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Nuclear forces in a manifestly Lorentz-invariant formulation of chiral EFT

Xiu-Lei Ren*

Institut für Kernphysik & Cluster of Excellence PRISMA$^+$, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany

E-mail: xiulei.ren@uni-mainz.de

We report on the recent development of the chiral nucleon-nucleon interaction using time-ordered perturbation theory in the framework of manifestly Lorentz-invariant chiral effective field theory. We follow the standard philosophy suggested by Weinberg, and study the chiral force up to next-to-next-to-leading order.

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1. Introduction

The study frontiers of nuclear force has moved from the diverse phenomenological models to the QCD-based theories and the lattice QCD simulations in the last two decades. Chiral perturbation theory, as an effective field theory (EFT) of low-energy QCD, has been suggested to formulate the nuclear force in 1990s by S. Weinberg [1, 2]. Based on this original idea, the chiral forces have been extensively studied: the nucleon-nucleon (NN) interaction has been formulated up to fifth order with very high precision; the more complicated three/four-nucleon forces have also been studied up to higher orders. For reviews of chiral forces see e.g. Refs. [3–5].

Along with the above development of chiral forces, how to correctly renormalize the NN scattering amplitude due to the singular behavior of chiral pion-exchange potentials at short distances has been intensively investigated/discussed but still remains controversial. A collection of different points of view on this issue can be found in Refs. [2, 6–38].

On the other hand, a clear consensus is that the singular chiral potential is an artifact of a naive extrapolation of the long-range potential to short distances. This is clear from the fact that the underlying theory does not have deeply bound states which are a characteristic feature of singular potentials. This fact triggers us to think about how to improve the ultraviolet (UV) behavior of chiral potential to obtain a renormalizable scattering amplitude. An initial idea, called a modified Weinberg approach, was proposed in Ref. [39] based on the manifestly Lorentz-invariant effective Lagrangian. The resulting one-pion-exchange (OPE) potential, which has the same infrared behavior as its non-relativistic counterpart, possesses a milder UV behavior which leads to a perturbatively renormalizable amplitude.

Following this idea, a systematic framework of formulating the nucleon-nucleon interaction by using the time-ordered perturbation theory (TOPT) in covariant chiral effective field theory (ChEFT) has been proposed in Ref. [40]. Diagrammatic rules of TOPT have been derived for particles with non-zero spin and interactions involving time derivatives. They can be applied to systematically derive chiral potentials at any chiral order. In this conference contribution the NN interaction up to next-to-next-to-leading order (NNLO) is presented, the renormalization issue of obtained potentials is discussed, and the resulting peripheral phase shifts are shown. More details can be found in our recent works [40, 41].

2. Theoretical framework

Our scheme follows the standard procedure of formulating chiral forces by using time-ordered perturbation theory, which was employed in the pioneered works [1, 2, 42] in the non-relativistic (heavy-baryon) chiral EFT. The detailed comparison of two frameworks is presented in Fig. 1. The take-home message is that our results should be consistent with the non-relativistic case when $1/m_N$ expansion is performed.

Below we explain our scheme point by point.

- **Effective Lagrangian:** Our starting point is the manifestly Lorentz invariant effective chiral Lagrangian, which is needed for calculating the NN potential up to NNLO

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(2)}_{NN}, \tag{1}$$

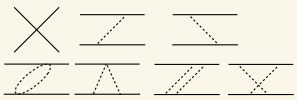
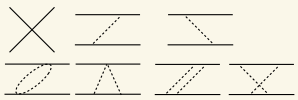
	Non-relativistic (Heavy-baryon)	**Manifestly Lorentz invariant**
Chiral Lagrangians	$N^\dagger \left[i(v\cdot D) + g_A (S\cdot u) \right] N$ $-\frac{1}{2} C_S \left(N^\dagger N\right)\left(N^\dagger N\right) - \frac{1}{2} C_T \left(N^\dagger \vec{\sigma} N\right)\left(N^\dagger \vec{\sigma} N\right) + \cdots$	$\bar{\Psi}_N \left\{ i\gamma_\mu D^\mu - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$ $+\frac{1}{2}\Big[C_S (\bar{\Psi}_N \Psi_N)(\bar{\Psi}_N \Psi_N) + C_A \left(\bar{\Psi}_N \gamma_5 \Psi_N\right)\left(\bar{\Psi}_N \gamma_5 \Psi_N\right)$ $+ C_V \left(\bar{\Psi}_N \gamma_\mu \Psi_N\right)\left(\bar{\Psi}_N \gamma^\mu \Psi_N\right) + C_{AV} \left(\bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N\right)\left(\bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N\right)$ $+ C_T \left(\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N\right)\left(\bar{\Psi}_N \sigma^{\mu\nu} \Psi_N\right) \Big] + \ldots$
Potential TOPT diagrams		
Scattering equations (T = V + VGT)	Lippmann-Schwinger eq.	Kadyshevsky eq.
Power counting	Weinberg p.c.	Weinberg p.c.

