

## Light meson decays at BESIII

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The BESIII experiment has accumulated the world largest  $J/\psi$  sample, its radiative or hadronic decays offer a unique opportunity to investigate physics in the low energy region. This report presents the recent highlights in  $\eta/\eta'$  decays at BESIII, including the first observation of decay  $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$ , the search of  $CP$  violation in  $\eta' \rightarrow \pi^+\pi^-e^+e^-$ , the study of the rare decays  $\eta' \rightarrow \pi^0\pi^0\pi^0\pi^0$  and  $\eta' \rightarrow \gamma\gamma\eta$ , and the precision measurement of the branching fraction of  $\eta$  and  $\eta'$  decays.

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## 1. Introduction

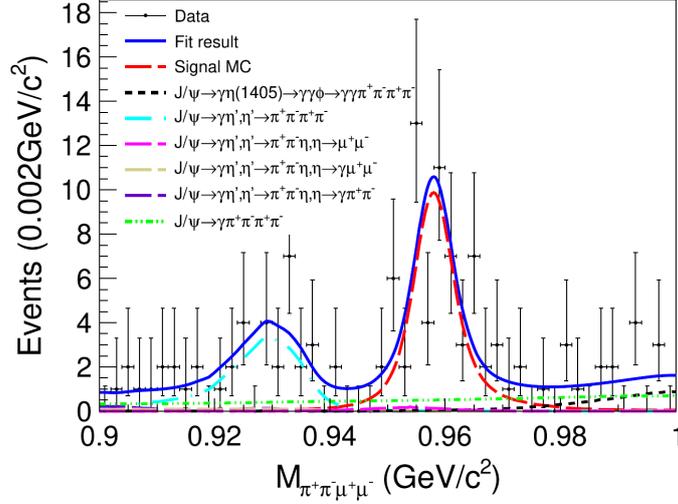
As admixtures of the lowest pseudoscalar singlet and octet,  $\eta$  and  $\eta'$  have attracted considerable theoretical and experimental attentions in a wide physics issues, e.g.,  $\eta - \eta'$  mixing parameters, the light quark mass ratio, the fundamental discrete symmetry violations, the hadronic contributions to the anomalous magnetic moment of the muon. In addition,  $\eta$  and  $\eta'$  decays offer unique opportunities to investigate their decay dynamics and test different chiral perturbation theory (ChPT) and the vector meson dominance (VMD) models. Moreover, their decays also provide a sensitive probe to test the potential of physics beyond the Standard Model (SM), including the searches for new light particles below GeV mass scale, such as hidden photons, light Higgs scalars and axion-like particles.

The BESIII detector [1] operates at BEPCII is an  $e^+e^-$  collider running at a center of mass energy between 2–4.9 GeV. In 2009 and 2012, the BESIII experiment accumulated 1.31 billion  $J/\psi$ . Based on this data sample, the BESIII collaboration has produced fruitful achievements related with light meson decays. The results reported in this proceeding are based on the data sample. In 2018 and 2019, the BESIII detector continued its data taking activities on the  $J/\psi$ , and make the number of  $J/\psi$  increased to 10 billion in total which are now available for physical analysis. Copious  $\eta$  and  $\eta'$  mesons are produced via the radiative and hadronic decays of  $J/\psi$ . Considering the radiative decays of  $J/\psi$ ,  $J/\psi \rightarrow \gamma\eta/\eta'$ , the total  $J/\psi$  sample corresponds to  $1.1 \times 10^7$   $\eta$  mesons and  $5.2 \times 10^7$   $\eta'$  mesons, respectively, which allow to analysis specific decays with the unprecedented statistics and to search for rare and forbidden decays.

