Chiral Dynamics: Theory and Experiment – A Tribute to Aron Bernstein

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I review and discuss the contributions of Aron Bernstein to the field of chiral dynamics.

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[*]Speaker

1. Introductory remarks

This talk is about Aron Bernstein and the footprints he left in the field of chiral dynamics over more than three decades. It is a personal recollection that intends to give a flavor about the works of a brilliant physicist and great human being. In fact, I came across Aron's name first in the late 1970s, when I was an undergraduate at Bochum studying QFT using Schweber's book [1] and noticed the paragraph on the experimental verification of Delbrück scattering on page 595 [2]. Later, we became very good friends and had numerous discussions on physics and other topics. I dearly miss him.

Let me start with a short CV of Aron. He was born April 6, 1931, and grew up in Brooklyn and Queens. In 1953, he obtained his B.Sc. in physics at Union College in Schenectady, New York, and 5 years later, he was awarded the PhD in physics from the University of Pennsylvania, working on Delbrück scattering. He went on as a postdoc to Princeton, where he started nuclear physics (NP) research, see the left panel of Fig. 1, and became an assistant professor at MIT in 1961, performing numerous low-energy NP investigations at the then operating Markle Cyclotron. With the support of Victor Weisskopf, he was promoted to associate professor in 1966 and became full professor in 1975. To my recollection, Aron became interested in low-energy QCD and chiral dynamics in the late 1980s/early 1990s. Together with Barry Holstein, he initiated the "Chiral Dynamics - Theory and Experiment" workshop series that started in 1994 at MIT, with a rather unusual format. Besides the plenary talks delivered by world-leading experts in chiral dynamics, there were true working groups where people sat together and discussed the interplay of theory and experiment to advance the field [3]. The second meeting took place in Mainz in 1997, followed by Jefferson Lab in 2000, Bonn in 2003 until this, the tenth meeting of this series in 2021 in Beijing (online). Aron spent many months in Mainz working at the MAMI accelerator on issues related to threshold pion photoproduction, see also Sec. 2, supported by a prestigious Humboldt research prize, and later was one of the initiators of the PrimEx experiment at Jefferson Lab to measure precisely the neutral pion lifetime and test the chiral anomaly of QCD, see also Sec. 3. Besides all his works in physics, which I can not possibly cover, he was a life-long arms control activist, a topic that was always very dear to his heart. Aron passed away on January 14, 2020, leaving an impressive legacy which I can
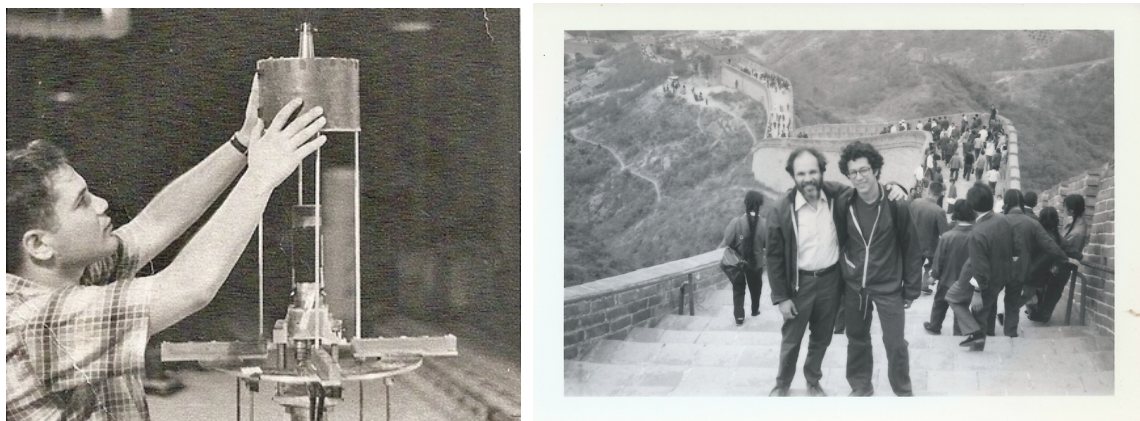

