c} B_A \\ \diagdown \\ \text{---} a \text{---} \\ \diagup \\ B_B \end{array} = G_{aAB} (S \cdot q), \quad \text{with } G_{aAB} = -\frac{1}{f_a} g_{aAB} + \mathcal{O}\left(\frac{1}{f_a^2}\right), \quad (21)$$

where A, B refer either to p or n (in SU(2)), or to the SU(3) indices of the octet baryons in the physical basis. Finally, g_{aAB} contains an expansion in chiral power counting

$$g_{aAB} = \underbrace{g_{aAB}^{(1)}}_{\text{LO, tree}} + \underbrace{g_{aAB}^{(2)}}_{\text{NLO, } 1/m_B} + \underbrace{g_{aAB}^{(3)}}_{\text{NNLO, } 1/m_B^2, \text{ one-loop}} + \dots \quad (22)$$

3.3 The case of SU(2) HBCHPT

The relevant pieces of the SU(2) HBCHPT Lagrangian up to next-to-next-to-leading order are

$$\begin{aligned} \mathcal{L}_{\pi N} = \bar{N} \bigg\{ & i(v \cdot \mathcal{D}) + g_A (S \cdot u) + g_0^i (S \cdot u_i) + \mathcal{L}_{1/m_N} + \mathcal{L}_{1/m_N^2} \\ & + d_{16}(\lambda) (S \cdot u) \langle \chi_+ \rangle + d_{16}^i(\lambda) (S \cdot u_i) \langle \chi_+ \rangle + d_{17} S^\mu \langle u_\mu \chi_+ \rangle + id_{18} S^\mu [\mathcal{D}_\mu, \chi_-] \\ & + id_{19} S^\mu [\mathcal{D}_\mu, \langle \chi_- \rangle] + \tilde{d}_{25}(\lambda) \left(v \cdot \overleftarrow{\mathcal{D}} \right) (S \cdot u) (v \cdot \mathcal{D}) + \tilde{d}_{25}^i(\lambda) \left(v \cdot \overleftarrow{\mathcal{D}} \right) (S \cdot u_i) (v \cdot \mathcal{D}) \\ & + \tilde{d}_{29}(\lambda) (S^\mu [(v \cdot \mathcal{D}), u_\mu] (v \cdot \mathcal{D}) + \text{h.c.}) + \tilde{d}_{29}^i(\lambda) (S^\mu [(v \cdot \mathcal{D}), u_{\mu,i}] (v \cdot \mathcal{D}) + \text{h.c.}) \bigg\} N, \end{aligned} \quad (23)$$

where \mathcal{L}_{1/m_N} and \mathcal{L}_{1/m_N^2} are the terms of the $1/m_N$ expansion, g_A and the g_0^i 's are the axial isovector and isoscalar coupling constants, and the $d_j^{(i)}$ and $\tilde{d}_j^{(i)}$ are low-energy constants, from which some depend on the scale λ and are needed for the renormalization of the one-pion loop

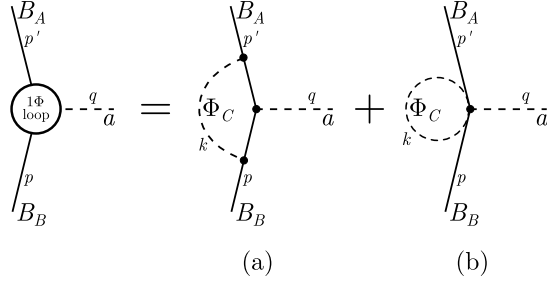


Figure 1: Loop contributions to the axion-baryon coupling. In SU(2), B_A and B_B are either proton or neutron, while in SU(3) A , B and C are SU(3) indices in the physical basis for baryons and mesons.

contributions shown in Fig. 1. The full renormalized NNLO result for the axion-nucleon coupling is (here given in nucleon rest frame, i.e. $v = (1, 0, 0, 0)^T$ and $q_0 = (v \cdot q) \ll m_N$)

