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Precision tests of fundamental physics with η and η' mesons

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Decays of the neutral and long-lived η and η' mesons provide a unique, flavor-conserving laboratory to test low-energy Quantum Chromodynamics and search for physics beyond the Standard Model. In these proceedings, we discuss recent progress in a dispersion-theoretical analysis of the η and η' transition form factors, which determine the corresponding pole contribution to hadronic light-by-light scattering in the anomalous magnetic moment of the muon. Subsequently, we consider discrete symmetry tests relevant for the search of potential new sources of CP-violation. We point out how stringent constraints from electric dipole moment measurements strongly limit the realistic opportunities to observe P- and CP-odd mechanisms in η or η' decays. For the far less investigated C- and CP-odd processes, we introduce a dispersion-theoretical formalism to assess these from Dalitz plot asymmetries in a model-independent way.[†]

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1. The η and η' mesons: properties, symmetries, quantum numbers

Decays of the lightest flavor-neutral mesons, the η and the η', post ample opportunities of various symmetry tests. Apart from parity, they both share the quantum numbers of the vacuum, $I^G(J^{PC}) = 0^+(0^{-+})$. The η with its mass $M_\eta = 547.86\,\text{MeV}$ is largely a (pseudo-) Goldstone boson of spontaneously broken chiral symmetry. Its width, $\Gamma_\eta = 1.31\,\text{keV}$, is tiny largely because many of its decay modes, strong or electromagnetic, are forbidden at leading order due to P, C, CP, isospin/G-parity, or angular momentum conservation [2]. The much heavier η', $M_{\eta'} = 957.78\,\text{MeV}$, is no Goldstone boson due to the $U(1)_A$ anomaly, however its width, $\Gamma_{\eta'} = 196\,\text{keV}$, still makes it much narrower than, say, the $\omega(782)$ or $\phi(1020)$ vector resonances of comparable masses.

Amongst stringent tests of Standard Model (SM) physics with η and η' decays, we mention various processes of odd intrinsic parity that are determined by the Wess–Zumino–Witten anomaly [3, 4]; the extraction of the light quark mass difference $m_u - m_d$ from $\eta \to 3\pi$ [5] (and references therein), which is less contaminated by electromagnetic effects than the extraction from meson masses (see Ref. [6] for recent work on the latter); theory input for the anomalous magnetic moment of the muon [7] via the $\eta^{(\prime)}$ transition form factors [8]; or scalar resonance dynamics of, e.g., the $f_0(500)$ or $a_0(980)$ resonances in decays such as $\eta^{(\prime)} \to \pi^0\gamma\gamma$, $\eta' \to \eta\gamma\gamma$ [9, 10]. These and several more topics are covered extensively in a recent review [11]. Due to the high current interest after the recent $(g-2)_\mu$ measurement at Fermilab [12], we here concentrate on new developments towards a dispersion-theoretical reconstruction of the η transition form factor [13], which we discuss in Sec. 2.

Possible probes of physics beyond the Standard Model (BSM) fall broadly into two categories. On the one hand, there are searches for weakly-interacting new light particles, such as dark photons, protophobic or leptophobic gauge bosons of new $U(1)$ symmetries, light Higgs-like scalars, or axion-like particles; we do not discuss these here, but refer to Ref. [11] and references therein. In these proceedings, we concentrate on tests of discrete symmetries, in particular searches for possible new sources of CP-violation. As the η and η' mesons are C and P eigenstates, their decays are a flavor-conserving laboratory for such symmetry tests, with little or no SM background. The different possible classes of violation and conservation of C, P, and T (always assuming CPT to be conserved) are enlisted in Table 1. While the Standard Model weak interactions are in class IV, violating all three discrete symmetries separately, they are in many circumstances close to class I,

class	violated	conserved	interaction
0		C, P, T, CP, CT, PT, CPT	strong, electromagnetic
I	C, P, CT, PT	T, CP, CPT	(weak, with no KM phase or flavor mixing)
II	P, T, CP, CT	C, PT, CPT	
III	C, T, PT, CP	P, CT, CPT	
IV	C, P, T, CP, CT, PT	CPT	weak

Table 1: Possible classes I–IV of interactions that violate discrete spacetime symmetries, assuming CPT invariance. Table taken from Ref. [11].

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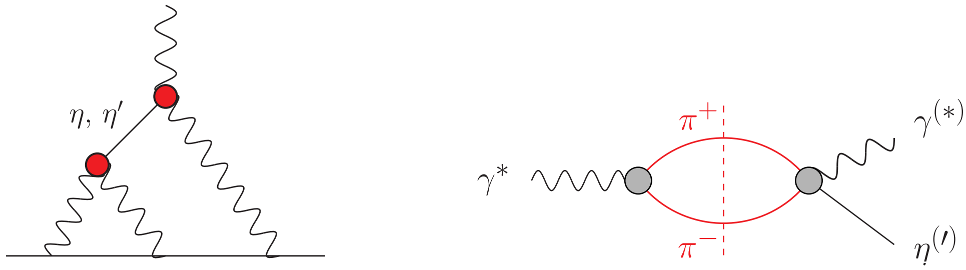

Figure 1: *Left:* $\eta^{(\prime)}$ pole contributions to $(g-2)_\mu$ via hadronic light-by-light scattering; the red blobs denote the $\eta^{(\prime)}$ transition form factors. *Right:* two-pion contribution to the $\eta^{(\prime)}$ transition form factor discontinuity.

violating C and P maximally, with CP almost conserved. In the following, we concentrate on class II (Sec. 3), P- and T- violating, but C-conserving interactions, which are closely connected to the physics of electric dipole moments (EDMs); and on class III (Sec. 4), T- and C-odd, but P-even interactions, which have been much less explored to date.

2. η and η' contributions to the anomalous magnetic moment of the muon

Pseudoscalar pole terms constitute the largest individual contributions to hadronic light-by-light scattering (HLbL) in the anomalous magnetic moment of the muon. Next to the dominant lightest π^0 exchange, η and η' yield important corrections, displayed diagrammatically in Fig. 1(left). The ultimate goal is to reconstruct the transition form factors

$$i\int \mathrm{d}^4x\, e^{iqx}\left\langle 0\left|T\{j_\mu(x)j_\nu(0)\}\right|\eta^{(\prime)}(q+k)\right\rangle = \epsilon_{\mu\nu\alpha\beta}q^\alpha k^\beta F_{\eta^{(\prime)}\gamma^*\gamma^*}(q^2,k^2) \tag{1}$$

from dispersion theory, in analogy to similar analyses for the π^0 [14–16]. Here, j_μ is the electromagnetic current. The most important contribution to its discontinuity in one of the variables is due to two-pion intermediate states, cf. Fig. 1(right). The resulting dispersion relation for the isovector ($I=1$) part, in unsubtracted form, reads [17]

$$F^{(I=1)}_{\eta^{(\prime)}\gamma^*\gamma^*}(q^2,k^2) = \frac{1}{96\pi^2}\int_{4M_\pi^2}^\infty \mathrm{d}t\,\frac{t\sigma_\pi^3(t)F_{\eta^{(\prime)}\pi\pi\gamma^*}(t,k^2)\,[F_\pi^V(t)]^*}{t-q^2}\,, \tag{2}$$

where $F_\pi^V(t)$ is the pion vector form factor, and $F_{\eta\pi\pi\gamma^*}(t,k^2)$ denotes the P-wave projection of the $\eta\to\pi^+\pi^-\gamma^*$ amplitude.