## 2. Observation of $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$

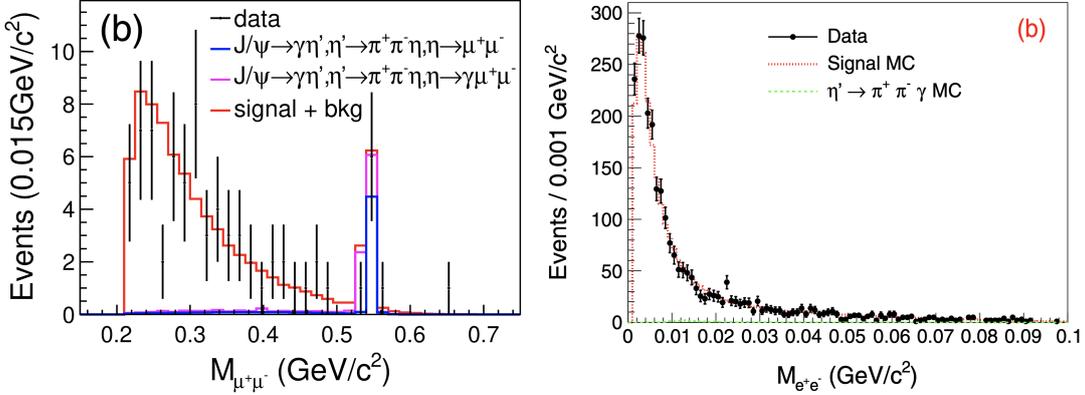
The processes  $\eta' \rightarrow \pi^+\pi^-l^+l^-$  (with  $l = e$  or  $\mu$ ) are expected to have similar decay mechanism as  $\eta' \rightarrow \pi^+\pi^-\gamma$ , with the radiative  $\gamma$  replaced with an offshell one and decays into a lepton pair [2]. Sizeable contribution from the box anomaly or excited  $\rho$  is found to be necessary to described the decay dynamic of  $\eta' \rightarrow \pi^+\pi^-\gamma$  [3], which makes those decays especially interested. Different theoretical models are used to investigate those decays, including the effective meson theory [4], the chiral unitary approach [5], and the hidden gauge model [6]. Due to the larger muon mass, the virtual photon conversion to dimuon is significantly suppressed relative to the conversion to dielectron. Therefore, the branching fraction of  $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$  is predicted about 2 orders of magnitude lower than for  $\eta' \rightarrow \pi^+\pi^-e^+e^-$ . In 2013, BESIII performed a study to search the processes  $\eta' \rightarrow \pi^+\pi^-l^+l^-$  (with  $l = e$  or  $\mu$ ) with a data sample of 225 million  $J/\psi$  events [7]. The branching fraction of  $\eta' \rightarrow \pi^+\pi^-e^+e^-$  is determined to be  $(2.11 \pm 0.12(stat) \pm 0.14(sys)) \times 10^{-3}$ , while no obvious  $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$  signal was observed and an upper limit  $\mathcal{B}(\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-) < 2.9 \times 10^{-5}$  at 90% confidence level (CL) is set, which is the most stringent result to date.

Using the radiative decay process  $J/\psi \rightarrow \gamma\eta'$  of  $1.31 \times 10^9$   $J/\psi$  events, BESIII recently observed a clean  $\eta'$  signal with the significance of  $8\sigma$  is observed for the first time in the invariant mass of  $\pi^+\pi^-\mu^+\mu^-$  [8], as shown in Fig. 1. The dominant background is from  $\eta' \rightarrow \pi^+\pi^-\pi^+\pi^-$ , corresponding to the left peak in Fig. 1. A global fit to the  $\pi^+\pi^-\mu^+\mu^-$  mass spectrum yields  $53 \pm 9$  signal events, with the corresponded branching fraction determined to be  $(1.97 \pm 0.33_{stat} \pm 0.19_{syst}) \times 10^{-5}$ . The result is in good agreement with theoretical predictions, which are predicted in the range of  $(1.5 - 2.5) \times 10^{-5}$  [4–6].



**Figure 1:** The  $\pi^+\pi^-\mu^+\mu^-$  invariant mass spectrum around the  $\eta'$  mass. The dots with error bars represent the data, the red line is signal MC, and the blue line is the total fit result. The other dotted lines represent different backgrounds.

In addition, the transition form factor for  $\eta'$  can be extracted from the electromagnetic dalitz decay processes  $\eta' \rightarrow \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-l^+l^-$  (with  $l = e$  or  $\mu$ ), which can contribute to the anomalous magnetic moment of the muon. Figure 2 shows the  $l^+l^-$  invariant mass distribution in the analysis of  $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$  and  $\eta' \rightarrow \pi^+\pi^-e^+e^-$ . With the full available  $J/\psi$  sample at BESIII, it's possible to access the transition form factor.



**Figure 2:** Left: The  $\mu^+\mu^-$  invariant mass spectrum for  $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$ . Right: The  $e^+e^-$  invariant mass spectrum for  $\eta' \rightarrow \pi^+\pi^-e^+e^-$ .