$$\begin{aligned}
 g_{aNN} = & g_a \left(1 + \frac{q_0}{2m_N} + \frac{q_0^2}{4m_N^2} \right) + \frac{\hat{g}_a}{6} \left(\frac{g_A M_\pi}{4\pi F_\pi} \right)^2 \\
 & \times \left[-1 + \left(\frac{q_0}{M_\pi} \right)^2 + \frac{2}{q_0 M_\pi^2} \left(\frac{\pi M_\pi^3}{2} - (M_\pi^2 - q_0^2)^{\frac{3}{2}} \arccos \frac{q_0}{M_\pi} \right) \right] \\
 & + 4M_\pi^2 \left[\left(\bar{d}_{16} \tau_3 + d_{17} \frac{m_u - m_d}{m_u + m_d} \right) c_{u-d} + \bar{d}_{16}^i c_i - (d_{18} + 2d_{19}) \frac{m_u m_d}{(m_u + m_d)^2} \right],
 \end{aligned} \tag{24}$$

where the first term is the leading order result with the first two terms of the $1/m_N$ expansion. The $\bar{d}_j^{(i)}$ denote the renormalized and scale independent low-energy constants, and

$$g_a = g_{ACu-d} \tau_3 + g_0^i c_i \mathbb{1}, \quad \hat{g}_a = -g_{ACu-d} \tau_3 + 3g_0^i c_i \mathbb{1}. \tag{25}$$

The couplings g_A and the g_0^i 's can be matched to the nucleon matrix elements, i. e.

$$g_A = \Delta u - \Delta d, \quad g_0^{u+d} = \Delta u + \Delta d, \quad g_0^q = \Delta q, \quad \text{for } q = s, c, b, t \tag{26}$$

where $s^\mu \Delta q = \langle p | \bar{q} \gamma^\mu \gamma_5 q | p \rangle$, with s^μ the spin of the proton. The leading order results for the axion-proton and axion-neutron couplings then can be written as

$$\begin{aligned}
 g_{app}^{(1)} &= -\frac{\Delta u + z\Delta d + w\Delta s}{1 + z + w} + \Delta u X_u + \Delta d X_d + \sum_{q=\{s,c,b,t\}} \Delta q X_q \\
 g_{ann}^{(1)} &= -\frac{z\Delta u + \Delta d + w\Delta s}{1 + z + w} + \Delta d X_u + \Delta u X_d + \sum_{q=\{s,c,b,t\}} \Delta q X_q
 \end{aligned} \tag{27}$$

The numerical results are discussed below when it comes to the SU(3) results.

3.4 The case of SU(3) HBCHPT

In principle, a complete $O(p^3)$ description of the axion-baryon coupling would also include terms from the $1/m_B$ expansion, the NNLO Lagrangian and the topologically same diagrams as in Fig. 1. Here, I restrict the discussion to the leading order calculation based on the Lagrangian (see Ref. [35] for more details)

$$\mathcal{L}_{\Phi B}^{(1)} = \langle i \bar{B} v^\mu [\mathcal{D}_\mu, B] \rangle + D \langle \bar{B} S^\mu \{u_\mu, B\} \rangle + F \langle \bar{B} S^\mu [u_\mu, B] \rangle + D^i \langle \bar{B} S^\mu u_{\mu,i} B \rangle, \tag{28}$$

which as in the SU(2) case comes with axial isovector (F , D) and isoscalar couplings (D^i). The leading order coupling of an axion to any baryon in the physical basis can be written as

$$g_{aAB}^{(1)} = \frac{1}{2} \left\{ D \left(c^{(3)} \left\langle \tilde{\lambda}_A^\dagger \{ \lambda_3, \tilde{\lambda}_B \} \right\rangle + c^{(8)} \left\langle \tilde{\lambda}_A^\dagger \{ \lambda_8, \tilde{\lambda}_B \} \right\rangle \right) + F \left(c^{(3)} \left\langle \tilde{\lambda}_A^\dagger [\lambda_3, \tilde{\lambda}_B] \right\rangle + c^{(8)} \left\langle \tilde{\lambda}_A^\dagger [\lambda_8, \tilde{\lambda}_B] \right\rangle \right) + 2c_i D^i \delta_{AB} \right\}, \quad (29)$$

where $\tilde{\lambda}_A$ are a set of traceless, non-Hermitian matrices, which are the generators of the physical basis. The coupling constants F , D , and D^i are again matched to the nucleon matrix elements

$$\begin{aligned} (D + F) &= g_A = \Delta u - \Delta d, & -(D - 3F) &= \Delta u + \Delta d - 2\Delta s, \\ D^1 &= \Delta u + \Delta d + \Delta s, & D^q &= \Delta q, \quad \text{for } q = c, b, t \end{aligned} \quad (30)$$