For real photons, $F_{\eta\pi\pi\gamma^*}(t,0)$ has been modeled using an Omnès representation [18],

$$F_{\eta^{(\prime)}\pi\pi\gamma^*}(t,0) = A_{\eta^{(\prime)}}\times P_{\eta^{(\prime)}}(t)\times\Omega(t)\,, \qquad \Omega(t) = \exp\left\{\frac{t}{\pi}\int_{4M_\pi^2}^\infty \mathrm{d}x\,\frac{\delta_1^1(x)}{x(x-t)}\right\}, \tag{3}$$

with a linear polynomial $P_\eta(t)$ shown to be sufficient to describe the $\eta\to\pi^+\pi^-\gamma$ data [19, 20] in the decay region. Left-hand cuts due to the lowest-lying $\pi\eta$ resonance, the $a_2(1320)$, induce curvature in a generalized "effective polynomial" [21], which is visible in the larger decay region of $\eta'\to\pi^+\pi^-\gamma$ [22]. Furthermore, for the latter decay, even ρ–ω mixing can be clearly distinguished [23], a further refined dispersive analysis of the (singly-virtual) η' transition form factor that takes this

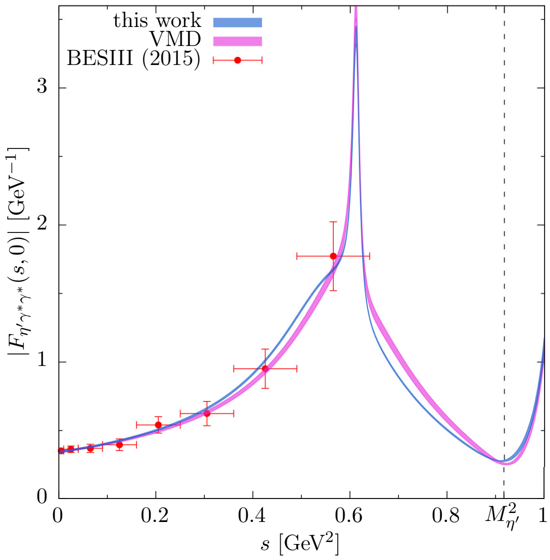

Figure 2: Dispersion theoretical prediction of the singly-virtual η' transition form factor, compared to data for $\eta' \to e^+e^-\gamma$ [25]. Figure taken from Ref. [24].

isospin-violating mixing effect into account consistently has been performed in Ref. [24], with the resulting prediction shown in Fig. 2. Also similar predictions for the η transition form factor [13, 17] compare favorably to Dalitz decay data for $\eta \to \ell^+\ell^-\gamma$ [26–29] [see Fig. 4(right) below].

However, to determine the $\eta^{(\prime)}$ pole contributions to $(g-2)_\mu$, also the *doubly*-virtual transition form factors need to be controlled. An interesting question to investigate in this context is to which extent the dependence on the two virtualities can be assumed to factorize at low-to-medium energies; it is well known that this factorization does not hold at high energies. With respect to the dispersion relation (2), this can be translated into a test of the assumption

$$F_{\eta\pi\pi\gamma^*}(t,k^2) = F_{\eta\pi\pi\gamma^*}(t,0) \times \tilde{F}_{\eta\gamma\gamma^*}(k^2)\,, \tag{4}$$

where for $F_{\eta\pi\pi\gamma^*}(t,0)$, parameterizations analogous to Eq. (3) have been tested vs. data on $e^+e^- \to \eta\pi^+\pi^-$ [30, 31], allowing for linear or quadratic polynomials, as well as the inclusion of the a_2 left-hand cut. The latter induces a natural breaking of the factorization assumption.

Figure 3 shows quality of the data fits (with a sum-of-Breit–Wigner description of $\tilde{F}_{\eta\gamma\gamma^*}(k^2)$), both for the total cross sections and the dipion invariant mass distributions [13]. However, the surprising observation is that extrapolation of the amplitude representation back to the real-photon point $k^2 = 0$ yields consistency with the $\eta \to \pi\pi\gamma$ decay data only for the naïve, factorizing quadratic polynomial assumption, while the a_2 left-hand cut leads to inconsistent values for the subtraction constants; see Fig. 4(left). Similarly, the resulting singly-virtual η transition form factor comes out too low in comparison to data [26–29] when based on the extrapolated $e^+e^- \to \eta\pi^+\pi^-$ data, cf. Fig. 4(right). Clearly, more theoretical work is required to understand the reasons for the attenuation mechanism of the physically well motivated, data-driven factorization breaking.

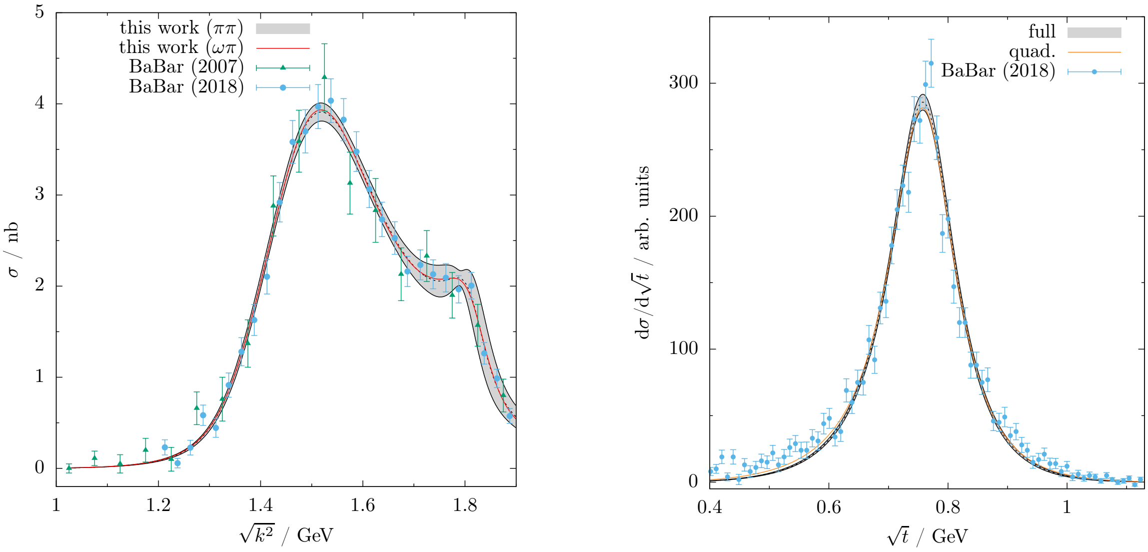

Figure 3: Total cross section for $e^+e^- \to \eta\pi^+\pi^-$ (left) and differential distribution with respect to the $\pi^+\pi^-$ invariant-mass distribution (right). Data from Refs. [30, 31]. Figures taken from Ref. [13].