### 3. The study of $\eta' \rightarrow \pi^+\pi^-e^+e^-$

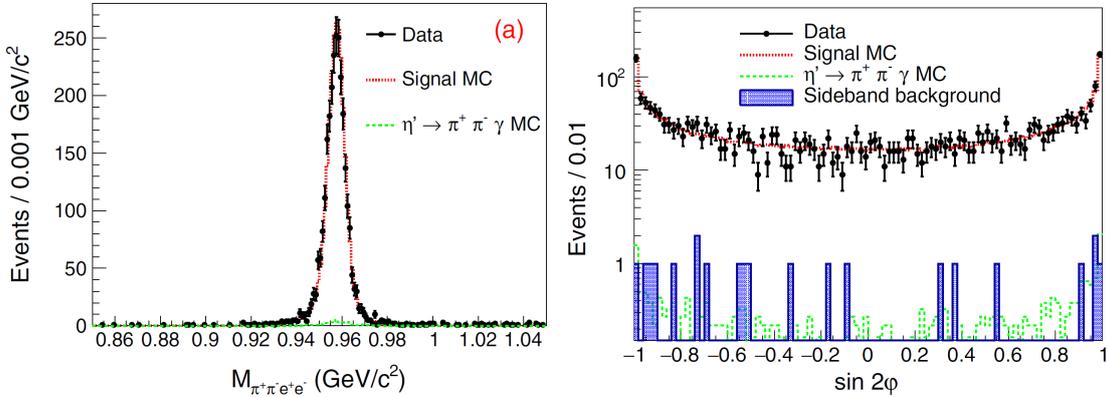
Similar as the decay  $\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$ ,  $\eta' \rightarrow \pi^+\pi^-e^+e^-$  is expected to proceed through an intermediate virtual photon  $\eta' \rightarrow \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-e^+e^-$ . It is possible to probe the electromagnetic structure of  $\eta'$  meson via this process by comparing the precision measurement of the branching ratio with the predictions from different theoretical approaches, such as VMD and ChPT models, .

Using a data sample of 1.31 billion  $J/\psi$  events, BESIII recently re-analyzed  $\eta' \rightarrow \pi^+\pi^-e^+e^-$  [9] via the process  $J/\psi \rightarrow \gamma\eta'$ . A global fit to the  $\pi^+\pi^-e^+e^-$  mass spectrum yields  $2584 \pm 52$  signal events, as shown in the left panel of Fig. 3. The corresponding branching ratio is determined to be  $\mathcal{B}(\eta' \rightarrow \pi^+\pi^-e^+e^-) = (2.42 \pm 0.05(stat) \pm 0.08(syst)) \times 10^{-3}$ . The statistical uncertainty has been improved by a factor of two compared to the previous BESIII result, which is superseded by this result. The result is consistent with but about one standard deviation higher than the predictions from various theoretical approach, including the hidden gauge model  $[(2.17 \pm 0.21) \times 10^{-3}]$  and the modified VMD model  $[(2.27 \pm 0.13) \times 10^{-3}]$  in reference [6], and the unitary chiral perturbation theory approach  $[(2.13^{+0.17}_{-0.31}) \times 10^{-3}]$  in reference [5].

A new aspect of this decay is the possibility of a new  $CP$ -violating mechanism [10] beyond the known CKM phase and flavor-changing process in Standard Model, which is analogue to the studies in  $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$  [11, 12]. The  $CP$ -violation effect could be induced from the interference between the parity-conserving magnetic transition and a possible parity-violating electric dipole type transition. The interference term can be extracted by an asymmetry  $A_\phi$  in the angular distribution defined as

$$A_\phi = \frac{N(\sin 2\phi > 0) - N(\sin 2\phi < 0)}{N(\sin 2\phi > 0) + N(\sin 2\phi < 0)}, \quad (1)$$

where  $\phi$  is the angle between the electrons and pions decay planes in the  $\eta'$  rest frame,  $N(x)$  is the acceptance-corrected number of events in the corresponding angular region. The distribution of  $\sin 2\phi$  is shown in the right panel of Fig 3. The asymmetry,  $A_\phi = (2.9 \pm 3.7_{stat} \pm 1.1_{syst})\%$ , is determined for the first time and consistent with zero, which means no  $CP$  violation observed in this process with the current statistics.



