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A primer analysis of π^0-η-η' mixing from $V \to P\gamma$ and $P \to V\gamma$ decays

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A primer analysis of π^0-η-η' mixing from $V \to P\gamma$ and $P \to V\gamma$ ($V = \rho$, K^*, ω, ϕ and $P = \pi$, K, η, η') decays is performed using an enhanced phenomenological model that includes isospin-symmetry breaking. When the model is contrasted with the most recent experimental data for these radiative transitions, estimations for the mixing angles amongst the three pseudoscalar states with vanishing third-component of isospin are obtained. The current experimental uncertainties allow for isospin-symmetry violations with a confidence level of approximately 2.5σ.

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1. Introduction

The flavour $SU(3)$ symmetry is broken by the strange quark being significantly heavier than the up and down quarks [1–3]. As a result of this, the physical states η and η' become a mixture of the pure octet $|\eta_8\rangle$ and singlet $|\eta_0\rangle$ mathematical states. Through an orthogonal transformation with mixing angle θ_P, the mass eigenstates $|\eta\rangle$ and $|\eta'\rangle$ can be expressed as a linear combination of $|\eta_8\rangle$ and $|\eta_0\rangle$ [1, 3],

$$|\eta\rangle = \cos\theta_P\,|\eta_8\rangle - \sin\theta_P\,|\eta_0\rangle\ , \quad |\eta'\rangle = \sin\theta_P\,|\eta_8\rangle + \cos\theta_P\,|\eta_0\rangle\ , \tag{1}$$

with $|\eta_8\rangle = \frac{1}{\sqrt{6}}\left|u\bar{u}+d\bar{d}-2s\bar{s}\right\rangle$ and $|\eta_0\rangle = \frac{1}{\sqrt{3}}\left|u\bar{u}+d\bar{d}+s\bar{s}\right\rangle$. Another commonly used basis for the description of the η-η' mixing is the quark-flavour basis, which becomes exact in the limit $m_s \to \infty$ [4],

$$|\eta\rangle = \cos\phi_P\,|\eta_{\rm NS}\rangle - \sin\phi_P\,|\eta_{\rm S}\rangle\ , \quad |\eta'\rangle = \sin\phi_P\,|\eta_{\rm NS}\rangle + \cos\phi_P\,|\eta_{\rm S}\rangle\ , \tag{2}$$

where $|\eta_{\rm NS}\rangle = \frac{1}{\sqrt{2}}\left|u\bar{u}+d\bar{d}\right\rangle$ and $|\eta_{\rm S}\rangle = |s\bar{s}\rangle$. The mixing angles θ_P and ϕ_P are related by $\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$.

The mixing of the η and η' mesons is heavily influenced by the $U(1)_A$ anomaly of QCD [5], which induces a significant amount of mixing in the η-η' sector [2]. The $U(1)_A$ anomaly forces the $|\eta\rangle$ and $|\eta'\rangle$ mass eigenstates, which one would naively expect to be almost *ideally mixed*, to be nearly flavour octet and singlet states. In addition, the $U(1)_A$ anomaly is responsible for the non-Goldstone nature of the singlet state, forcing it to be massive even in the chiral limit. As a result of the mixing, the $U(1)_A$ anomaly is transferred to both the η and η' mesons [3].

In the vector meson sector, where the spins of the quark-antiquark bound states are parallel, the mixing between the ω and ϕ mesons is usually described using the quark-flavour basis, as there is no anomaly affecting this sector [2, 6]. Accordingly, the mixing angle ϕ_V is small (about 3° to 4°), which is consistent with the OZI-rule and becomes rigorous in the limit $N_c \to \infty$ [2].

In 2001, Bramon et al. [6] introduced an additional source of flavour $SU(3)$-symmetry breaking by including a quantum mechanical extension for the $VP\gamma$ radiative decays. The phenomenological model assumed isospin symmetry and the expectation that, even though gluon annihilation channels induce η-η' mixing, they play a negligible role in $VP\gamma$ transitions, respecting, therefore, the OZI-rule [6]. The $VP\gamma$ decay couplings were expressed in terms of the mixing angles and relative spatial wavefunction overlaps; then, using experimental estimations for the decay couplings, the best fit values for the free parameters of the model were obtained. The quality of their fits was very good (e.g. $\chi^2_{\rm min}/\text{d.o.f.} = 0.7$) and the estimations for the mixing angles were found to be $\phi_P = (37.7 \pm 2.4)^\circ$ and $\phi_V = (3.4 \pm 0.2)^\circ$ using the experimental data available at the time. An important conclusion that was drawn is that the $SU(3)$-breaking effects originated from flavour dependence through the relative spatial wavefunction overlaps cannot be neglected.