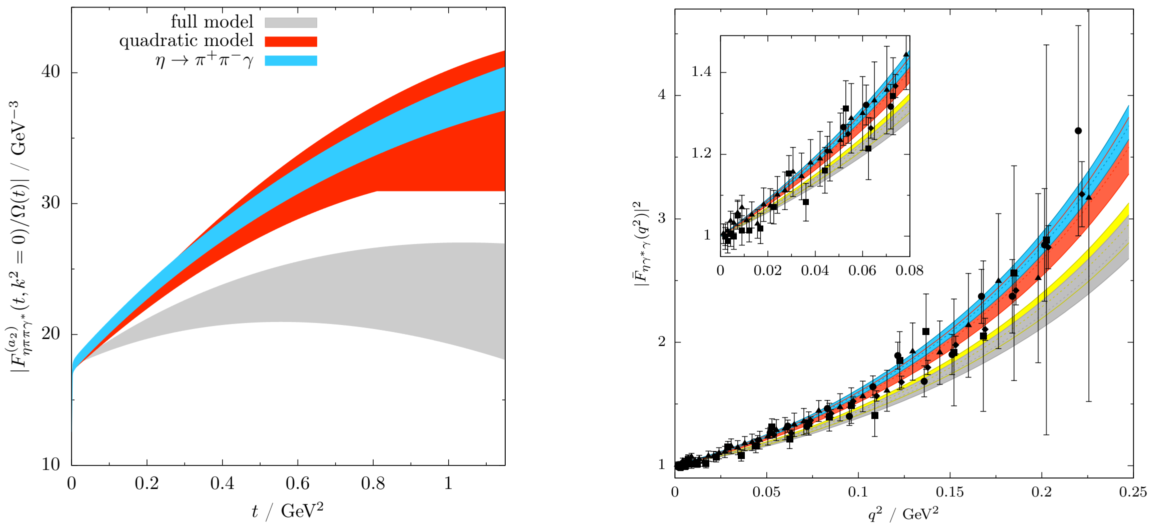

Figure 4: Extrapolated amplitudes $F_{\eta\pi\pi\gamma^*}(t, k^2 = 0)$, normalized by the Omnès function (left), as well as the resulting singly-virtual η transition form factors compared to data [26–29] (right), both based on different models fitted to $e^+e^- \to \eta\pi^+\pi^-$ data. Figures taken from Ref. [13].

3. *P*- and *CP*-violation

Permanent EDMs such as of neutron or proton are signs of P- and CP-violating interactions, or class II in the nomenclature of Table 1. As the weak contribution to these is vanishingly small, experimental limits in particular on the neutron EDM have been employed to constrain the (renormalizable) dimension-four P- and CP-odd operator known as the QCD θ-term,

$$\mathcal{L}_\theta = \frac{g_s^2\theta}{32\pi^2} G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} \,. \tag{5}$$

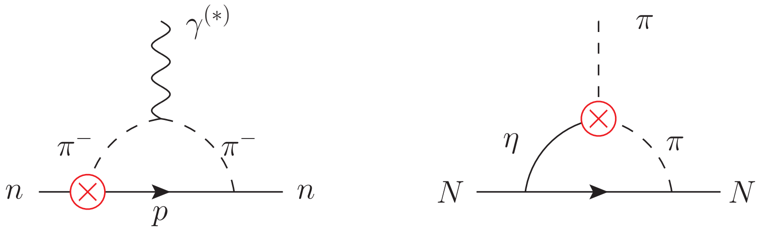

Figure 5: Pion loop contribution to the neutron EDM via a CP-violating pion–nucleon coupling (left), and loop-induced CP-violating pion–nucleon vertex function generated through an $\eta \to \pi\pi$ coupling (right). The red crosses indicated CP-violating vertices. Figures taken from Ref. [11].

At the same time, the θ-term induces a coupling for the decay $\eta \to \pi\pi$ [32, 33]:

$$\mathcal{B}(\eta \to \pi^+\pi^-) = \frac{\bar{g}_{\eta\pi\pi}^2}{16\pi M_\eta \Gamma_\eta}\sqrt{1-\frac{4M_{\pi^\pm}^2}{M_\eta^2}}\,, \qquad \bar{g}_{\eta\pi\pi} = \frac{2\bar{\theta}M_\pi^2 m_u m_d}{\sqrt{3}F_\pi(m_u+m_d)^2}\,, \tag{6}$$

where $\bar{\theta} = \theta + \arg\det(\mathcal{M})$ is related to the Lagrangian parameter θ via the phase of the determinant of the quark mass matrix $\mathcal{M}$. The best experimental upper limit on this decay, $\mathcal{B}(\eta \to \pi^+\pi^-) < 4.4 \times 10^{-6}$ [34], implies $|\bar{\theta}| < 4 \times 10^{-4}$, which however is a much weaker limit than the one derived from the neutron EDM, $|\bar{\theta}| \lesssim 10^{-10}$ (see, e.g., Ref. [35] or references therein). Given the suggested suppression of $\bar{\theta}$, various P- and CP-odd operators of dimension six may be competitive in size [36, 37], so one may wonder whether a tiny EDM may not still be reconciled with a signal in $\eta \to \pi\pi$. However, this possibility has been debunked frequently [11, 38–41]. Figure 5(left) shows how the leading nonanalytic contribution to the neutron EDM is, in fact, generated from the θ-term: it generates a CP-violating pion–nucleon coupling, and the photon coupling to the charged-pion loop induces a logarithmically (in M_π) enhanced contribution to the EDM. A CP-odd $\eta \to \pi\pi$ coupling in turn induces such a CP-odd pion–nucleon coupling at one loop, see Fig. 5(right), such that, at the two-loop level, any $\eta \to \pi\pi$ decay generates an EDM contribution. The estimated relation between neutron EDM d_n and coupling $\bar{g}_{\eta\pi\pi}$ reads [11]

$$d_n \approx 7 \times 10^{-16} \left(\frac{\bar{g}_{\eta\pi\pi}}{\text{GeV}}\right) e\,\text{cm}\,, \tag{7}$$

which results in the limit $\mathcal{B}(\eta \to \pi^+\pi^-) < 2 \times 10^{-17}$, eleven orders of magnitude below the current measurement [34], and way beyond experimental reach in the foreseeable future.