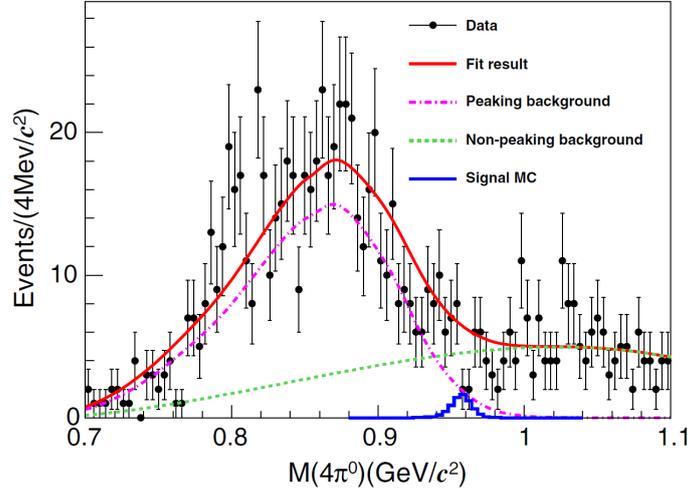
**Figure 3:** The  $\pi^+\pi^-e^+e^-$  invariant mass spectrum around  $\eta'$  mass (left) and the  $\sin 2\phi$  distribution in the signal region (right). The dots with error bars are data, the red histograms are signal MC, and other histograms are representing for backgrounds.

#### 4. The rare decay of $\eta' \rightarrow \pi^0\pi^0\pi^0\pi^0$

The decay  $\eta' \rightarrow \pi^0\pi^0\pi^0\pi^0$  is highly suppressed because of the  $S$ -wave  $CP$  violation. In ChPT, the  $S$ -wave is induced by the  $CP$ -violating QCD  $\theta$ -term with an expected branching fraction at the level of  $10^{-23}$  [13, 14]. While the  $P$ -wave mechanism via two  $\rho$  intermediate resonances cannot

contribute to the  $4\pi^0$  final states. The higher-order contributions involving a  $D$ -wave  $CP$ -conserving effect through the pion-pion charge-exchange rescattering loop predicts the branching fraction to be  $\sim 4 \times 10^{-8}$  [15] based on ChPT and VMD model but not strictly due to the lack of knowledge at such a high order in the chiral expansion. In addition, an alternative mechanism through two  $f_2$  tensor mesons is found to be completely negligible in comparison to the pion loop [15]. Therefore, a search for the decay  $\eta' \rightarrow \pi^0\pi^0\pi^0\pi^0$  is useful to check the reliability.

The most stringent upper limit  $3.2 \times 10^{-4}$  at 90% CL was from the GAMS- $4\pi$  Collaboration [16]. Using a data sample of 1.31 billion  $J/\psi$  events, BESIII Collaboration performed the search for this decay via the radiative decay of  $J/\psi$  [17]. The invariant mass spectrum of  $\pi^0\pi^0\pi^0\pi^0$  is shown in Fig. 4, and no significant  $\eta'$  signal is observed. With a Bayesian approach, the upper limit on the branching fraction is determined to be  $\mathcal{B}(\eta' \rightarrow \pi^0\pi^0\pi^0\pi^0) < 4.94 \times 10^{-5}$  at 90% CL, which is still far to reach the theoretical predication with a level of  $10^{-8}$ . Further studies with 10 billion  $J/\psi$  events are still necessary to search for the process  $\eta' \rightarrow 4\pi^0$ .

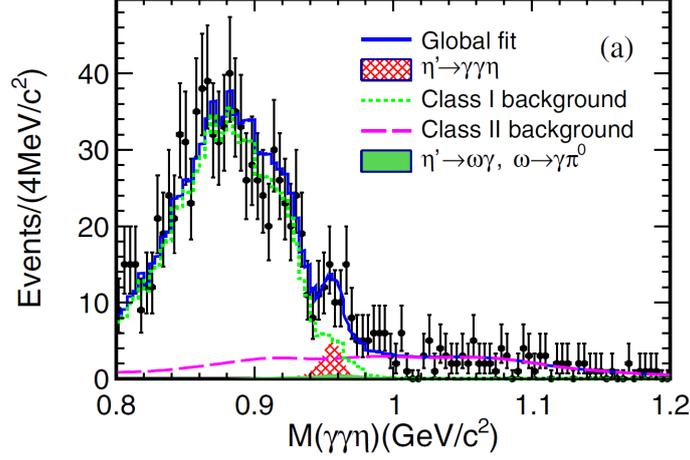


**Figure 4:**  $\pi^0\pi^0\pi^0\pi^0$  invariant mass spectrum in  $\eta'$  signal region. The black dots with error bars are data, the red line is fit result without signal, the blue histogram is the expected signal contribution shown with arbitrary normalization, the other lines represent contributions from different backgrounds.