Feldmann et al. discussed in Ref. [7] the effects of isospin-symmetry breaking, which is induced by the mass difference between the u and d quarks, as well as QED effects, using the theoretical framework first presented in Ref. [8]. Mathematically, they expressed the admixtures of the η and η' to the physical π^0 as [7]

$$\left|\pi^0\right\rangle = |\pi_3\rangle + \epsilon\,|\eta\rangle + \epsilon'\,|\eta'\rangle\ , \tag{3}$$

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where $|\pi_3\rangle$ denotes the $I_3 = 0$ state of the pseudoscalar isospin triplet. By assuming a mixing angle of $\phi = 39.3^\circ$ for the η-η' system, they found through the diagonalisation of the associated mass matrix that the mixing between the π^0 and η mesons was $\epsilon = 1.4\%$, whilst the π^0-η' mixing was $\epsilon' = 0.37\%$ (no errors associated to these theoretical estimations were provided).

Kroll, as a continuation of the previous work, highlighted in Ref. [9] that isospin-symmetry breaking is of order $(m_d - m_u)/m_s$ due to the effect of the $U(1)_A$ anomaly, which is embodied in the divergence of the singlet axial-vector current [10, 11]. As a result of the mixing, the $U(1)_A$ anomaly is transferred to the π^0, η and η' physical states. A simple generalisation of the quark-flavour mixing scheme (e.g. [2, 7]) allowed him to write the following theoretical expressions for the mixing parameters ϵ and ϵ' [9],

$$\epsilon(z) = \cos\phi\left[\frac{1}{2}\frac{m_{dd}^2 - m_{uu}^2}{m_\eta^2 - m_{\pi^0}^2} + z\right] , \quad \epsilon'(z) = \sin\phi\left[\frac{1}{2}\frac{m_{dd}^2 - m_{uu}^2}{m_{\eta'}^2 - m_{\pi^0}^2} + z\right] , \tag{4}$$

where the parameter z is the quotient of decay constants $z = (f_u - f_d)/(f_u + f_d)$ and the quark mass difference $m_{dd}^2 - m_{uu}^2$ was estimated from the K^0-K^+ mass difference. Assuming again a mixing angle in the η-η' sector of $\phi = 39.3^\circ$ and making use of the $f_u = f_d$ limit, he found the following numerical estimations for the mixing parameters ϵ and ϵ',

$$\hat{\epsilon} = \epsilon(z = 0) = (1.7 \pm 0.2)\% , \quad \hat{\epsilon}' = \epsilon'(z = 0) = (0.4 \pm 0.1)\% . \tag{5}$$

Escribano et al. analysed in Ref. [12] the second-class current decays $\tau^- \to \pi^-\eta^{(\prime)}\nu_\eta$ and found estimations for the π^0-η and π^0-η' mixing parameters from theory, making use of scalar and vector form factors at next-to-leading order in χPT. The analytic expressions that they found are consistent with those from Kroll shown in Eq. (4) up to high-order isospin corrections. The numerical estimations that they obtained are

$$\begin{aligned}
\epsilon_{\pi\eta} &= \mathrm{c}\phi_{\eta\eta'}\frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_\eta^2 - m_{\pi^-}^2}\left[1 - \frac{m_\eta^2 - m_{\pi^-}^2}{M_S^2}\right] = (9.8 \pm 0.3)\times 10^{-3} , \\
\epsilon_{\pi\eta'} &= \mathrm{s}\phi_{\eta\eta'}\frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_{\eta'}^2 - m_{\pi^-}^2}\left[1 - \frac{m_{\eta'}^2 - m_{\pi^-}^2}{M_S^2}\right] = (2.5 \pm 1.5)\times 10^{-4} .
\end{aligned} \tag{6}$$

where $\mathrm{c}\phi_{\eta\eta'}$ and $\mathrm{s}\phi_{\eta\eta'}$ stand for $\cos\phi_{\eta\eta'}$ and $\sin\phi_{\eta\eta'}$; also, an η-η' mixing angle of $\phi_{\eta\eta'} = (41.4 \pm 0.5)^\circ$ was assumed, together with a scalar mass limit of $M_S = 980$ MeV.