Alternative η decays to investigate P- and CP-violation that have been suggested in the past are $\eta \to \pi^+\pi^-\gamma^{(*)}$ [42, 43], where, e.g., an asymmetry in the angle between the $\pi^+\pi^-$ and $\ell^+\ell^-$ decay planes would be an experimental CP-odd signature [43]. Such an effect could be generated by the interference of magnetic (M) and electric (E) transitions,

$$\mathcal{A}\big(\eta \to \pi^+\pi^-\gamma^{(*)}\big) = M\,\epsilon_{\mu\nu\alpha\beta}\,p_+^\mu p_-^\nu k^\alpha \epsilon^\beta + E\,\big((\epsilon\cdot p_+)(k\cdot p_-) - (\epsilon\cdot p_-)(k\cdot p_+)\big)\,. \tag{8}$$

In the SM, $M \propto e/F_\pi^3$ is given by the chiral anomaly at leading order [18, 21, 44], while $E = 0$. In Refs. [42, 43], nonvanishing contributions to E are constructed based on a four-quark operator of the form $\big(\bar{s}\,i\sigma_{\mu\nu}\gamma_5(p-k)^\nu s\big)\,(\bar{q}\gamma^\mu q)$, where the light-quark vector current $(\bar{q}\gamma^\mu q)$ hadronizes

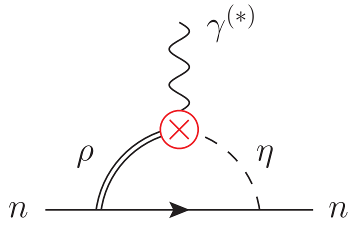

Figure 6: One-loop contribution to the neutron EDM, induced by a CP-odd $\eta \to \rho\gamma$ vertex. Figure taken from Ref. [11].

into a P-wave $\pi\pi$ pair, while the strange-quark operator induces the $\eta \to \gamma^{(*)}$ transition. However, also this mechanism is heavily restricted through indirect EDM constraints: the P-wave $\pi\pi$ pair develops, via strong final-state interactions, resonant enhancement as the $\rho(770)$, such that, in a simplified manner of speaking, the E-term induces a CP-odd $\eta \to \rho\gamma$ vertex. Via the mechanism shown in Fig. 6, this once more generates a neutron EDM at one loop. As a result, the corresponding coupling can be estimated to be $E/M \lesssim 10^{-11}$ [11], which generates asymmetries way beyond any experimental accessibility at any time soon.

A loophole in this overall seemingly rather bleak picture of possible searches for CP-violation in $\eta^{(\prime)}$ decays has been identified in Ref. [45], where new symmetry tests in decays with dimuon pairs in the final state such as $\eta \to \mu^+\mu^-$, $\eta \to \mu^+\mu^-\gamma$, and $\eta \to \mu^+\mu^-e^+e^-$ have been investigated; the corresponding CP-odd observables in the first two of these necessarily involve muon polarization. These are induced by scalar–pseudoscalar quark–muon operators of the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2v^2}\operatorname{Im} c^{2222}_{\ell edq}\Big[(\bar{\mu}\mu)(\bar{s}i\gamma_5 s) - (\bar{\mu}i\gamma_5\mu)(\bar{s}s)\Big] + [u\text{-}, d\text{-quarks}]\,. \tag{9}$$

Their possible contribution to EDMs is suppressed to the two-loop level, as the muon (pseudo)scalar bilinears cannot directly couple to a single photon on account of vanishing VS and VP two-point functions in QCD+QED (due to C-parity). As a result, EDM constraints on such operators are significantly weaker, in particular for strange quarks, whose matrix elements in nucleons are suppressed. The resulting constraint on the coefficient was found to be $|\operatorname{Im} c^{2222}_{\ell edq}| < 0.04$ [45], which results in CP-odd asymmetries in particular in $\eta \to \mu^+\mu^-$ that may be testable at REDTOP [46].

The analysis of Ref. [45] has since been generalized to $\eta \to \pi^0\mu^+\mu^-$ and related decays [47] as well as to $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ [48], but so far, $\eta \to \mu^+\mu^-$ seems to remain the most promising channel to observe these operators.

4. C- and CP-violation

Class III interactions in the sense of Table 1 are harder to motivate theoretically from the point of view of a fundamental underlying theory. Effective operators on the quark level appear at mass dimension seven and involve both four-quark or fermion–gauge-boson operators such as [49–51]

$$\frac{1}{\Lambda^3}\bar{\psi}_f\gamma_5 D_\mu\psi_f\,\bar{\psi}_{f'}\gamma^\mu\gamma_5\psi_{f'} + \text{h.c.}\,, \qquad \frac{1}{\Lambda^3}\bar{\psi}_f\sigma_{\mu\nu}\lambda_a\psi_f G^{\mu\lambda}_a F^\nu_\lambda\,. \tag{10}$$

As these involve fermion helicity flips, they are actually of mass dimension eight when formulated in terms of fields invariant under the SM symmetry group (in other words, the prefactor $1/\Lambda^3$ would

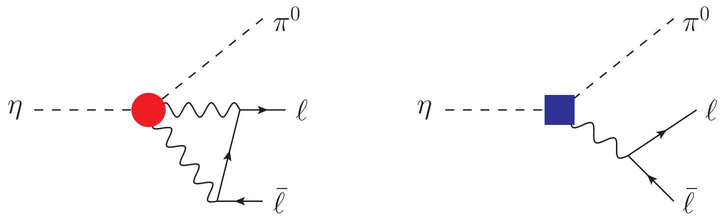

Figure 7: C-conserving two-photon (left) and C-violating single-photon (right) mechanism for the decay $\eta \to \pi^0\ell^+\ell^-$. Figure taken from Ref. [11].

have to be interpreted as $v/\Lambda^4_{\mathrm{BSM}}$, where v is the Higgs vacuum expectation value). Electroweak radiative corrections mix classes II and III, such that indirect EDM constraints will also limit the prefactors of the operators (10).

As the flavor-neutral light pseudoscalars $\eta^{(\prime)}$, π^0 are charge-conjugation eigenstates with eigenvalues $C = +1$, any decay that involves only these mesons as well as an *odd* number of photons is directly a test of C-conservation. Examples of such C-forbidden decays are π^0, $\eta^{(\prime)} \to 3\gamma$ or $\eta^{(\prime)} \to 2\pi^0\gamma^{(*)}$. This is more complicated for the seemingly simplest such decays, $\eta \to \pi^0\gamma^{(*)}$ as well as analogous η' decays: as a consequence of gauge invariance (and angular momentum conservation), the corresponding matrix elements can be decomposed according to

$$\langle \pi^0(k)|j_\mu(0)|\eta(p)\rangle = \big[q^2(p+k)_\mu - (M_\eta^2 - M_\pi^2)q_\mu\big]F_{\eta\pi^0}(q^2) \tag{11}$$