With this matching one exactly reproduces Eq. (27) for the axion-proton and axion-neutron coupling, which means that at leading order the SU(2) and SU(3) results for these couplings are identical. Inserting [43]

$$\begin{aligned} \Delta u &= 0.847(50) & \Delta d &= -0.407(34) & \Delta s &= -0.035(13) \\ z &= 0.485(19) & w &= 0.025(1) \end{aligned} \quad (31)$$

one gets the numerical results

$$\begin{aligned} g_{a\Sigma^+\Sigma^+}^{(1)} &= -0.543(34) + 0.847(50)X_u - 0.035(13)X_d - 0.407(34)X_s \\ g_{a\Sigma^-\Sigma^-}^{(1)} &= -0.242(21) - 0.035(13)X_u + 0.847(50)X_d - 0.407(34)X_s \\ g_{a\Sigma^0\Sigma^0}^{(1)} &= -0.396(25) + 0.417(25)X_u + 0.395(25)X_d - 0.407(35)X_s \\ g_{app}^{(1)} &= -0.430(36) + 0.847(50)X_u - 0.407(34)X_d - 0.035(13)X_s \\ g_{a\Xi^-\Xi^-}^{(1)} &= 0.140(15) - 0.035(13)X_u - 0.407(34)X_d + 0.847(50)X_s \\ g_{ann}^{(1)} &= -0.002(30) - 0.407(34)X_u + 0.847(50)X_d - 0.035(13)X_s \\ g_{a\Xi^0\Xi^0}^{(1)} &= 0.267(23) - 0.407(34)X_u - 0.035(13)X_d + 0.847(50)X_s \\ g_{a\Lambda\Lambda}^{(1)} &= 0.126(25) - 0.147(25)X_u - 0.125(25)X_d + 0.677(35)X_s \\ g_{a\Sigma^0\Lambda}^{(1)} &= -0.153(10) + 0.463(25)X_u - 0.476(25)X_d + 0.013(1)X_s \end{aligned} \quad (32)$$

with the model dependent axion-quark couplings X_q . In particular, the constant first terms in each expression represent the KSVZ axion-baryon couplings (where $X_q = 0$), while for the DFSZ one finds

$$\begin{aligned} g_{a\Sigma^+\Sigma^+}^{(1),\text{DFSZ}} &= -0.690(36) + 0.430(21) \sin^2 \beta, & g_{a\Sigma^-\Sigma^-}^{(1),\text{DFSZ}} &= -0.095(29) - 0.158(21) \sin^2 \beta \\ g_{a\Sigma^0\Sigma^0}^{(1),\text{DFSZ}} &= -0.400(29) + 0.143(12) \sin^2 \beta, & g_{app}^{(1),\text{DFSZ}} &= -0.577(38) + 0.430(21) \sin^2 \beta \\ g_{a\Xi^-\Xi^-}^{(1),\text{DFSZ}} &= 0.287(25) - 0.158(21) \sin^2 \beta, & g_{ann}^{(1),\text{DFSZ}} &= 0.269(34) - 0.406(21) \sin^2 \beta \\ g_{a\Xi^0\Xi^0}^{(1),\text{DFSZ}} &= 0.531(29) - 0.406(21) \sin^2 \beta, & g_{a\Lambda\Lambda}^{(1),\text{DFSZ}} &= 0.310(29) - 0.233(12) \sin^2 \beta \\ g_{a\Sigma^0\Lambda}^{(1),\text{DFSZ}} &= -0.308(13) + 0.309(16) \sin^2 \beta. \end{aligned} \quad (33)$$

4. Conclusion

The numerical results at the end of the last section have some notable consequences. First, the coupling of the axion to the neutron can vanish (or might at least be strongly suppressed) in both models (in the DFSZ at $\sin^2 \beta \approx 2/3$), whereas the axion-proton coupling is non-vanishing in any case. Second, the coupling to hyperons are of similar strength as to nucleons, which suggests that it might be advisable to include interactions of axions with these particles also in studies dedicated to neutron star cooling due to axion bremsstrahlung.

In this brief review, I have only shown the $\mathcal{O}(p^3)$ results including one-pion loops for the SU(2) case, while one-meson loops have also been studied for the SU(3) case (see Ref. [35]). However, the numerical results for these cases suffer from the fact that many of the involved low energy constants (especially but not exclusively the isoscalar ones) are entirely unknown. Future work might fill this gap, which would make a more precise determination of the axion-baryon couplings possible.

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