Figure 1: Comparison of theoretical frameworks of chiral nuclear force

where the superscripts denote the chiral orders. The lowest order purely pionic Lagrangian reads [43],

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \tag{2}$$

with the pion decay constant $f_\pi = 92.4$ MeV, $u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$, $\chi_+ = u^\dagger \chi u + u \chi u^\dagger$, $u = \exp(i\Phi/2f_\pi)$ and $\chi = \mathrm{diag}(M_\pi^2, M_\pi^2)$. The pion-nucleon part of the effective Lagrangian up to second order reads [44, 45]

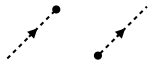
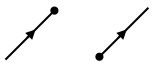
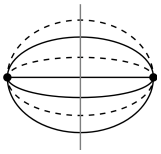
$$\begin{aligned}
\mathcal{L}^{(1)}_{\pi N} &= \bar{\Psi}_N \left[i \not{D} - m_N + \frac{g_A}{2} \gamma^\mu \gamma^5 u_\mu \right] \Psi_N, \\
\mathcal{L}^{(2)}_{\pi N} &= \bar{\Psi}_N \Big\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4 m_N^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \left\langle u^\mu u_\mu \right\rangle \\
&\quad - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \Big\} \Psi_N \,,
\end{aligned} \tag{3}$$

with the axial vector coupling g_A, $D_\mu \Psi_N = \partial_\mu \Psi_N + [\Gamma_\mu, \Psi_N]$, and $\Gamma_\mu = \frac{1}{2} \left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger \right)$. The four low-energy constants (LECs) $c_1, ..., c_4$ are fixed as $c_1 = -0.74$ GeV^{-1}, $c_2 = 1.81$ GeV^{-1}, $c_3 = -3.61$ GeV^{-1}, and $c_4 = 2.17$ GeV^{-1}, which are obtained from the order-Q^2 matching to the πN subthreshold parameters, determined by the Roy-Steiner analysis of πN scattering [46], using covariant ChEFT [47]. The nucleon-nucleon contact interaction Lagrangian up to the second order has the form [48–50]

$$\begin{aligned}
\mathcal{L}^{(0)}_{NN} =& \frac{1}{2} \Big[C_S (\bar{\Psi}_N \Psi_N)(\bar{\Psi}_N \Psi_N) + C_A \left(\bar{\Psi}_N \gamma_5 \Psi_N \right) \left(\bar{\Psi}_N \gamma_5 \Psi_N \right) \\
&+ C_V \left(\bar{\Psi}_N \gamma_\mu \Psi_N \right) \left(\bar{\Psi}_N \gamma^\mu \Psi_N \right) + C_{AV} \left(\bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N \right) \left(\bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N \right) \\
&+ C_T \left(\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N \right) \left(\bar{\Psi}_N \sigma^{\mu\nu} \Psi_N \right) \Big], \\
\mathcal{L}^{(2)}_{NN} =& \sum_{i=1} \bar{\Psi}_N \bar{\Psi}_N O_i \Psi_N \Psi_N,
\end{aligned} \tag{4}$$

where the operators O_i are listed in Refs. [49, 50]. The introduced unknown LECs can be fixed through the description of NN scattering observables or phase shifts.

- **Diagrammatic rules:** In Ref. [40], we derived the rules for time-ordered diagrams corresponding to manifestly Lorentz-invariant Lagrangians, especially for the rules with spin-1/2 fermions. As far as we know, such rules were not available in the literature before. In the following table, we briefly summarize these diagrammatic rules of TOPT:

▸ External lines		
Spin 0 boson (in, out)		1
Spin 1/2 fermion (in, out)		$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$
▸ Internal lines		
Spin 0 (anti-)boson		$\frac{1}{2\,\epsilon_q}$ $\quad \epsilon_q \equiv \sqrt{\mathbf{q}^2+M^2}$
Spin 1/2 fermion		$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p})$ $\quad \omega_p \equiv \sqrt{\mathbf{p}^2+m^2}$
anti-fermion		$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$
▸ Intermediate state		
A set of lines between two vertices		$\frac{1}{E - \sum\limits_i \omega_{p_i} - \sum\limits_j \epsilon_{q_j} + i\epsilon}$

Besides, interaction vertices are obtained via the standard Feynman rules, while one has to take special care of the zeroth components of momenta appearing in vertices. Further, each one-loop diagram with internal momentum k contains a three-dimensional integration $\int \frac{d^3k}{(2\pi)^3}$. More details of diagrammatic rules are given in Ref. [40]. We utilize those rules to obtain expressions of each time-ordered diagram.