(cf. the similar, although flavor-changing, decays $K \to \pi\ell^+\ell^-$ [52, 53]), which vanish for real photons $q^2 = 0$. Potential nonvanishing, C-odd transitions can therefore only be assessed in the corresponding dilepton decays $\eta \to \pi^0\ell^+\ell^-$, for which, however, a C-conserving two-photon exchange, see Fig. 7, forms an irreducible SM background. The corresponding SM branching ratios have been recalculated recently [54], $\mathcal{B}(\eta \to \pi^0 e^+e^-) = 2.1(5) \times 10^{-9}$ and $\mathcal{B}(\eta \to \pi^0\mu^+\mu^-) = 1.2(3) \times 10^{-9}$ (as well as similar orders of magnitude for $\eta' \to \pi^0\ell^+\ell^-$ and $\eta' \to \eta\ell^+\ell^-$), based on a vector-meson-dominance model for the corresponding two-photon decays [9]. These are to be compared to the current experimental upper limits, $\mathcal{B}(\eta \to \pi^0 e^+e^-) < 7.5 \times 10^{-6}$ [55] and $\mathcal{B}(\eta \to \pi^0\mu^+\mu^-) < 5 \times 10^{-6}$ [56]. Evidence for a C-odd signal can hence only be claimed if measured branching ratios significantly exceed the theoretical SM prediction, or in the nowadays rather unlikely case that interference effects between C-even and C-odd mechanisms allow us to observe Dalitz plot asymmetries in differential decay distributions.

Obviously, the idea of testing C-odd interactions via certain asymmetries (instead of via rates of decays that are strictly possible only if C is violated) is even more appealing for decays with much larger branching ratios than these dilepton decays with their heavily suppressed SM rates. Indeed, with precisely this argument, it has been suggested to search for C-violation in Dalitz plot asymmetries in $\eta \to \pi^+\pi^-\pi^0$ [57], one of the dominant η decay modes, precisely for the reasons that such an asymmetry scales *linearly* with (supposedly very small) BSM physics parameters, while decay rates for, e.g., $\eta \to 3\gamma$ scale with such parameters *squared*.[1] Subsequently, analogous suggestions have also been made for similar η' decays, $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \pi^+\pi^-\pi^0$,

[1]The same problem obviously also affects the P- and CP-violating decays discussed in the previous section: decay rates for, e.g., $\eta^{(\prime)} \to \pi\pi$ or $\eta \to 4\pi^0$ [58] are quadratically suppressed in the small couplings.

with the rationale that these are partly sensitive to underlying short-distance operators of different isospin [59, 60].

The decay $\eta \to \pi^+\pi^-\pi^0$ manifestly breaks G-parity. Within the SM, charge conjugation is preserved and hence isospin has to be broken; with electromagnetic effects strongly suppressed [61–63], this makes $\eta \to 3\pi$ an ideal process to extract the light quark mass difference $m_u - m_d$ [5]. Allowing for BSM effects, two additional amplitudes that break C-invariance can be added, either conserving or breaking isospin, such that the full decay amplitude is written as

$$\mathcal{M}_c(s,t,u) = \mathcal{M}_0^{\not{C}}(s,t,u) + \xi\,\mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_2^{\not{C}}(s,t,u)\,, \qquad \xi = \frac{(M_{K^+}^2 - M_{K^0}^2)_{\text{QCD}}}{3\sqrt{3}F_\pi^2}\,, \tag{12}$$

where the subscript denotes the total (three-pion) isospin of the final state, and ξ parameterizes the isospin breaking in the SM amplitude. Each of these amplitudes can subsequently be analyzed in the so-called Khuri–Treiman framework [64], which allows us to take all iterated pion–pion rescattering effects into account consistently. For this purpose, the amplitudes are decomposed into single-variable amplitudes (SVAs) according to reconstruction theorems [57, 65, 66] up to (and including) P-waves:

$$\begin{aligned}
\mathcal{M}_1^{C}(s,t,u) &= \mathcal{F}_0(s) + (s-u)\,\mathcal{F}_1(t) + (s-t)\,\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)\,,\\
\mathcal{M}_0^{\not{C}}(s,t,u) &= (t-u)\,\mathcal{G}_1(s) + (u-s)\,\mathcal{G}_1(t) + (s-t)\,\mathcal{G}_1(u)\,,\\
\mathcal{M}_2^{\not{C}}(s,t,u) &= 2(u-t)\,\mathcal{H}_1(s) + (u-s)\,\mathcal{H}_1(t) + (s-t)\,\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)\,,
\end{aligned} \tag{13}$$

where the subscripts at the SVAs denote isospin (which in turn identifies the angular momentum uniquely via Bose symmetry). Obviously, $\mathcal{M}_{0,2}^{\not{C}}$ are odd under the exchange $t \leftrightarrow u$. For the SVAs, inhomogeneous Omnès solutions can be constructed,

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}x}{x^n}\,\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(x)}{|\Omega_I(x)|\,(x-s)}\right), \tag{14}$$

$\mathcal{A} \in \{\mathcal{F}, \mathcal{G}, \mathcal{H}\}$, where the Omnès functions $\Omega_I(s)$ are given in terms of known pion–pion phase shift input, and the subtraction polynomials $P_{n-1}(s)$ yield the free parameters of the dispersive representation. A minimal subtraction scheme for $\mathcal{M}_1^C$ depends on three (real) constants only; it allows us to fit data for $\eta \to \pi^+\pi^-\pi^0$ [67] and $\eta \to 3\pi^0$ [68] very well, as well as to fulfill constraints from chiral perturbation theory at $\mathcal{O}(p^4)$ [5, 69]. Employing the same minimal assumptions, it can be shown [59, 60] that $\mathcal{M}_{0,2}^{\not{C}}$ both depend on one single (real) coupling constant each, which can be matched unambiguously onto leading Taylor invariants:

$$\mathcal{M}_0^{\not{C}}(s,t,u) = i\,g_0\,(s-t)(u-s)(t-u) + \mathcal{O}(p^8)\,, \qquad \mathcal{M}_2^{\not{C}}(s,t,u) = i\,g_2\,(t-u) + \mathcal{O}(p^4)\,. \tag{15}$$

The effective coupling constants $g_{0,2}$ can be constrained from the most recent KLOE Dalitz plot data [67], which overall restricts C-violation to the permille level. While the kinematical dependence of the two C-odd amplitudes and their different interference patterns, cf. Fig. 8, can clearly not be resolved (all C-/CP-violating signals vanish within 1–2σ), the small phase space severely reduces the sensitivity to the isoscalar amplitude $\mathcal{M}_0^{\not{C}}$ [57, 59, 60].