Figure 1: Left panel: Aron during this postdoc time at Princeton. Right panel: Aron with his son on the Great Wall in the late 1970s. Pictures courtesy of the Bernstein family.
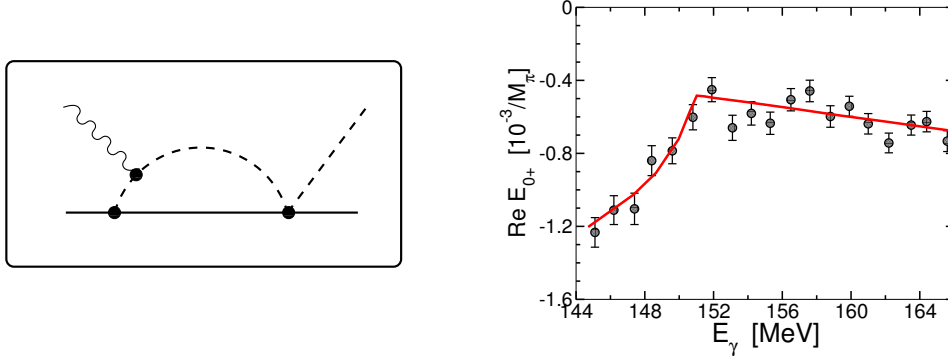
Figure 2: Left panel: The triangle diagram in neutral pion photoproduction. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order. The black dots represent leading order interactions. Crossed diagram not shown. Right panel: The real part of the electric dipole amplitude E_{0+} in the threshold region. E_γ is the photon energy in the lab frame. The data are from Ref. [15] and the fourth order baryon CHPT calculation is from Ref. [16].

only recount in small parts in the following.

2. Threshold pion photoproduction off nucleons

Consider the reaction $\gamma p \rightarrow \pi^0 p$ in the threshold region, where the pion three-momentum is very small, $\vec{q}_\pi \simeq 0$. At threshold, a low-energy theorem (LET) had been derived in [4, 5],

$$E_{0+,\text{thr}} = -\frac{eg_{\pi N}}{8\pi m_p}\left[\mu - \frac{1}{2}(3 + \kappa_p)\mu^2\right] = -2.3 \cdot \frac{10^{-3}}{M_\pi}\,, \quad \mu = \frac{M_\pi}{m_p} \simeq \frac{1}{7}\,, \tag{1}$$

in terms of the pion-nucleon coupling constant $g_{\pi N}$, the proton anomalous magnetic moment, μ_p, the proton mass m_p and the pion mass M_π. This LET was challenged by the measurements at Saclay [6] and Mainz [7], which gave much smaller values for $E_{0+,\text{thr}}$ than given in Eq. (1). A flurry of theoretical papers was published to resurrect the LET. This is were Aron entered the scene. In the paper "Threshold pion photoproduction and chiral invariance," co-authored with Barry Holstein, he argued that the Saclay and Mainz measurements were subject to corrections that brought the results in agreement with the LET [8] – "all roads lead to Rome", as they stated. But when it comes to chiral dynamics, all ways indeed lead to Bern! In 1991, Véronique Bernard, Jürg Gasser, Norbert Kaiser and I (BGKM) reanalyzed the LET in baryon chiral perturbation theory (CHPT), and found that the Taylor expansion in the energy is not well behaved, that is the so-called triangle diagram, see the left panel of Fig. 2, starts to contribute already at $O(\mu^2)$ and not at $O(\mu^3)$ as expected from the power counting for one-loop diagrams. In fact, the authors of Ref. [5] were aware of such IR effects (non-analyticities in the amplitudes) but did not consider the triangle diagram. So the LET really reads (in fact, the old and incorrect LET was later called a LEG (low-energy guess) [9])

$$E_{0+,\text{thr}} = -\frac{eg_{\pi N}}{8\pi m_p}\left[\mu - \left(\frac{3 + \kappa_p}{2} + \frac{m_p^2}{16F_\pi^2}\right)\mu^2 + O(\mu^3)\right]\,, \tag{2}$$

where the new term at $O(\mu^2)$ is numerically sizable and the issue of convergence of the series arises. Thus, one needs to work out the $O(\mu^3)$ corrections, which was achieved by BKM in
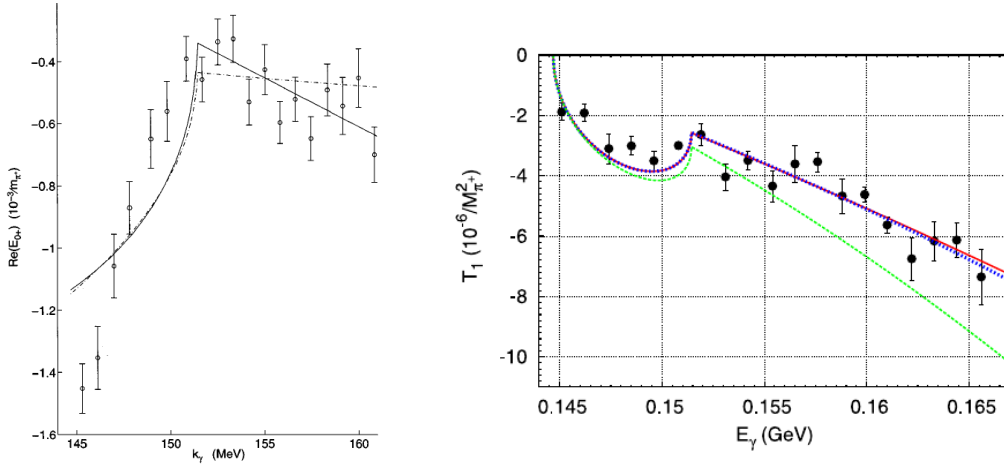
Figure 3: Left panel: Re E_{0+} versus the photon energy. The circles represent the multipole fit, the solid line represents the unitary fit based on Eq. (3) and the dashed-dotted line is the fit based on CHPT. Figure from Ref. [24]. Right panel: T_1 extracted from the data of Ref. [15] using the following approaches: SP (dotted), SPD (solid) and SP with the D-waves added without refitting the data (dashed). Figure from Ref. [31]