- **Power counting:** We apply the standard Weinberg power counting (PC) to derive the chiral potential [1, 2]. The chiral order ν of time-ordered diagrams is defined as

$$\nu = 2l + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2 \right), \tag{5}$$

where l is the number of loops, V_i is the number of vertices of type i, d_i is the number of derivatives acting on pion fields and/or spatial components of derivatives acting on nucleon fields, or pion-mass insertions, and n_i denotes the number of nucleon fields involved in vertex i. The sum in the above equation runs over all vertices contained in given diagram.

- **Potential:** The effective potential is defined as the sum of the two-nucleon irreducible TOPT diagrams. Up to NNLO, we have those time-ordered diagrams in Fig. 2, organized by the Weinberg power counting. Next is to employ the obtained TOPT rules to evaluate their contributions to the NN potential.

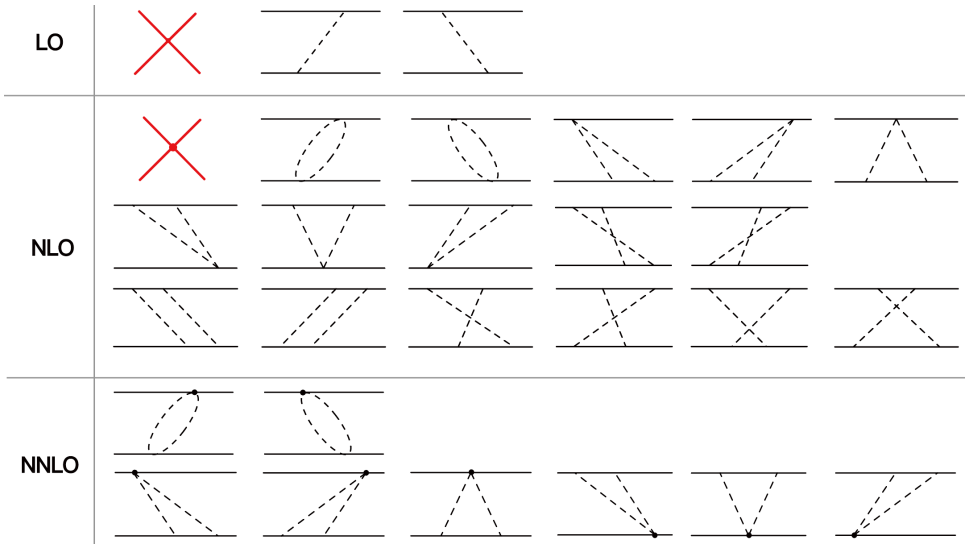


Figure 2: Time-ordered diagrams contributing to the NN potential up to NNLO.

- **Scattering equation:** In the integral equation of NN scattering amplitude, $T = V + VGT$, by applying the TOPT rules the two-nucleon Green function can be written as

$$G = \frac{m_N^2}{\omega_k^2} \frac{1}{E - 2\omega_k + i\epsilon}, \tag{6}$$

where the two-internal nucleon lines give the pre-factor in terms of the energy denominator. Thus, our integral scattering equation can be written as

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{\omega_k^2} \frac{1}{E - 2\omega_k + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p}), \tag{7}$$

which is just the Kadyshevsky equation of NN scattering, proposed by Kadyshevsky in 1968 [51]. In comparison with the canonical Lippmann-Schwinger equation (LSE), one finds that the Kadyshevsky equation has a relatively milder ultraviolet behavior, since the Green function of Kadyshevsky equation tends to be $\sim 1/k^3$ with the internal momentum $k \to \infty$, while the counterpart of LSE is to be $\sim 1/k^2$. The benefit of this mild UV behavior will be presented in the LO study.