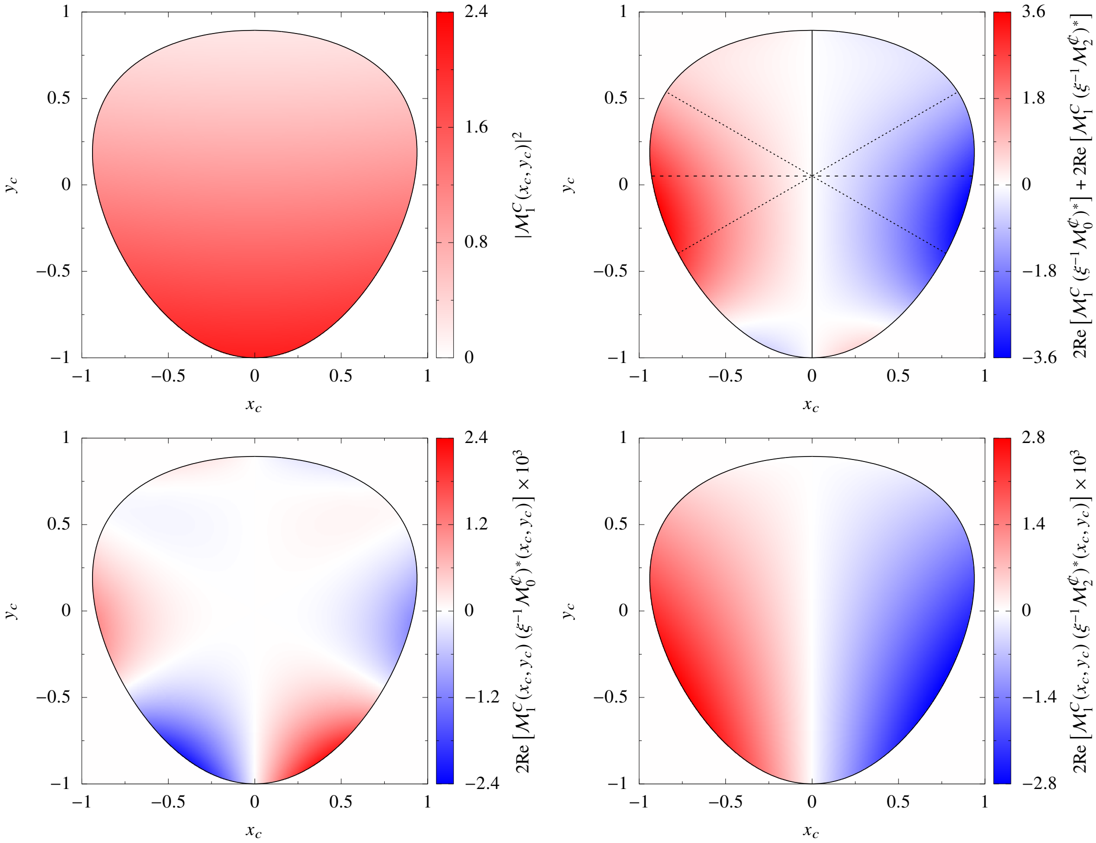

Figure 8: C-even (top left) and C-odd (top right) contributions to the Dalitz-plot distribution for $\eta \to \pi^+\pi^-\pi^0$ for the central fit result. The interferences of $\mathcal{M}_1^C$ with $\mathcal{M}_0^{\not{C}}$ (bottom left) and $\mathcal{M}_2^{\not{C}}$ (bottom right) give rise to significantly different mirror-symmetry-breaking patterns.

In principle, this kinematic suppression could be overcome in the decay $\eta' \to \pi^+\pi^-\pi^0$, in which one would expect C-violation to be caused by essentially the same fundamental operators; and indeed, the relative theoretical sensitivity to g_0 would be enhanced by about two orders of magnitude compared to $\eta \to \pi^+\pi^-\pi^0$ [59]. However, as this is a comparably rare decay ($\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0) \approx 3.6 \times 10^{-3}$), an analysis on C-violation based on the existing data by BESIII [70] is not too promising right now.

This is somewhat different for $\eta' \to \eta\pi^+\pi^-$ [71], for which a Khuri–Treiman-type dispersive analysis of the SM amplitude has been performed already [72]. Here, the SM decay conserves isospin, while a C-odd contribution would change the isospin by 1. Charge asymmetries in the $\eta' \to \eta\pi^+\pi^-$ Dalitz plot therefore test a BSM operator of different isospin, which is independent of the ones constrained in $\eta \to \pi^+\pi^-\pi^0$. A Khuri–Treiman analysis of the corresponding amplitudes has also been performed in Refs. [59, 60], and while the resulting constraints are not quite as strong yet, C-violation is restricted to the percent level in this channel. We also refer to that paper [59] as well as the corresponding dedicated contribution at this conference [60] for further graphical representations of the various Dalitz plot asymmetries.

5. Summary

Decays of η and η' mesons allow for a vast range of physics investigations, both within the Standard Model and beyond. Here we have concentrated on a dispersion-theoretical analysis of the $\eta^{(\prime)}$ transition form factors, as well as possible new patterns of discrete symmetry violation. While the singly-virtual $\eta^{(\prime)}$ transition form factors that determine the Dalitz decays $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ seem to be very well under control and calculable with high precision, the extension to the doubly-virtual case still poses some challenges, in particular the understanding of the breaking of a simple factorization assumption in both arguments at low-to-medium energies. Electric dipole moments set rigorous limits on P- and CP-violation in $\eta^{(\prime)}$ decays, putting most candidate channels way beyond foreseeable experimental reach; the exception being certain strange-quark–muon operators of dimension six that may be testable in particular in $\eta \to \mu^+\mu^-$. Very little theory work has been done on P-conserving, but C- and CP-violating mechanisms so far, however, a dispersion-theoretical framework has now been established to investigate charge asymmetries in $\eta^{(\prime)} \to \pi^+\pi^-\pi^0$ and $\eta' \to \eta\pi^+\pi^-$ Dalitz plots. New experimental results from forthcoming facilities such as the JLab Eta Factory [73] or REDTOP [46] are eagerly awaited.

References

[1] B. Kubis, *Precision tests of fundamental symmetries with η and η' decays*, *PoS* **Regio2021** (2022) 013.

[2] B.M.K. Nefkens and J.W. Price, *The Neutral decay modes of the eta meson*, *Phys. Scripta T* **99** (2002) 114 [nucl-ex/0202008].

[3] J. Wess and B. Zumino, *Consequences of anomalous Ward identities*, *Phys. Lett. B* **37** (1971) 95.

[4] E. Witten, *Global Aspects of Current Algebra*, *Nucl. Phys. B* **223** (1983) 422.

[5] G. Colangelo, S. Lanz, H. Leutwyler and E. Passemar, *Dispersive analysis of $\eta \to 3\pi$*, *Eur. Phys. J. C* **78** (2018) 947 [1807.11937].

[6] D. Stamen, D. Hariharan, M. Hoferichter, B. Kubis and P. Stoffer, *Kaon electromagnetic form factors in dispersion theory*, *Eur. Phys. J. C* **82** (2022) 432 [2202.11106].

[7] T. Aoyama et al., *The anomalous magnetic moment of the muon in the Standard Model*, *Phys. Rept.* **887** (2020) 1 [2006.04822].

[8] P. Masjuan and P. Sánchez-Puertas, *Pseudoscalar-pole contribution to the $(g_\mu - 2)$: a rational approach*, *Phys. Rev. D* **95** (2017) 054026 [1701.05829].