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1994 [10]. In fact, to verify the proper LET, progress was made in two strongly intertwined strands. On the theory side, BKM were improving the theory and the fits to the Mainz data [11], whereas on the experimental side, Reinhard Beck, his students and Aron were performing improved experiments [12]. Note also the experimental activity at Saskatoon [13, 14]. This culminated on the experimental side with the 2001 paper by Schmidt et al. [15] and on the theory side with the improved fourth order calculation by Bernard, Kubis and myself [16], see the right panel of Fig. 2. As we will see later, this is still state-of-the-art in the determination of $E_{0+,\text{thr}}$. More details on this intriguing story are given in the review [17]. It is also worth noticing in this context that new P-wave LETs [10, 11] were successfully tested, but this is a different story not told here.

In fact, Aron was even more interested in the unitary cusp due to the opening of the $\pi^+ n$ threshold just 6.76 MeV above the $\pi^0 p$ threshold. Cusps had been predicted by Wigner already in 1948 [18] but had proven to be elusive in experiment, and are still a hot topic nowadays, see e.g. [19]. In the context of neutral pion photoproduction, a cusp was predicted by Höhler and Müllensiefen in 1959 [20],

$$E_{0+}^{\gamma p \to \pi^0 p}(k_\gamma) = A_0(k_\gamma) + i\beta\, q_+ \, , \;\; \beta = E_{0+}^{\gamma p \to \pi^+ n} \cdot a_{\text{CEX}}^{\pi^+ n \leftrightarrow \pi^0 p} \, , \tag{3}$$

with q_+ the momentum of the charged pion in the $\pi^+ n$-channel and the strength of the cusp β featuring the pion-nucleon charge exchange (CEX) scattering length $a_{\text{CEX}}^{\pi^+ n \leftrightarrow \pi^0 p}$. Further, $E_{0+}^{\gamma p \to \pi^+ n}$ is given by the famous Kroll-Ruderman term. This formula was rediscovered in the 1980s by Fäldt [21], Laget [22] and others. Aron realized early the importance of measuring β to get a handle on this πN scattering length as a test of chiral dynamics, see his talk at the Blaubeuren meeting in 1995 [23]. In the paper "Observation of a unitary cusp in the threshold $\gamma p \to \pi^0 p$ reaction" [24] Aron and coworkers performed a rigorous multipole analysis of the threshold data from MAMI and indeed observed the unitary cusp, see the left panel of Fig. 3. Clearly, from these data any precision determination of the thought-after πN scattering length was not possible, but in this paper it was

also shown how the measurement of polarization observables in $\vec{\gamma}p \to \pi^0 p$ would make such a determination possible. For an update on the prediction for the cusp, see Ref. [25].

From the beginning of his considerations of neutral pion photoproduction, Aron was always intrigued to get a handle on isospin-breaking effects. This was on the one hand inspired by Steve Weinberg's 1977 paper on the "Problem of Mass", where he predicted a huge isospin breaking effect in the scattering lengths for neutral pion scattering off protons and neutrons, $a(\pi^0 p) = 1.4 \cdot 10^{-15}$cm versus $a(\pi^0 n) = 1.9 \cdot 10^{-15}$cm, e.g. these differ by more than 30% while isospin conservation would say that they should be equal [26]. One the other hand, Aron was fascinated by the work of Jürg Gasser and Heiri Leutwyler on the determination of the light quark masses [27, 28], a topic closely related to strong isospin breaking. In the paper "Light quark mass difference and isospin breaking in electromagnetic pion production" [29], he developed a three-channel generalization of the Fermi-Watson theorem to relate isospin breaking in photopion production and πN scattering. In particular, he proposed to measure the target asymmetry T to determine isospin violation in the cusp strength β respectively in $a_{\text{CEX}}^{\pi^+ n \leftrightarrow \pi^0 p}$. Furthermore, this paper contains a proposal to measure $a(\pi^0 p)$ via the imaginary part of the E_{0+} amplitude below the $\pi^+ n$ threshold, a very daunting enterprise but a first step to check Weinberg's prediction. I consider this paper Aron's photoproduction masterpiece. His view on the status of pion photoproduction and the opportunities at MAMI and HIγS is nicely summarized in the 2009 review article [30].