It must be noted that Kroll's mixing parameters ϵ and ϵ' in Ref. [9] (cf. Eq. (3)) were defined in the quark-flavour basis whilst Escribano et al.'s $\epsilon_{\pi\eta}$ and $\epsilon_{\pi\eta'}$ in Ref. [12] were defined making use of the octet-singlet basis. Despite this difference, it can be easily shown that, given that both authors used the same $SO(3)$ rotation matrix *structure*, one can write $\epsilon = \epsilon_{\pi\eta}$ and $\epsilon' = \epsilon_{\pi\eta'}$, which are valid as first order approximations.

2. Methodology

From the effective Lagrangian that is commonly used to describe $VP\gamma$ radiative decays, a set of expressions for the theoretical decay couplings is found in terms of the free parameters of the model.

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Table 1: Comparison between estimations for the seven free parameters from the model presented in Ref. [6], using the PDG 2000 and the most up-to-date experimental data.

Parameter	Estimation from [6]	Current Estimation
g	0.70 ± 0.02 GeV^{-1}	0.70 ± 0.01 GeV^{-1}
$\frac{m_s}{\overline{m}}$	1.24 ± 0.07	1.17 ± 0.06
ϕ_P	$(37.7 \pm 2.4)^\circ$	$(41.4 \pm 0.5)^\circ$
ϕ_V	$(3.4 \pm 0.2)^\circ$	$(3.3 \pm 0.1)^\circ$
$z_{\rm NS}$	0.91 ± 0.05	0.84 ± 0.02
$z_{\rm S}$	0.89 ± 0.07	0.76 ± 0.04
$z_{\rm K}$	0.91 ± 0.04	0.89 ± 0.03
$\chi^2_{\rm min}$/d.o.f.	0.7	4.6

Next, using experimental data from Ref. [13], the corresponding experimental decay couplings are calculated and, finally, an optimization fit can be performed.

In the framework of the conventional quark model, the flavour symmetry-breaking mechanism associated to differences in the effective magnetic moments of light and strange quarks in magnetic dipolar transitions is introduced via constituent quark mass differences. This is implemented by means of a multiplicative $SU(3)$-breaking term, i.e. $1 - s_e \equiv \overline{m}/m_s$, in the s-quark entry of the quark-charge matrix Q [1]. A second source of flavour symmetry breaking, connected to the differences in the spatial extensions of the meson state wavefunctions, is also considered [6]. This symmetry-breaking mechanism is introduced through additional multiplicative factors in the theoretical coupling constants, accounting for the corresponding relative wavefunction overlaps, and are left as free parameters in the fit.

The isospin violation in the pseudoscalar sector is investigated in this framework. The mixing in this case requires an $SO(3)$ rotation matrix relating the π^0, η and η' mass eigenstates to the $SU(3)$ mathematical states, with three mixing angles. Additional wavefunction overlap factors are introduced to the model and gluon annihilation channels, which might contribute to the mixing, are neglected[1].

3. The mixing of the η-η' revisited

The analysis carried out in Ref. [6] for the estimation of the mixing angle in the η-η' sector is reproduced in this section using the most up-to-date experimental data [13]. The theoretical $VP\gamma$ decay couplings are confirmed to be those presented in Ref. [6]. The relationship between the decay couplings and the decay widths is given by

$$\Gamma(V \to P\gamma) = \frac{1}{3}\frac{g^2_{VP\gamma}}{4\pi}|\boldsymbol{p}_\gamma|^3 = \frac{1}{3}\Gamma(P \to V\gamma)\,, \tag{7}$$

where $\boldsymbol{p}_\gamma$ is the linear momentum of the outgoing photon. Using Eq. (7) together with the experimental data for the total decay widths, branching ratios and meson masses from Ref. [13], one can

[1]This is a necessary simplification to reduce the number of free parameters in the model; otherwise, the statistical fit would not be possible given the limited number of available decay channels.