## 5. Search for the rare decay of $\eta' \rightarrow \gamma\gamma\eta$

The dominant contributions in the doubly radiative decays of  $\eta' \rightarrow \gamma\gamma\pi^0$  and  $\eta' \rightarrow \gamma\gamma\eta$  are expected from vector meson exchange processes. Within the frameworks of the VMD and linear  $\sigma$  model, the branching fractions of  $\eta' \rightarrow \gamma\gamma\pi^0$  and  $\eta' \rightarrow \gamma\gamma\eta$  are predicted to be  $2.91(21) \times 10^{-3}$  and  $1.17(8) \times 10^{-4}$ , respectively [18]. With 1.31 billion  $J/\psi$  events, the branching fraction of  $\eta' \rightarrow \gamma\gamma\pi^0$  including the intermediate contributions from  $\omega(\rho^0) \rightarrow \gamma\pi^0$  was determined to be  $(32.0 \pm 0.7 \pm 2.3) \times 10^{-4}$  [19] by BESIII experiment. While the nonresonant branching fraction of  $\eta' \rightarrow \gamma\gamma\pi^0$  was determined to be  $(6.16 \pm 0.64 \pm 0.67) \times 10^{-4}$ , in agreement with theoretical prediction with hypothesis that this decay is dominated by the VMD process. However, the  $\eta' \rightarrow \gamma\gamma\eta$  decay has not been observed to date. The most stringent upper limit  $8 \times 10^{-4}$  at the 90% CL was from GAMS- $4\pi$  Collaboration [20].

Using a data sample of 1.31 billion  $J/\psi$  events, a search for the decay  $\eta' \rightarrow \gamma\gamma\eta$  is performed by BESIII experiment via the radiative decay of  $J/\psi$ ,  $J/\psi \rightarrow \gamma\eta'$  [21]. A global fit to the  $\gamma\gamma\eta$  invariant mass spectrum, shown in Fig. 5, yields  $24.9 \pm 10.3$   $\eta' \rightarrow \gamma\gamma\eta$  events, with a statistical significance of  $2.6\sigma$ , and the branching fraction is calculated to be  $(8.25 \pm 3.41 \pm 0.72) \times 10^{-5}$ , which need to be confirmed with higher statistics from BESIII and other experiments. Here the first error is statistical and the second systematical. An upper limit of the branching fraction is also set as  $1.3 \times 10^{-4}$  at 90% CL, which is consistent with a recent theoretical prediction of  $2 \times 10^{-4}$  [18] within the frame work of the linear  $\sigma$  model and the VMD model.



**Figure 5:**  $\gamma\gamma\eta$  invariant mass distribution in  $\eta'$  signal region. The black dots with error bars are data, the blue line is the global fit result, the shaded histogram is signal, and the others are backgrounds.

## 6. Precision measurement of the branching fractions of $\eta$ and $\eta'$ decays

Precise measurements of the branching fractions of  $\eta$  and  $\eta'$  decays are important for a wide variety of physics topics. For example, the decay widths of  $\eta/\eta' \rightarrow \gamma\gamma$  are related to the quark content of the two mesons, the branching fractions of  $\eta/\eta' \rightarrow 3\pi$  can provide valuable information on the light quark mass difference, the branching fractions of  $\eta/\eta' \rightarrow \gamma\pi^+\pi^-$  are related to the details of chiral dynamics. Due to the difficulty of tagging  $\eta$  and  $\eta'$  inclusive decays, the exclusive branching fractions of  $\eta$  and  $\eta'$  in the Particle Data Group (PDG) [22] are all relative measurements. Recently, the BESIII Collaboration developed an approach to tag the  $\eta$  [25] and  $\eta'$  [26] inclusive decays and then measured the absolute branching fraction of  $J/\psi \rightarrow \gamma\eta$  and  $J/\psi \rightarrow \gamma\eta'$ , respectively, and the exclusive branching fractions of  $\eta$  and  $\eta'$  for the first time.