3. Leading order study

At leading order, following the TOPT rules, the non-derivative four-fermion contact term is written as,

$$\begin{aligned} V_{\rm C}^{(0)} &= C_S (\bar{u}_4 u_2)(\bar{u}_3 u_1) + C_A (\bar{u}_4 \gamma_5 u_2)(\bar{u}_3 \gamma_5 u_1) \\ &+ C_V (\bar{u}_4 \gamma_\mu u_2)(\bar{u}_3 \gamma^\mu u_1) + C_{AV} (\bar{u}_4 \gamma_\mu \gamma_5 u_2)(\bar{u}_3 \gamma^\mu \gamma_5 u_1) \\ &+ C_T (\bar{u}_4 \sigma_{\mu\nu} u_2)(\bar{u}_3 \sigma_{\mu\nu} u_1), \end{aligned} \tag{8}$$

which also contains higher order contributions according to the Weinberg PC. Therefore, we perform the expansion for the nucleon energies via $\sqrt{\omega_p + m_N} = \sqrt{2m_N} + O(v^2)$ and keep the lowest order terms resulting in our final LO contact term,

$$\tilde{V}_{\rm C}^{(0)} = (C_S + C_V) - (C_{AV} - 2C_T)\,\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \tag{9}$$

which is consistent with the non-relativistic result [1] and has two independent parameters to be fixed.

Using the TOPT rules, the OPE potential reads as

$$\begin{aligned} V_{1\pi}^{(0)} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{1}{2\epsilon_q}\Bigg[& \frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)\,(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega_p + \omega_{p'} + \epsilon_q - E - i\epsilon} \\ & + \frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)\,(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega_p + \omega_{p'} + \epsilon_q - E - i\epsilon}\Bigg], \end{aligned} \tag{10}$$

which, similarly to the contact term, contains the higher order contributions according to Weinberg PC. Then we expand the nucleon energies in the numerator, while keeping them untouched in denominator, which is consistent with the Kadyshevsky equation. Finally we obtain the following expression of the OPE potential:

$$\tilde{V}_{1\pi}^{(0)} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{1}{\epsilon_q}\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)\,(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega_p + \omega_{p'} + \epsilon_q - E - i\epsilon}. \tag{11}$$

It is worth to notice that our OPE potential has a mild UV behavior, for fixed p' and large p leading to $\tilde{V}_{1\pi}^{(0)} \sim 1/p$, while the OPE potential in non-relativistic ChEFT for the same kinematics behaves like ~ 1.

In combination with the Kadyshevsky equation, which also has milder UV behavior, we found a distinctive feature of our LO study, that the iterations of the OPE potential within the integral equation lead to ultraviolet finite diagrams. This can be easily checked via the UV behavior of the once-iterated OPE potential for the fixed p, p' and the large internal momentum k:

$$\int \frac{d^3k}{(2\pi)^3}\,\tilde{V}_{1\pi}^{(0)}\,G(E)\,\tilde{V}_{1\pi}^{(0)} \xrightarrow{k\to\infty} \begin{cases} \text{This work:} & \displaystyle\int \frac{d^3k}{(2\pi)^3}\,\frac{1}{k}\,\frac{1}{k^3}\,\frac{1}{k} \equiv \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{k^5}, \\ \text{Non-rel.:} & \displaystyle\int \frac{d^3k}{(2\pi)^3}\,1\,\frac{1}{k^2}\,1 \equiv \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{k^2}. \end{cases} \tag{12}$$

Therefore, our once-iterated OPE potential is UV convergent and all the sub-diagrams of iterated OPE potential are also finite. As a consequence, the ultraviolet cutoff can be safely removed at LO, avoiding the finite-cutoff artefacts inherent to the conventional non-relativistic framework. Furthermore, our OPE potential also results in the unique solutions of the NN scattering amplitudes for all partial waves.

After performing the partial wave decomposition of the LO potential $V_{\rm LO} = \tilde{V}_C^{(0)} + \tilde{V}_{1\pi}^{(0)}$ and the Kadyshevsky equation Eq. (7), we obtain the phase shifts of NN scattering (Fig. 3). Two parameters of contact term are fixed by reproducing the scattering lengths of ${}^1S_0$ and ${}^3S_1$ channels, respectively. Our LO calculation provides a reasonable description of the Nijmengen phase shifts. While, the