However, Aron was still not completely satisfied with the theory of threshold pion photoproduction. With Cesar Fernández-Ramírez and Bill Donnelly he investigated the effects of the D-waves in neutral pion photoproduction [31, 32]. As usual, one expands the differential cross section in the threshold region in terms of Legendre polynomials $P_i(\theta)$,

$$\frac{d\sigma}{d\Omega} \propto T_0 + T_1 P_1(\theta) + T_2 P_2(\theta) + T_3 P_3(\theta) + T_4 P_4(\theta) + \ldots, \tag{4}$$

where the T_i depend on the energy. If one takes the S- and P-waves into account, then only the $T_{0,1,2}$ contribute (these are often called A, B, C), whereas adding the D-waves leads to the appearance of the the quantities T_3 and T_4. For the T_1-coefficient, this amounts to

$$T_1 = 2 \operatorname{Re} \left[E_{0+} P_1^* \right] + \delta T_1 , \tag{5}$$

with $P_1 = 3E_{1+} + M_{1+} - M_{1-}$ the conventional combination of electric and magnetic P-waves and δT_1 emerges due to the P/D-wave interference. This effect is particularly pronounced above the $\pi^+ n$ threshold as shown in the right panel of Fig. 3. So the inclusion of the D-waves improves the accuracy and their effect is most visibly seen in the polarization observables.

All these efforts resulted in the MAMI proposal A2-10/09 with the title "Measurement of Polarized Target and Beam Asymmetries in Pion-Photoproduction on the Proton: Test of Chiral Dynamics" of the A2 collaboration, led by Michael Ostrick, Dave Hornidge, Wouter Deconinck and Aron, which was quite ambitious, proposing precise measurements of $\vec{\gamma}p \to \pi^0 p$ from threshold up to the region of the Δ-resonance, with the aim of providing stringent tests of the chiral dynamics of QCD. After lots of work this resulted in the paper "Accurate Test of Chiral Dynamics in the $\vec{\gamma}p \to \pi^0 p$ Reaction" [33]. In that paper, precision measurements of the differential cross sections $d\sigma/d\Omega$ and the linearly polarized photon asymmetry $\Sigma = (d\sigma_\perp - d\sigma_\parallel)/(d\sigma_\perp - d\sigma_\parallel)$ were reported. In fact, the energy dependence of the photon asymmetry Σ had never been measured, e.g. in
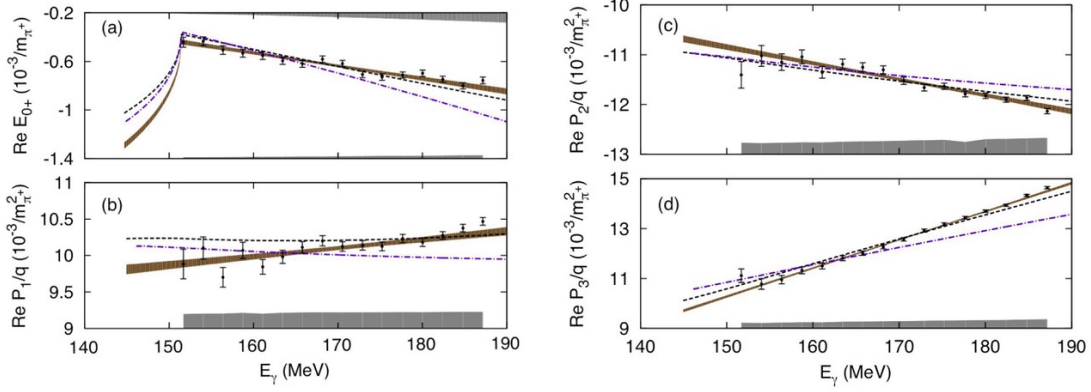
Figure 4: S- and P-wave multipoles versus the photon energy in the lab frame. Left panel: (a) Re E_{0+}. (b) Re P_1/q. Right panel: (c) Re P_2/q. (d) Re P_3/q. The data are depicted by the black circles with error bars. The green bands represent the empirical fits based on the unitary SP approximation. The dashed black line denotes the heavy baryon CHPT result and the blue dotted line is based on covariant baryon CHPT [34]. Figure courtesy of Dave Hornidge.