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obtain experimental values for the decay couplings. From these and the corresponding theoretical counterparts, an optimisation fit can be performed. Making use of a standard minimisation software package, the optimal values for the seven free parameters of the model are presented in Table 1. One can see that the fitted values obtained in the present work are in good agreement with those found by Bramon et al. in Ref. [6]. The current associated standard errors are smaller, which is due to the fact that the uncertainties associated to the experimental measurements have decreased over the years. The most recent empirical data seems to favour a somewhat bigger η-η' mixing angle ϕ_P, which is consistent with other recent results (e.g. Refs. [14–16]). As well as this, the most up-to-date experimental data grants more relevance to the secondary source of flavour $SU(3)$-symmetry breaking, as the $z_{\rm NS}$ and $z_{\rm S}$ spatial wavefunction overlap factors are further from unity.

That being said, the quality of the fit for the current estimations is poor with a $\chi^2_{\rm min}/\text{d.o.f.} \simeq 23.1/5 \simeq 4.6$, while in Ref. [6], using the data available at the time, the quality of the fit was excellent, i.e. $\chi^2_{\rm min}/\text{d.o.f.} = 0.7$. This, again, is connected to the improved quality of the most recent data [13]. Based on this goodness-of-fit test, one ought to come to the conclusion that the current experimental data no longer supports the model presented in Ref. [6].

4. Enhanced model for the π^0-η-η' mixing

The phenomenological model presented above is enhanced in this section by incorporating isospin-breaking effects, enabling the investigation of the mixing phenomena between the π^0, η and η' pseudoscalar mesons. This improved model considers that the physical pseudoscalar mesons with vanishing third-component of isospin are an admixture of some pure mathematical states and the mixing is, thus, implemented by a three-dimensional rotation amongst them. In addition, the mechanisms of flavour $SU(3)$-symmetry breaking that have been discussed in section 2 are enhanced to account for violations of isospin. In the vector meson sector, a single mixing angle is still considered, as this sector is anomaly-free.

In order to find the theoretical decay couplings associated to the different $VP\gamma$ radiative transitions, one starts with the effective Lagrangian that is used to calculate amplitudes in $V \to P\gamma$ and $P \to V\gamma$ decay processes [1],

$$\mathscr{L}_{VP\gamma} = g_e \epsilon_{\mu\nu\alpha\beta} \partial^\mu A^\nu \text{Tr}[Q(\partial^\alpha V^\beta P + P \partial^\alpha V^\beta)] \,, \tag{8}$$

where g_e is a generic electromagnetic coupling constant, $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor, A_μ is the electromagnetic field, V_μ and P are the matrices for the vector and pseudoscalar meson fields, respectively, and Q is the quark-charge matrix $Q = diag\{2/3, -1/3, -1/3\}$ [1].

Next, the following $SO(3)$ rotation matrix correlating the pseudoscalar $I_3 = 0$ physical states with the pure quark-flavour basis states is selected

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}\text{c}\phi_{23} + \epsilon_{13}\text{s}\phi_{23} & \text{c}\phi_{23} & -\text{s}\phi_{23} \\ -\epsilon_{13}\text{c}\phi_{23} - \epsilon_{12}\text{s}\phi_{23} & \text{s}\phi_{23} & \text{c}\phi_{23} \end{pmatrix} \begin{pmatrix} \pi_3 \\ \eta_{\rm NS} \\ \eta_{\rm S} \end{pmatrix} , \tag{9}$$

where ϵ_{12} and ϵ_{13} are first order approximations to the corresponding ϕ_{12} and ϕ_{13} mixing angles, as isospin-breaking corrections are small [8]. It must be stressed that the particular *structure* that we

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have selected for the $SO(3)$ rotation matrix is down to the fact that it enables an enhanced *resolution* against the statistical uncertainties associated to both mixing parameters ϵ_{12} and ϵ_{13} simultaneously, once the optimisation fits are performed[2].