Taking advantage of the excellent momentum resolution of charged tracks in the main drift chamber (MDC), the  $\eta$  and  $\eta'$  inclusive decays can be well reconstructed by requiring the radiative photon from  $J/\psi$  converted into  $e^+e^-$  pairs in the beam pipe or MDC inner wall. The branching fractions of  $J/\psi \rightarrow \gamma\eta$  and  $J/\psi \rightarrow \gamma\eta'$  are determined to be  $(1.067 \pm 0.005(stat) \pm 0.023(syst)) \times 10^{-3}$  and  $(5.27 \pm 0.03(stat) \pm 0.05(syst)) \times 10^{-3}$ , respectively, by analyzing the converted events. The results are consistent with the world average values, but with a significantly improved precision.

Four dominant decays of  $\eta$ ,  $\eta \rightarrow \gamma\gamma$ ,  $\pi^0\pi^0\pi^0$ ,  $\pi^+\pi^-\pi^0$ ,  $\gamma\pi^+\pi^-$  are selected via  $J/\psi$  decays to  $\gamma\eta$  with the radiative photon detected by the electromagnetic calorimeter. Similarly, five dominant

decays of  $\eta'$ ,  $\eta' \rightarrow \gamma\pi^+\pi^-$ ,  $\eta\pi^+\pi^-$ ,  $\eta\pi^0\pi^0$ ,  $\gamma\omega$ , and  $\gamma\gamma$  are selected via  $J/\psi$  decays to  $\gamma\eta'$ . Together with the  $\eta/\eta'$  sample tagged by photon conversion, the absolute branching fractions for those decay channels can be determined by

$$\mathcal{B}(\eta/\eta' \rightarrow X) = \frac{N_{\eta/\eta' \rightarrow X}^{obs}}{\varepsilon_{\eta/\eta' \rightarrow X}} \times \frac{\varepsilon_{J/\psi \rightarrow \gamma\eta/\eta'}}{N_{J/\psi \rightarrow \gamma\eta/\eta'}^{obs}}.$$

Here  $N_{\eta/\eta' \rightarrow X}^{obs}$  and  $\varepsilon_{\eta/\eta' \rightarrow X}$  are the observed signal number and efficiency for the specific process  $\eta/\eta' \rightarrow X$ . While  $N_{J/\psi \rightarrow \gamma\eta/\eta'}^{obs}$  and  $\varepsilon_{J/\psi \rightarrow \gamma\eta/\eta'}$  are the event number and efficiency for the  $\eta/\eta'$  sample tagged by photon conversion. The first independent branching fraction measurements for  $\eta$  and  $\eta'$  are presented in Table 1 and Table 2, respectively. Furthermore, the relative branching fractions for  $\eta$  and  $\eta'$  decays are also presented in Table 1 and Table 2, together with a comparison with CLEO's results [23, 24], which are all in agreement with CLEO's result within one or two standard deviations but with improved precision.

**Table 1:** Summary of the measured branching fractions for  $\eta$  decays.

Decay mode	$\mathcal{B}(\eta \rightarrow X)(\%)$		$\mathcal{B}/\mathcal{B}(\eta \rightarrow \gamma\gamma)$	
	BESIII [25]	PDG [22]	BESIII [25]	CLEO [24]
$\eta \rightarrow \gamma\gamma$	$39.86 \pm 0.04 \pm 0.99$	$39.41 \pm 0.20$	-	-
$\eta \rightarrow \pi^0\pi^0\pi^0$	$31.96 \pm 0.07 \pm 0.84$	$32.68 \pm 0.23$	$0.802 \pm 0.002 \pm 0.014$	$0.884 \pm 0.022 \pm 0.019$
$\eta \rightarrow \pi^+\pi^-\pi^0$	$23.04 \pm 0.03 \pm 0.54$	$22.92 \pm 0.28$	$0.578 \pm 0.001 \pm 0.008$	$0.587 \pm 0.011 \pm 0.009$
$\eta \rightarrow \gamma\pi^+\pi^-$	$4.38 \pm 0.02 \pm 0.10$	$4.22 \pm 0.08$	$0.110 \pm 0.001 \pm 0.002$	$0.103 \pm 0.004 \pm 0.004$