Ref. [15] this quantity was averaged and given for just one energy. These data allowed for a precise determination of the energy dependence of the real parts of the S- and all three P-wave amplitudes for the first time and provided the most stringent test of the predictions of CHPT and its energy region of agreement with experiment, see Fig. 4. There is one drawback with these data, as a close inspection of Fig. 4 reveals: Statistically significant data could only be taken starting from the $\pi^+ n$ threshold, that is why the energy dependence of Re E_{0+} shown in Fig. 2 still defines the state-of-the-art from the $\pi^0 p$ threshold up to the cusp. The reason for this was the increase in energy of the MAMI accelerator, which benefited many experiments but not the extraction of the multipoles at and very close to the threshold.

One last issue in neutral pion photoproduction concerns the range of validity of chiral perturbation theory. From the beginning of our discussions on this topic, Aron always asked me how far above threshold one could go with heavy baryon CHPT? My answer, which was largely based on intuition, always was that $E_\gamma^{\max} = 170$ MeV would be the limit, may be 180 MeV if one is optimistic. Remember that the threshold is located at $E_\gamma^{\mathrm{thr}} = 144.7$ MeV. Note also that in all our papers, BKM and associates showed the multipoles up to $E_\gamma^{\max} = 165$ MeV, see e.g. [10, 11, 16]. This question was finally answered by Aron himself in collaboration with Cesar Fernández-Ramírez. In the paper "Upper Energy Limit of Heavy Baryon Chiral Perturbation Theory in Neutral Pion Photoproduction" [35], they compared HBCHPT at $O(p^4)$ [10, 11] with a unitarized version (U-HBCHPT), in which Im E_{0+} was taken from the exact unitarity relation Eq. (3), and the S- and P-wave fit to the precise data from Ref. [33]. The outcome is shown in Fig. 5. There is very good agreement up to $E_\gamma^{\max} = 170$ MeV and marked deviations start to appear for higher maximal energies. So my intuition was not that bad and Aron put the answer on firm grounds.

Further progress was made by the A2 collaboration in Ref. [36]. They presented measurements of the polarization-dependent cross sections σ_T associated with the target asymmetry T for the reaction $\gamma \vec{p} \to \pi^0 p$ with transverse polarization from the π^0 threshold to energies of 190 MeV. This allowed for the first time for a direct extraction of the imaginary part of the E_{0+} multipole.
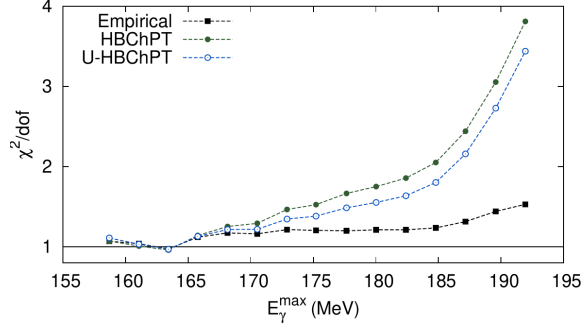
Figure 5: χ^2/dof energy dependence fits from the photon energy $E_\gamma^{\min} = 151.7\,\text{MeV}$ up to the variable maximum energy $E_\gamma^{\max}$. The black squares, open blue circles and full green circles represent the empirical, the HBCHPT and the U-HBCHPT fit, in order. The lines are drawn to guide the eye. Figure adapted from Ref. [35].

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Within uncertainties, this result is in agreement with the fourth order HBCHPT prediction as well as relativistic predictions from Refs. [34, 37]. This story of neutral pion photoproduction can be considered a flagship for the fruitful interplay of theory and experiment, exactly what always has been on Aron's mind. I will come back to this issue at the end of the talk.

A few remarks on neutral pion electroproduction, that is the reaction $\gamma^\star(Q^2)p \to \pi^0 p$, are in order. Here, due to the virtuality Q^2 of the virtual photon, the kinematics is more complicated as compared to the real photon case. But this is also a blessing as new longitudinal multipoles arise that allow for further tests of chiral dynamics, described in terms of appropriate LETs [38, 39]. The HBCHPT formalism for this reaction was developed by BKM and Harry Lee in Refs. [40, 41]. Early experiments from NIKHEF [42, 43] and Mainz [44, 45] showed some agreements but also disagreements with the HBCHPT predictions. Aron and I were involved in the Bigbite collaboration proposal on "Precision Measurements of Electroproduction of π^0 near threshold: A Test of Chiral QCD Dynamics" at Jefferson Lab [46] , which intended to settle all these questions. The experiment took some time to finally produce interesting data that were published in 2015, co-authored by Aron [47]. The outcome of this work is that the S-wave cross section a_0 is in reasonable agreement with the CHPT predictions, but the P-wave coefficient b in the threshold cross section $A_0^{T+L} = a_0 + b|\vec{p}_\pi|^2$, see Ref. [47] for details, starts to deviate from the predictions already at low virtualities $Q^2 \simeq 0.07\,\text{GeV}^2$. Clearly, more experimental and theoretical work in this field is needed.