The transformations that map Kroll's ϵ and ϵ' in the quark-flavour basis (cf. Eq. (3) and Ref. [9]) and Escribano et al.'s $\epsilon_{\pi\eta}$ and $\epsilon_{\pi\eta'}$ in the octet-singlet basis (cf. Ref. [12])) to the ϵ_{12} and ϵ_{13} in the quark-flavour basis used in this letter (cf. Eq. (9)) are[3]

$$\begin{pmatrix}\epsilon_{12}\\ \epsilon_{13}\end{pmatrix}=\begin{pmatrix}\mathrm{c}\phi_P & \mathrm{s}\phi_P\\ -\mathrm{s}\phi_P & \mathrm{c}\phi_P\end{pmatrix}\begin{pmatrix}\epsilon\\ \epsilon'\end{pmatrix}\,,\qquad \begin{pmatrix}\epsilon_{12}\\ \epsilon_{13}\end{pmatrix}=\frac{1}{\sqrt{3}}\begin{pmatrix}\mathrm{c}\theta_P-\sqrt{2}\,\mathrm{s}\theta_P & \mathrm{s}\theta_P+\sqrt{2}\,\mathrm{c}\theta_P\\ -\mathrm{s}\theta_P-\sqrt{2}\,\mathrm{c}\theta_P & \mathrm{c}\theta_P-\sqrt{2}\,\mathrm{s}\theta_P\end{pmatrix}\begin{pmatrix}\epsilon_{\pi\eta}\\ \epsilon_{\pi\eta'}\end{pmatrix}. \tag{10}$$

At this point, one can obtain the expressions for the theoretical decay couplings of the enhanced phenomenological model. These are

$$\begin{aligned}
g_{\rho^0\pi^0\gamma}&=g\left(\frac{1}{3}+\epsilon_{12}z_{\rm NS}\right)\,,\quad g_{\rho^+\pi^+\gamma}=g\frac{z_+}{3}\,,\\
g_{\rho^0\eta\gamma}&=g\left[\left(z_{\rm NS}-\frac{\epsilon_{12}}{3}\right)c\phi_{23}+\frac{\epsilon_{13}}{3}s\phi_{23}\right]\,,\\
g_{\omega\pi^0\gamma}&=g\left[\left(1+\frac{\epsilon_{12}}{3}z_{\rm NS}\right)c\phi_V+\frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}\epsilon_{13}s\phi_V\right]\,,\\
g_{\eta'\rho^0\gamma}&=g\left[\left(z_{\rm NS}-\frac{\epsilon_{12}}{3}\right)s\phi_{23}-\frac{\epsilon_{13}}{3}c\phi_{23}\right]\,,\\
g_{\omega\eta\gamma}&=g\left\{\left[\left(\frac{z_{\rm NS}}{3}-\epsilon_{12}\right)c\phi_{23}+\epsilon_{13}s\phi_{23}\right]c\phi_V-\frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}s\phi_{23}s\phi_V\right\}\,,\\
g_{\eta'\omega\gamma}&=g\left\{\left[\left(\frac{z_{\rm NS}}{3}-\epsilon_{12}\right)s\phi_{23}-\epsilon_{13}c\phi_{23}\right]c\phi_V+\frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}c\phi_{23}s\phi_V\right\}\,,\\
g_{\phi\pi^0\gamma}&=g\left[\left(1+\frac{\epsilon_{12}}{3}z_{\rm NS}\right)s\phi_V-\frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}\epsilon_{13}c\phi_V\right]\,,\\
g_{\phi\eta\gamma}&=g\left\{\left[\left(\frac{z_{\rm NS}}{3}-\epsilon_{12}\right)c\phi_{23}+\epsilon_{13}s\phi_{23}\right]s\phi_V+\frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}s\phi_{23}c\phi_V\right\}\,,\\
g_{\phi\eta'\gamma}&=g\left\{\left[\left(\frac{z_{\rm NS}}{3}-\epsilon_{12}\right)s\phi_{23}-\epsilon_{13}c\phi_{23}\right]s\phi_V-\frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}c\phi_{23}c\phi_V\right\}\,,\\
g_{K^{*0}K^0\gamma}&=-\frac{1}{3}g\left(1+\frac{\overline{m}}{m_s}\right)z_{K^0}=-\frac{1}{3}g\left(1+z_{\rm S}\frac{\overline{m}}{m_s}\right)z'_{K^0}\,,\\
g_{K^{*+}K^+\gamma}&=\frac{1}{3}g\left(2-\frac{\overline{m}}{m_s}\right)z_{K^+}=\frac{1}{3}g\left(2-z_{\rm S}\frac{\overline{m}}{m_s}\right)z'_{K^+}\,,
\end{aligned} \tag{11}$$