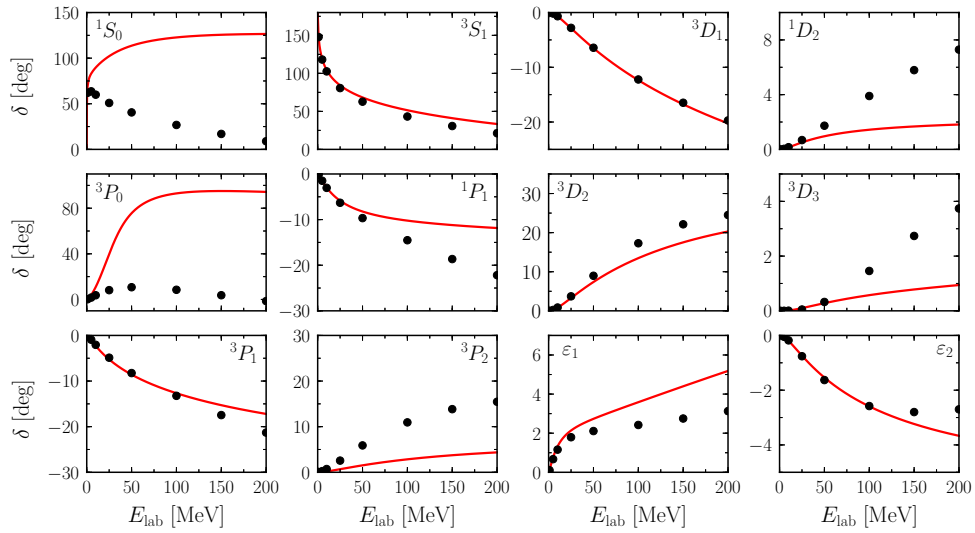


Figure 3: Nucleon-nucleon phase shifts and mixing angles at LO. Red lines denote our results, and solid dots are the Nijmegen partial wave analysis [52].

large discrepancies of the $^1S_0$ and $^3P_0$ channels indicate that part of the sub-leading corrections must be treated non-perturbatively and require studies beyond leading order.

Furthermore, we also studied the hyperon-nucleon interaction, e.g., for ΛN system, by extending our scheme to the SU(3) sector [53]. We found similar global description of phase shifts for ΛN scattering at LO.

4. Beyond leading order studies

In principle, we have two strategies to include the higher-order contributions in our framework. One is to keep the non-perturbative treatment of our (non-singular) LO potential and treat the high-order contributions perturbatively. This allows us to systematically remove all divergences from the NN scattering amplitude. Second strategy is to treat the full effective potential (LO + higher orders) non-perturbatively. The milder UV behavior of the potential and the scattering equation will permit a relatively large range of admissible cutoffs in solving the Kadyshevsky equation, which will be helpful for the few-/many-body problems.

As a first step, we focus on the second strategy and formulate the chiral nuclear potential up to NNLO, which reads as

$$V_{\rm NNLO} = V_C^{(0)} + V_C^{(2)} + V_{1\pi}^{(0)} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)}. \tag{13}$$

Here the LO contact term $V_C^{(0)}$ is given in Eq. (8). According to the Weinberg PC, the nucleon energy can be expanded up to $O(p^2)$, $\sqrt{\omega_p + m_N} = \sqrt{2m_N} + \frac{p^2}{4\sqrt{2}m_N^{3/2}}$. For simplicity, we just keep the full form of Dirac spinors to take into account the higher order contributions. The contact terms with two derivatives $V_C^{(2)}$, in principle, can be obtained via the second order Lagrangian $\mathcal{L}_{NN}^{(2)}$. Keeping only the lowest order term in the expansion of the nucleon energy, we obtain the same

form as in the non-relativistic case

$$V_C^{(2)} = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{k}^2 + \left(C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{k}^2\right) (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + i C_5 \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}) + C_6 (\boldsymbol{q} \cdot \boldsymbol{\sigma}_1) (\boldsymbol{q} \cdot \boldsymbol{\sigma}_2) + C_7 (\boldsymbol{k} \cdot \boldsymbol{\sigma}_1) (\boldsymbol{k} \cdot \boldsymbol{\sigma}_2), \tag{14}$$

with $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$ and $\boldsymbol{k} = (\boldsymbol{p} + \boldsymbol{p}')/2$ and 7 unknown LECs to be fixed. Including 5 parameters in $V_C^{(0)}$, it seems that there are 12 LECs in contact terms up to NNLO. But all of them are not totally independent, which can be clearly seen in the partial wave basis [54]. Actually, we have 9 independent LECs, which is the same number of contact terms as in the non-relativistic case.