3. The chiral anomaly and the neutral pion lifetime

As it is well known, the decay $\pi^0 \to 2\gamma$ has revealed anomalous symmetry breaking, meaning that quantum corrections break a symmetry of the classical theory, see the work by Adler [48], Bell and Jackiw [49] in 1969 and many others to follow. Calculating the pertinent VVA triangle diagram and its crossed partner, one obtains a precise prediction at leading order for the width (lifetime) of the neutral pion

$$\Gamma_{\pi^0\gamma\gamma}^{\text{anom}} = \frac{\alpha^2 M_\pi^3}{64\pi^3 F_\pi^2} = 7.76\,\text{eV} \ . \tag{6}$$
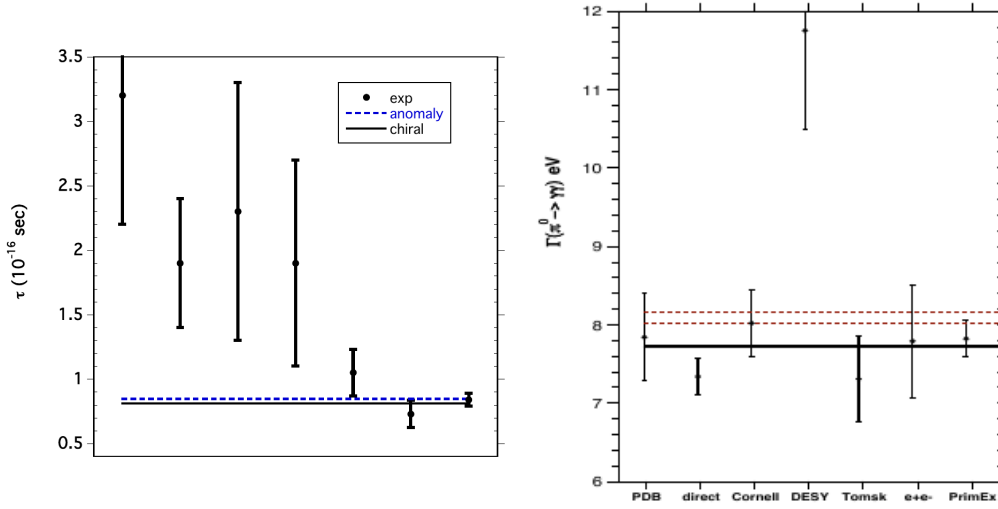
Figure 6: Left panel: Early measurements of the neutral pion lifetime in comparison to the anomaly prediction and the leading chiral corrections due to pseudoscalar meson mixing. Figure courtesy of Aron Bernstein. Right panel: Preliminary PrimEx results presented by Aron in his 2009 Chiral Dynamics talk. Figure from Ref. [59]

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Early measurements were rather inconclusive, related to the fact that this lifetime is difficult to measure, see the left panel of Fig. 6 and the detailed discussion in the nice review by Aron and Barry Holstein [50]. On the theoretical side, the first calculations of corrections to this leading order result in the framework of chiral perturbation theory based on the Wess-Zumino-Witten Lagrangian were done in Refs. [51, 52], leading to a few percent correction mostly due to π^0-η-η' mixing. Aron was intrigued by Bachir Moussallam's work on $\pi^0, \eta, \eta' \to 2\gamma$ decays in CHPT when he had a sabbatical at MIT and worked on chiral sum rules and higher-order corrections to π^0, η and η' decays [53]. Based on these observations and early works, Aron was one of the initiators of the PrimEx proposal [54]. This experiment intended to measure the neutral pion lifetime with an error of 1.5%, a very significant improvement in accuracy compared to earlier measurements. Consequently, the race was on to improve the theoretical predictions based on CHPT with dynamical photons or CHPT combined with the $1/N_c$ expansion, which led to two publications which appeared within one month in 2002, on the one side the work by Balasubramanian Ananthanarayan and Bachir Moussallam [55] and on the other side the one of José Goity, Aron and Barry Holstein [56], with very similar results

$$\Gamma_{\pi^0\gamma\gamma} = \begin{cases} 8.06 \pm 0.02 \pm 0.06 \text{ eV } [55], \\ 8.10 \pm 0.08 \text{ eV } [56]. \end{cases} \tag{7}$$

This was later improved to two-loop accuracy in Ref. [57] with the result $\Gamma_{\pi^0\gamma\gamma} = 8.09 \pm 0.11$ eV. For more details on these theoretical developments, I refer to Kampf's talk at this workshop [58].