**Table 2:** Summary of the measured branching fractions for  $\eta'$  decays.

Decay mode	$\mathcal{B}(\eta' \rightarrow X)(\%)$		$\mathcal{B}/\mathcal{B}(\eta' \rightarrow \eta\pi^+\pi^-)$	
	BESIII [26]	PDG [22]	BESIII [26]	CLEO [23]
$\eta' \rightarrow \gamma\pi^+\pi^-$	$29.90 \pm 0.03 \pm 0.55$	$28.9 \pm 0.5$	$0.725 \pm 0.002 \pm 0.010$	$0.677 \pm 0.024 \pm 0.011$
$\eta' \rightarrow \eta\pi^+\pi^-$	$41.24 \pm 0.08 \pm 1.24$	$42.6 \pm 0.7$	-	-
$\eta' \rightarrow \eta\pi^0\pi^0$	$21.36 \pm 0.10 \pm 0.92$	$22.8 \pm 0.8$	$0.518 \pm 0.003 \pm 0.021$	$0.555 \pm 0.043 \pm 0.013$
$\eta' \rightarrow \gamma\omega$	$2.489 \pm 0.018 \pm 0.074$	$2.62 \pm 0.13$	$0.0604 \pm 0.0005 \pm 0.0012$	$0.055 \pm 0.007 \pm 0.001$
$\eta' \rightarrow \gamma\gamma$	$2.331 \pm 0.012 \pm 0.035$	$2.22 \pm 0.08$	$0.0565 \pm 0.0003 \pm 0.0015$	$0.053 \pm 0.004 \pm 0.001$

## 7. Conclusion

The BESIII collaboration has produced fruitful results related with light meson decays, including the studies of the decay dynamics, tests of discrete symmetries, searches for rare decays, and many other interesting results not covered in this proceeding. The BESIII experiment has accumulated 10 billion  $J/\psi$  events in total, which is a unique worldwide sample, allows to study the light mesons with unprecedented statistics. Ongoing analyses will produce more precise results in the next years.

## References

- [1] M. Ablikim *et al.* (BESIII Collaboration), Nucl. Instrum. Meth. A **614**, 345 (2010).
- [2] E. Witten, Nucl. Phys. B **223**, 422 (1983).
- [3] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **120**, 242003 (2018).
- [4] A. Faessler, C. Fuchs and M. I. Krivoruchenko, Phys. Rev. C **61**, 035206 (2000).
- [5] B. Borasoy and R. Nissler, Eur. Phys. J. A **33**, 95 (2007).
- [6] T. Petri, Other thesis (2010), 1010.2378.
- [7] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **87**(9), 092011 (2013).
- [8] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **103**(7), 072006 (2021).
- [9] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **103**(9), 092005 (2021).
- [10] D.-N. Gao, Mod. Phys. Lett. A **17**, 1583 (2002).
- [11] A. Alavi-Harati *et al.* (KTeV Collaboration), Phys. Rev. Lett. **84**, 408 (2000).
- [12] L. Iconomidou-Fayard *et al.* (NA48 Collaboration), Int. J. Mod. Phys. A **17**, 3012 (2002).
- [13] A. Pich and E. de Rafael, Nucl. Phys. B **367**, 313 (1991).
- [14] K. Ottnad, B. Kubis, U. G. Meissner and F. K. Guo, Phys. Lett. B **687**, 42 (2010).
- [15] F.-K. Guo, B. Kubis and A. Wirzba, Phys. Rev. D **85**, 014014 (2012).
- [16] S. V. Donskov *et al.* (GAMS-4 $\pi$  Collaboration), Mod. Phys. Lett. A **29**(40), 1450213 (2014).
- [17] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **101**(3), 032001 (2020).
- [18] R. Escobedo, S. González-Solís, R. Jora and E. Royo, Phys. Rev. D **102**(3), 034026 (2020).
- [19] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **96**(1), 012005 (2017).
- [20] S. V. Donskov (GAMS-4 $\pi$  Collaboration), Phys. Atom. Nucl. **78**(9), 1043 (2015).
- [21] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **100**(5), 052015 (2019).
- [22] P. A. Zyla *et al.*, *Review of Particle Physics*, PTEP **2020**(8), 083C01 (2020).
- [23] T. K. Pedlar *et al.* (CLEO Collaboration), Phys. Rev. D **79**, 111101 (2009).
- [24] A. Lopez *et al.* (CLEO Collaboration), Phys. Rev. Lett. **99**, 122001 (2007).
- [25] M. Ablikim *et al.* (BESIII Collaboration), arXiv **2109.12812**.
- [26] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **122**, 142002 (2019).