As for the pion exchange contributions, we have the OPE potential, $V_{1\pi}^{(0)}$, given in Eq. (10). Since plugging the OPE potential into the Kadyshevsky equation one encounters the pole in the denominator of the half-off-shell $V_{1\pi}^{(0)}(p', k)$, it is convenient to eliminate the energy-dependence of the OPE potential by performing an expansion in powers of $E - \omega_p - \omega_{p'}$. Up to NNLO, we obtain an equivalent energy-independent potential

$$V_{\not E} = V_{1\pi,\not E}^{(0)} + V_{2\pi,\not E}^{(2)} + \ldots, \tag{15}$$

with the energy-independent OPE potential

$$V_{1\pi,\not E}^{(0)} = -\frac{g_A^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{4 f_\pi^2} \frac{1}{\varepsilon_q^2} \left(\bar{u}_3 \gamma_\mu \gamma_5 q^\mu u_1\right) \left(\bar{u}_4 \gamma_\nu \gamma_5 q^\nu u_2\right), \tag{16}$$

where we follow the same treatment of the LO contact term by keeping the full form of the Dirac spinors. The second term $V_{2\pi,\not E}^{(2)}$, which is obtained by cancelling the energy denominator of the Green function in the Kadyshevsky equation, is written as

$$V_{2\pi,\not E}^{(2)} = \frac{1}{2} \left(\frac{g_A^2}{4 f_\pi^2}\right)^2 (3 - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \int \frac{d^3 k}{(2\pi)^3} \frac{m_N^2}{\boldsymbol{k}^2 + m_N^2} \frac{\varepsilon_{p'-k} + \varepsilon_{p-k}}{\varepsilon_{p'-k}^3 \, \varepsilon_{p-k}^3} \times \left[\boldsymbol{\sigma}_1 \cdot (\boldsymbol{p}' - \boldsymbol{k}) \boldsymbol{\sigma}_1 \cdot (\boldsymbol{k} - \boldsymbol{p})\right] \left[\boldsymbol{\sigma}_2 \cdot (\boldsymbol{p}' - \boldsymbol{k}) \boldsymbol{\sigma}_2 \cdot (\boldsymbol{k} - \boldsymbol{p})\right], \tag{17}$$

where we have kept the LO terms in the expansion of the Dirac spinors. Notice that the $V_{2\pi,\not E}^{(2)}$ can be thought as the part of the TPE potential at NLO.

For the two-pion-exchange contributions $V_{2\pi}^{(2,3)}$ the time-ordered diagrams are shown in Fig. 2 up to NNLO. We formulate the expressions of those one-loop diagrams according to our TOPT rules and keep the LO terms in the expansion of the Dirac spinors. We found that the football diagrams at NNLO do not contribute. In order to renormalize the one-loop diagrams, we employ the subtractive renormalization by subtracting the divergent and power counting breaking terms. The details are reported in our most-recent work [41].

Since the complicated for of the TPE potential in our scheme, it is desirable to verify its correctness. The higher partial waves (e.g. D, F ... waves) do not have the contributions from the contact terms at NNLO, therefore they provide an ideal place to perform the consistency check by comparing with the non-relativistic TPE potential when the nucleon mass m_N is taken to infinity. In Fig. 4 we present such comparison for the phase shifts of D and F waves by using our irreducible TPE potential

$$V_{2\pi,irr} = V_{2\pi,\not E}^{(2)} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)}, \tag{18}$$

for the physical nucleon mass $m_N^{\rm Phys}$ and for a large nucleon mass $m_N = 1000\, m_N^{\rm Phys}$. One can see that our results are consistent with the ones of the non-relativistic TPE potential. Thus, our TPE potential at NNLO passes the consistency check, which partially guarantees its correctness.

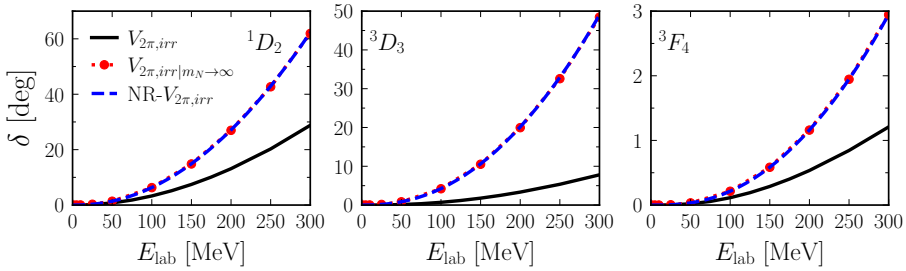


Figure 4: The phase shifts of $^1D_2$, $^3D_3$ and $^3F_4$ partial waves from the TPE potential up to NNLO. The black solid (red dotted) lines denote our results with the physical nucleon mass (the infinite nucleon mass, such as $m_N = 1000\, m_N^{\rm Phys}$). The blue dashed lines present the results of non-relativistic TPE potential.