Aron presented preliminary results of the PrimEx experiment at the 6th Chiral Dynamics meeting in Bern in 2009, see the right panel of Fig. 6 and the corresponding PrimEx paper was published in 2011 [60]. It was highlighted in Aron's opening talk at the 7th Chiral Dynamics

meeting in 2012 at Jefferson Laboratory [61]. The measured value was

$$\Gamma_{\pi^0\gamma\gamma} = 7.82 \pm 0.14 \pm 0.17 \text{ eV} , \qquad (8)$$

so the achieved accuracy was 2.8%, somewhat above the targeted value. After this groundbreaking result, Aron teamed up again with Barry Holstein to write the nice review on "Neutral pion lifetime measurements and the QCD chiral anomaly," published in *Review of Modern Physics* in 2013 [50]. This is another testimony of Aron's legacy! The 1.5% accuracy for the pion lifetime measurement was finally achieved by the the PrimEx-II experiment, which reported $\Gamma_{\pi^0\gamma\gamma} = 7.80 \pm 0.06 \pm 0.11$ eV in 2020 [62]. More details on this achievement are given in Gasparian's talk at this workshop [63].

Aron was also interested in the decays of pseudoscalar mesons (PS), $PS \to \gamma\gamma^\star$, and the radii of the PS mesons. In his contribution to the 8th Chiral Dynamics meeting in Pisa, he derived the radii of pseudoscalar mesons the decays $PS \to \gamma^\star(Q^2)\gamma$ at low Q^2, [64]

$$F(Q^2) = F_{PS}(0)\left(1 - Q^2\langle\frac{1}{6}r^2\rangle + \ldots\right) , \quad F_{PS}(0) = \left(\frac{4\Gamma(PS \to 2\gamma)}{\pi M_{PS}^3 \alpha^2}\right)^{1/2} . \qquad (9)$$

His intention was to stimulate new, accurate experiments and further calculations based on CHPT and lattice QCD. Note that these transition form factors gained prominence in the theoretical analysis of the muon anomalous magnetic moment.

4. Summary and an experimental challenge

The imprints Aron left in the field of chiral dynamics are best summarized by the talks he gave at the Chiral Dynamics meetings over more than two decades, as summarized in Table 1. These cover the issues discussed here plus other topics in chiral dynamics, that were less central to Aron's work. Also, he was one of the founding fathers of this successful workshop series and over the years enthusiastically helped to shape the program of each single meeting.

Year	Place	Proc.	Title of Aron's talk
1994	MIT	[3]	none - main organizer
1997	Mainz	[65]	*Introduction to Chiral Dynamics: Theory and Experiment*
2000	JLab	[66]	*Experimental Chiral Dynamics*
2003	Bonn	[67]	*Hadron Deformation and Chiral Dynamics*
2006	Duke	[68]	*Opening Remarks: Experimental Tests of Chiral Symmetry Breaking*
2009	Bern	[69]	*Lifetime Measurement of the π^0 Meson and the QCD Chiral Anomaly*
2012	JLab	[70]	*Outlook*
2015	Pisa	[71]	*The $\pi^0, \eta, \eta' \to \gamma\gamma^\star$ Decay Rates and Radii*
2018	Duke	[72]	none - could not attend, but very active organizer

Table 1: Summary of the first to ninth "Chiral Dynamics - Theory & Experiment" workshop with its year, location, the reference to the proceedings and Aron's talk or role.

In Aron's spirit, I will end this contribution with a challenge to the experimentalists. To be specific, consider now the reaction $\gamma n \to \pi^0 n$ in the threshold region. In the classical dipole picture,