[9] R. Escribano, S. Gonzàlez-Solís, R. Jora and E. Royo, *Theoretical analysis of the doubly radiative decays $\eta^{(\prime)} \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$*, *Phys. Rev. D* **102** (2020) 034026 [1812.08454].

[10] J. Lu and B. Moussallam, *The $\pi\eta$ interaction and a_0 resonances in photon–photon scattering*, *Eur. Phys. J. C* **80** (2020) 436 [2002.04441].

[11] L. Gan, B. Kubis, E. Passemar and S. Tulin, *Precision tests of fundamental physics with η and η' mesons*, *Phys. Rept.* **945** (2022) 2191 [2007.00664].

[12] Muon $g-2$ collaboration, *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, *Phys. Rev. Lett.* **126** (2021) 141801 [2104.03281].

[13] S. Holz, J. Plenter, C.-W. Xiao, T. Dato, C. Hanhart, B. Kubis et al., *Towards an improved understanding of $\eta \to \gamma^*\gamma^*$*, *Eur. Phys. J. C* **81** (2021) 1002 [1509.02194].

[14] M. Hoferichter, B. Kubis, S. Leupold, F. Niecknig and S.P. Schneider, *Dispersive analysis of the pion transition form factor*, *Eur. Phys. J. C* **74** (2014) 3180 [1410.4691].

[15] M. Hoferichter, B.-L. Hoid, B. Kubis, S. Leupold and S.P. Schneider, *Pion-pole contribution to hadronic light-by-light scattering in the anomalous magnetic moment of the muon*, *Phys. Rev. Lett.* **121** (2018) 112002 [1805.01471].

[16] M. Hoferichter, B.-L. Hoid, B. Kubis, S. Leupold and S.P. Schneider, *Dispersion relation for hadronic light-by-light scattering: pion pole*, *JHEP* **10** (2018) 141 [1808.04823].

[17] C. Hanhart, A. Kupść, U.-G. Meißner, F. Stollenwerk and A. Wirzba, *Dispersive analysis for $\eta \to \gamma\gamma^*$*, *Eur. Phys. J. C* **73** (2013) 2668 [1307.5654].

[18] F. Stollenwerk, C. Hanhart, A. Kupść, U.-G. Meißner and A. Wirzba, *Model-independent approach to $\eta \to \pi^+\pi^-\gamma$ and $\eta' \to \pi^+\pi^-\gamma$*, *Phys. Lett. B* **707** (2012) 184 [1108.2419].

[19] WASA-at-COSY collaboration, *Exclusive Measurement of the $\eta \to \pi^+\pi^-\gamma$ Decay*, *Phys. Lett. B* **707** (2012) 243 [1107.5277].

[20] KLOE collaboration, *Measurement of $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$ with the KLOE Detector*, *Phys. Lett. B* **718** (2013) 910 [1209.4611].

[21] B. Kubis and J. Plenter, *Anomalous decay and scattering processes of the η meson*, *Eur. Phys. J. C* **75** (2015) 283 [1504.02588].

[22] BESIII collaboration, *Precision Study of $\eta' \to \gamma\pi^+\pi^-$ Decay Dynamics*, *Phys. Rev. Lett.* **120** (2018) 242003 [1712.01525].

[23] C. Hanhart, S. Holz, B. Kubis, A. Kupść, A. Wirzba and C.-W. Xiao, *The branching ratio $\omega \to \pi^+\pi^-$ revisited*, *Eur. Phys. J. C* **77** (2017) 98 [1611.09359].

[24] S. Holz, C. Hanhart, M. Hoferichter and B. Kubis, *A dispersive analysis of $\eta' \to \pi^+\pi^-\gamma$ and $\eta' \to \ell^+\ell^-\gamma$*, *Eur. Phys. J. C* **82** (2022) 434 [2202.05846].

[25] BESIII collaboration, *Observation of the Dalitz Decay $\eta' \to \gamma e^+e^-$*, *Phys. Rev. D* **92** (2015) 012001 [1504.06016].

[26] NA60 collaboration, *Study of the electromagnetic transition form-factors in $\eta \to \mu^+\mu^-\gamma$ and $\omega \to \mu^+\mu^-\pi^0$ decays with NA60*, Phys. Lett. B **677** (2009) 260 [0902.2547].

[27] A2 collaboration, *New determination of the η transition form factor in the Dalitz decay $\eta \to e^+e^-\gamma$ with the Crystal Ball/TAPS detectors at the Mainz Microtron*, Phys. Rev. C **89** (2014) 044608 [1309.5648].

[28] P. Adlarson et al., *Measurement of the $\omega \to \pi^0 e^+e^-$ and $\eta \to e^+e^-\gamma$ Dalitz decays with the A2 setup at MAMI*, Phys. Rev. C **95** (2017) 035208 [1609.04503].

[29] NA60 collaboration, *Precision study of the $\eta \to \mu^+\mu^-\gamma$ and $\omega \to \mu^+\mu^-\pi^0$ electromagnetic transition form-factors and of the $\rho \to \mu^+\mu^-$ line shape in NA60*, Phys. Lett. B **757** (2016) 437 [1608.07898].

[30] BaBar collaboration, *The $e^+e^- \to 2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$ and $K^+K^-\pi^+\pi^-\eta$ Cross Sections Measured with Initial-State Radiation*, Phys. Rev. D **76** (2007) 092005 [0708.2461].

[31] BaBar collaboration, *Study of the process $e^+e^- \to \pi^+\pi^-\eta$ using initial state radiation*, Phys. Rev. D **97** (2018) 052007 [1801.02960].

[32] R.J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, *Chiral Estimate of the Electric Dipole Moment of the Neutron in Quantum Chromodynamics*, Phys. Lett. B **88** (1979) 123.

[33] A. Pich and E. de Rafael, *Strong CP violation in an effective chiral Lagrangian approach*, Nucl. Phys. B **367** (1991) 313.

[34] KLOE-2 collaboration, *Upper limit on the $\eta \to \pi^+\pi^-$ branching fraction with the KLOE experiment*, JHEP **10** (2020) 047 [2006.14710].

[35] K. Ottnad, B. Kubis, U.-G. Meißner and F.-K. Guo, *New insights into the neutron electric dipole moment*, Phys. Lett. B **687** (2010) 42 [0911.3981].

[36] J. de Vries, E. Mereghetti, R.G.E. Timmermans and U. van Kolck, *The Effective Chiral Lagrangian From Dimension-Six Parity and Time-Reversal Violation*, Annals Phys. **338** (2013) 50 [1212.0990].

[37] J. Bsaisou, U.-G. Meißner, A. Nogga and A. Wirzba, *P- and T-Violating Lagrangians in Chiral Effective Field Theory and Nuclear Electric Dipole Moments*, Annals Phys. **359** (2015) 317 [1412.5471].

[38] M. Gorchtein, *Nucleon EDM and rare decays of η and η' mesons*, 0803.2906.