Before we perform the non-perturbative treatment for the full NNLO potential $V_{\rm NNLO}$, it would be also interesting to illustrate the pion exchange contributions to the higher partial waves perturbatively. At one-loop order in the Born series expansion, the T-matrix of NN scattering is

$$T(p',p) = V^{(0)}_{1\pi,\not{E}}(p',p) + V_{2\pi,irr}(p',p) + V_{2\pi,it}(p',p), \tag{19}$$

where $V_{2\pi,it}$ denotes the once-iterated OPE

$$V_{2\pi,it}(p',p) = V^{(0)}_{1\pi,\not{E}}\, G\, V^{(0)}_{1\pi,\not{E}} = \int \frac{d^3k}{(2\pi)^3} \frac{m_N^2}{\omega_k^2} \frac{V^{(0)}_{\rm OPE,\not{E}}(p',k)\, V^{(0)}_{\rm OPE,\not{E}}(k,p)}{E - 2\,\omega_k + i\epsilon}\,. \tag{20}$$

In Fig. 5, we present the phase shifts and mixing angles of D, F, G waves. Besides the LO, NLO and NNLO results in our scheme, the non-relativistic counterpart at NNLO is also given using the same values of $c_{1,2,3,4}$. One can draw the following conclusion (more detailed discussion can be found in Ref. [41]):

- **D waves:** The leading correction of TPE potential gives the correct direction of improvement for all D-wave phase shifts based on the LO results. However its contribution is relatively small. Including the sub-leading TPE potential, we found sizable improvement for $^3D_3$ and ϵ_2. But the good agreement with the data observed at LO and NLO is worsened for $^1D_2$ and $^3D_2$ at energies $E_{\rm lab} > 100$ MeV. In comparison with the non-relativistic NNLO results, which are exploding beyond $E_{\rm lab} > 50$ MeV for all D waves, the improvement delivered by our approach is visible, particularly for the $^3D_3$ partial wave.

- **F waves:** For the $^1F_3$, $^3F_3$, and $^3F_4$ partial waves, our results at NNLO are in a good agreement with the empirical phase shifts up to $E_{\rm lab} = 150$ MeV. Beyond that energy, the NNLO correction becomes too strong leading to deviation from the empirical phases, in comparison with the NLO results. Similar behavior is also observed for the non-relativistic NNLO results. While in the $^3F_2$ channel the correct tendency achieved at NLO is altered by including the sub-leading TPE potential.