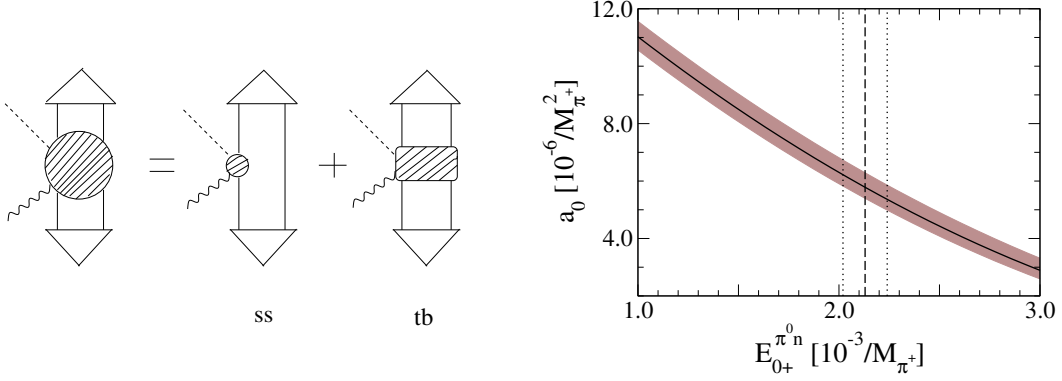**Figure 7:** Left panel: Feynman diagrams for $\gamma d \to \pi^0 d$. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order. The triangle represents the deuteron wave function. The single scattering (ss) amplitude is sensitive to pion photoproduction on the proton and the neutron, whereas the three-body (tb) amplitude subsumes all corrections from meson-exchange currents in terms of pion exchanges and local four-nucleon vertices. Right panel: Sensitivity of the S-wave cross section a_0 to the single-neutron multipole $E_{0+}^{\pi^0 n}$ in π^0 production off ^{3}He. The vertical dashed line gives the CHPT prediction for $E_{0+}^{\pi^0 n}$ and the vertical dotted lines indicate a 5% error in this quantity. The shaded band gives an estimate of the theory error from the few-body calculation combined with the one in the neutron amplitude. Figure from Ref. [80]

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we have $E_{0+}(\gamma n \to \pi^0 n) = 0$ [73]. In CHPT, one obtains the counter-intuitive prediction [10]

$$E_{0+}^{\pi^0 n} = 2.1 > |E_{0+}^{\pi^0 p}| \simeq 1.2 \,, \tag{10}$$

in the canonical units. This is truly remarkable as $|E_{0+}^{\pi^0 n}| \simeq 2|E_{0+}^{\pi^0 p}|$ shows that quantum effects clearly defy intuition. Since free neutron targets are not available, a first test of the prediction can be made on the deuteron, which is an isoscalar target and the S-wave amplitude for neutral pion photoproudction takes the form

$$E_d = E_d^{ss} + E_d^{tb} \,, \quad E_d^{ss} \propto E_{0+}^{\pi^0 p} + E_{0+}^{\pi^0 n} \,, \tag{11}$$

where "ss" denotes the single scattering amplitude and "tb" are the three-body corrections in Weinberg's notation [74], see also the left panel of Fig. 7. Such a few-body calculation was performed by Silas Beane, Véronique Bernard, Harry Lee, Bira van Kolck and me. In a hybrid calculation with precise wave functions from semi-phenomenological models and amplitudes from HBCHPT, in Ref. [75] it was shown that the three-body terms are dominant but converge quickly, leading to the prediction $E_d^{th} = -1.8 \pm 0.2$, based on the CHPT predictions for $E_{0+}^{\pi^0 p}$ and $E_{0+}^{\pi^0 n}$. This is consistent with the old Saclay data that were reanalyzed in 1987, leading to $E_d^{exp} = -1.7 \pm 0.2$ [76]. However, both values are presumably afflicted with larger uncertainties. This was later extended to neutral pion electroproduction off the deuteron in Refs. [77, 78]. The topic of extracting the S-wave multipole $E_{0+}^{\pi^0 n}$ in photoproduction off light nuclei was taken up by Mark Lenkewitz, Evgeny Epelbaum, Hans-Werner Hammer and me in Refs. [79, 80]. There, the sensitivity of the threshold cross section a_0 for $\gamma + {}^3\mathrm{H} \to \pi^0 + {}^3\mathrm{H}$ to the neutron S-wave amplitude was worked out with operators and wave functions from chiral EFT. The S-wave cross section is defined as

$$a_0 = \frac{|\vec{k}_\gamma|}{|\vec{q}_\pi|} \frac{d\sigma}{d\Omega}\bigg|_{\vec{q}_\pi = 0} = |E_{0+}|^2 \,, \tag{12}$$

with $\vec{k}_\gamma$ and $\vec{q}_\pi$ the photon and the pion three-momentum, respectively. The resulting cross section a_0 as as function of the elementary neutron S-wave amplitude is shown in the right panel of Fig. 7, indicating that ^{3}He is a promising candidate to test the CHPT prediction for $E_{0+}^{\pi^0 n}$. Therefore, such an experiment should be performed – a true challenge for my experimental colleagues. Aron would have loved to see/perform such an experiment.

I hope that with this contribution I could express my deep appreciation for Aron's work in the field of chiral dynamics, with this meeting serving as yet one more testimony. On the personal side, let me finish by noting that over the years, Aron has not only been an esteemed colleague but also has become a good friend.

Acknowledgments

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