[39] T. Gutsche, A.N. Hiller Blin, S. Kovalenko, S. Kuleshov, V.E. Lyubovitskij, M.J. Vicente Vacas et al., *CP-violating decays of the pseudoscalars η and η' and their connection to the EDM of the neutron*, Phys. Rev. D **95** (2017) 036022 [1612.02276].

[40] A.S. Zhevlakov, M. Gorchtein, A.N. Hiller Blin, T. Gutsche and V.E. Lyubovitskij, *Bounds on rare decays of η and η' mesons from the neutron EDM*, Phys. Rev. D **99** (2019) 031703 [1812.00171].

[41] A.S. Zhevlakov, T. Gutsche and V.E. Lyubovitskij, *Updated limits on the CP violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings derived from the neutron EDM*, Phys. Rev. D **99** (2019) 115004 [1904.08154].

[42] C.Q. Geng, J.N. Ng and T.H. Wu, *CP violation in the decay $\eta \to \pi^+\pi^-\gamma$*, Mod. Phys. Lett. A **17** (2002) 1489 [hep-ph/0201191].

[43] D.-N. Gao, *The CP-violating asymmetry in $\eta \to \pi^+\pi^-e^+e^-$*, Mod. Phys. Lett. A **17** (2002) 1583 [hep-ph/0202002].

[44] J. Bijnens, A. Bramon and F. Cornet, *Three Pseudoscalar Photon Interactions in Chiral Perturbation Theory*, Phys. Lett. B **237** (1990) 488.

[45] P. Sánchez-Puertas, *CP violation in η muonic decays*, JHEP **01** (2019) 031 [1810.13228].

[46] REDTOP collaboration, *The REDTOP Experiment: Rare η/η' Decays To Probe New Physics*, 2203.07651.

[47] R. Escribano, E. Royo and P. Sánchez-Puertas, *New-physics signatures via CP violation in $\eta^{(\prime)} \to \pi^0\mu^+\mu^-$ and $\eta' \to \eta\mu^+\mu^-$ decays*, JHEP **05** (2022) 147 [2202.04886].

[48] M. Zillinger, B. Kubis and P. Sánchez-Puertas, *CP violation in $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ decays*, 2210.14925.

[49] I.B. Khriplovich, *What do we know about T odd but P even interaction?*, Nucl. Phys. B **352** (1991) 385.

[50] M.J. Ramsey-Musolf, *Electric dipole moments and the mass scale of new T violating, P conserving interactions*, Phys. Rev. Lett. **83** (1999) 3997 [hep-ph/9905429].

[51] A. Kurylov, G.C. McLaughlin and M.J. Ramsey-Musolf, *Constraints on T odd, P even interactions from EDMs, revisited*, Phys. Rev. D **63** (2001) 076007 [hep-ph/0011185].

[52] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portoles, *The Decays $K \to \pi\ell^+\ell^-$ beyond leading order in the chiral expansion*, JHEP **08** (1998) 004 [hep-ph/9808289].

[53] B. Kubis and R. Schmidt, *Radiative corrections in $K \to \pi\ell^+\ell^-$ decays*, Eur. Phys. J. C **70** (2010) 219 [1007.1887].

[54] R. Escribano and E. Royo, *A theoretical analysis of the semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$*, Eur. Phys. J. C **80** (2020) 1190 [2007.12467].

[55] WASA-AT-COSY collaboration, *Search for C violation in the decay $\eta \to \pi^0 e^+e^-$ with WASA-at-COSY*, Phys. Lett. B **784** (2018) 378 [1802.08642].

[56] R.I. Dzhelyadin et al., *Search for Rare Decays of η and η' Mesons and for Light Higgs Particles*, Phys. Lett. B **105** (1981) 239.

[57] S. Gardner and J. Shi, *Patterns of CP violation from mirror symmetry breaking in the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot*, Phys. Rev. D **101** (2020) 115038 [1903.11617].

[58] F.-K. Guo, B. Kubis and A. Wirzba, *Anomalous decays of η' and η into four pions*, Phys. Rev. D **85** (2012) 014014 [1111.5949].

[59] H. Akdag, T. Isken and B. Kubis, *Patterns of C- and CP-violation in hadronic η and η' three-body decays*, JHEP **02** (2022) 137 [2111.02417].

[60] H. Akdag, T. Isken and B. Kubis, *Dispersive representation of C- and CP-violation in $\eta \to \pi^+\pi^-\pi^0$ and $\eta' \to \eta\pi^+\pi^-$*, PoS **CD2021** (2022) 025.

[61] D.G. Sutherland, *Current algebra and the decay $\eta \to 3\pi$*, Phys. Lett. **23** (1966) 384.

[62] R. Baur, J. Kambor and D. Wyler, *Electromagnetic corrections to the decays $\eta \to 3\pi$*, Nucl. Phys. B **460** (1996) 127 [hep-ph/9510396].

[63] C. Ditsche, B. Kubis and U.-G. Meißner, *Electromagnetic corrections in $\eta \to 3\pi$ decays*, Eur. Phys. J. C **60** (2009) 83 [0812.0344].

[64] N.N. Khuri and S.B. Treiman, *Pion–Pion Scattering and $K^\pm \to 3\pi$ Decay*, Phys. Rev. **119** (1960) 1115.

[65] J. Kambor, C. Wiesendanger and D. Wyler, *Final state interactions and Khuri–Treiman equations in $\eta \to 3\pi$ decays*, Nucl. Phys. B **465** (1996) 215 [hep-ph/9509374].

[66] A.V. Anisovich and H. Leutwyler, *Dispersive analysis of the decay $\eta \to 3\pi$*, Phys. Lett. B **375** (1996) 335 [hep-ph/9601237].

[67] KLOE-2 collaboration, *Precision measurement of the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot distribution with the KLOE detector*, JHEP **05** (2016) 019 [1601.06985].

[68] A2 collaboration, *High-statistics measurement of the $\eta \to 3\pi^0$ decay at the Mainz Microtron*, Phys. Rev. C **97** (2018) 065203 [1803.02502].

[69] J. Gasser and H. Leutwyler, *$\eta \to 3\pi$ to One Loop*, Nucl. Phys. B **250** (1985) 539.

[70] BESIII collaboration, *Amplitude Analysis of the Decays $\eta' \to \pi^+\pi^-\pi^0$ and $\eta' \to \pi^0\pi^0\pi^0$*, Phys. Rev. Lett. **118** (2017) 012001 [1606.03847].

[71] BESIII collaboration, *Measurement of the matrix elements for the decays $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \eta\pi^0\pi^0$*, Phys. Rev. D **97** (2018) 012003 [1709.04627].

[72] T. Isken, B. Kubis, S.P. Schneider and P. Stoffer, *Dispersion relations for $\eta' \to \eta\pi\pi$*, Eur. Phys. J. C **77** (2017) 489 [1705.04339].

[73] L. Gan, *Test Fundamental Symmetries via Precision Measurements of π^0, η, and η' Decays*, JPS Conf. Proc. **13** (2017) 020063.