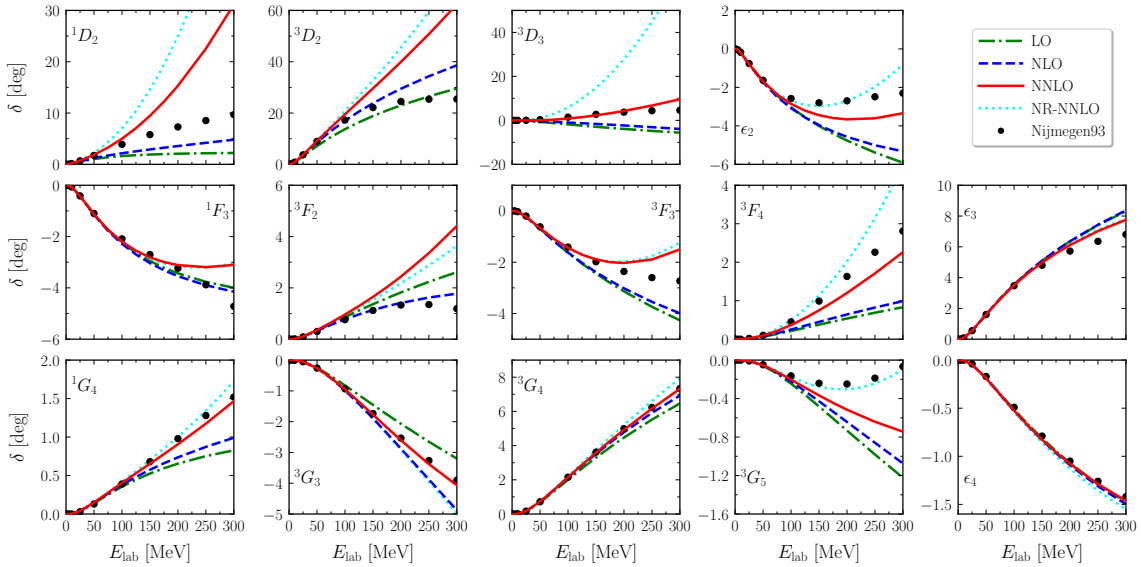


Figure 5: D, F, G-wave phase shifts and the mixing angles $\epsilon_{2,3,4}$ for laboratory energies below 300 MeV. The green dot-dashed curve denotes the LO result, the blue dashed and red solid lines are the NLO and NNLO results, respectively. The non-relativistic results at NNLO are shown as the cyan dotted lines. Solid dots are the Nijmegen partial wave analysis [52].

- **G waves:** Our NNLO results provide a rather good description of the Nijmegen $^1G_4$, $^3G_3$, $^3G_4$ phase shifts and the mixing angle ϵ_4, and are slightly better in comparison with the NNLO results of the non-relativistic ChEFT. However, the situation with the $^3G_5$ partial wave is different. The non-relativistic result at NNLO presents a good agreement with data, but it is accidental. The relativistic corrections of sub-leading TPE potential $\propto c_i/m_N$ are of the same size as the difference between our NNLO and NR-NNLO lines in Fig. 5, as shown in Ref. [55]. Our formalism has already included these relativistic corrections $\propto \sum_{n=1} c_i/m_N^n$. Thus, we expect the convergence of the covariant chiral EFT approach for $^3G_5$ to be superior as compared to the non-relativistic framework.

Furthermore, in Ref. [41], we also present the results for the H and I waves and our NNLO result is globally similar to the one of the non-relativistic approach.

Next task is to iterate the full NNLO potential via the Kadyshevsky equation and determine the 9 unknown LECs appeared in the contact terms by fitting the Nijmegen phase shifts of S and P waves. We are now working on this analysis, and the results will be reported soon [54].

5. Conclusion and perspectives

In this talk we presented a systematic framework to formulate the nucleon-nucleon interaction up to next-to-next-to-leading order based on the TOPT using the manifestly Lorentz-invariant effective chiral Lagrangian. We derived the diagrammatic rules of TOPT, particularly for the momentum-dependent vertices and the propagators of spin-1/2 fermions. Employing those rules, one can systematically formulate the NN potential at different orders in the chiral expansion. At

leading order, we found that NN scattering amplitude is renormalizable and the scattering equations have unique solutions for all partial waves. To include the beyond leading order contributions, we can treat them perturbatively to maintain a renormalizable scattering amplitude. On the other hand, the higher-order contributions can be iterated together with the leading order interaction by substituting into the Kadyshevsky equation. Here, we focused on the first case and presented the chiral potential up to next-to-next-to-leading order by including the full one-loop two-pion-exchange contributions. We have checked that our TPE potential is consistent with its non-relativistic counterpart in the large-nucleon-mass limit. Using the Born series truncated at one-loop order, we calculated the phase shifts and mixing angles of the partial waves with the angular momentum $l \geq 2$ and found an improved description of the phase shifts for some D waves, especially for ${}^3D_3$.

Currently we are working on the description of the low partial waves of the NN scattering amplitude and the deuteron properties by performing the non-perturbative treatment for the full NNLO potential. The results will be released soon [54]. A side-work along this line is to investigate the energy-independent potential at NNLO by using the same treatment of the energy-independent one-pion exchange potential. This representation of the chiral potential could be more convenient for many-body calculations. We can also treat the NLO/NNLO corrections perturbatively. Based on our non-singular LO potential, perturbative treatments allows us to obtain a renormalizable scattering amplitude and all divergences can be systematically removed. In the long run, it would be interesting to apply the symmetry preserving regularization in our framework.

Furthermore, we would like to emphasize that the values of $c_{1,2,3,4}$, appeared in the second order Lagrangian $\mathcal{L}^{(2)}_{\pi N}$ are crucial for the sub-leading TPE potential. Currently we use the values of $c_{1,2,3,4}$ given by the Roy-Steiner analysis of πN scattering data [46, 47]. To be consistent with our framework, we plan to determine these values by using TOPT in covariant ChEFT. Thus far, we have extended the TOPT framework to the meson-baryon scattering and investigated the πN and and $\bar{K}N$ systems at LO [56, 57]. The NLO study is work in progress.

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