where the wavefunction overlap parameters have been redefined as relative overlap factors [6]: $z_{\rm NS}\equiv Z_{\rm NS}/Z_3$, $z_{\rm S}\equiv Z_{\rm S}/Z_3$, $z_+\equiv Z_+/Z_3$, $z_{K^0}\equiv Z_{K^0}/Z_3$ and $z_{K^+}\equiv Z_{K^+}/Z_3$. The generic electromagnetic coupling constant g_e in Eq. (8) has been replaced by $g=Z_3 g_e$ on the right hand side equalities of Eq. (11). In some instances, the overlap factors in the strange sector have been redefined to $z'_{K^0}=z_{K^0}(1+\overline{m}/m_s)/(1+z_{\rm S}\overline{m}/m_s)$ and $z'_{K^+}=z_{K^+}(2-\overline{m}/m_s)/(2-z_{\rm S}\overline{m}/m_s)$ in order

[2]This point will become clearer later when the results are discussed.

[3]Given that these are orthogonal transformations, to move from one definition to the other in the opposite direction, one only needs to multiply by the transposed matrices.

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to avoid redundant free parameters. It is worth highlighting that Eq. (11) reduces to the couplings shown in Ref. [6] in the good $SU(2)$ limit, as expected.

A fit of the theoretical decay couplings from Eq. (11) to the experimental data for ten free parameters provides the following estimations

$$\begin{aligned} g &= 0.69 \pm 0.01\ \text{GeV}^{-1}\ , & z_+ &= 0.95 \pm 0.05\ , \\ \phi_{23} &= (41.5 \pm 0.5)^\circ\ , & \phi_V &= (4.0 \pm 0.2)^\circ\ , \\ \epsilon_{12} &= (2.3 \pm 1.0)\ \%\ , & \epsilon_{13} &= (2.5 \pm 0.9)\ \%\ , \\ z_{\text{NS}} &= 0.89 \pm 0.03\ , & z_{\text{S}}\overline{m}/m_s &= 0.65 \pm 0.01\ , \\ z'_{\text{K}^0} &= 1.01 \pm 0.04\ , & z'_{\text{K}^+} &= 0.76 \pm 0.04\ . \end{aligned} \tag{12}$$

The quality of the fit is relatively good, with $\chi^2_{\text{min}}/\text{d.o.f.} \simeq 4.6/2 = 2.3$. The fitted values for the mixing angles ϕ_{23} and ϕ_V are in very good agreement with recent published results (e.g. [5, 14]). The g and $m_s/\overline{m}$ (see Eq. (13) below for an estimation of the latter) are also consistent with those from other studies but, as highlighted by Bramon et al. in Ref. [6], these parameters are largely dependent on the particular model used; hence, comparison provides limited value.

An important point to notice from Eq. (12) is that the estimations for ϵ_{12} and ϵ_{13} are very small but not compatible with zero with a confidence level of 2.3σ and 2.8σ, respectively, assuming a Gaussian distribution for the error. The ϵ_{12} and ϵ_{13} values from our fit can be translated to Kroll's and Escribano et al.'s definitions for their $SO(3)$ rotation matrix yielding $\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)$ % and $\epsilon' = \epsilon_{\pi\eta'} = (3.4 \pm 0.9)$ %. It can be observed that our mixing parameters ϵ and $\epsilon_{\pi\eta}$ are compatible with zero, whilst our parameters ϵ' and $\epsilon_{\pi\eta'}$ are not consistent with zero with a confidence level of 3.8σ. Clearly, all mathematical representations for the physical states are equivalent; however, the specific rotation matrix selected in Eq. (9) enables the simultaneous ascertainment that *both* parameters controlling the mixing in the π^0-η and π^0-η' sectors are incompatible with zero.

In addition, it is worth noting from our results that the contribution to the physical state $|\pi^0\rangle$ from the mathematical state $|\eta_8\rangle$ is significantly smaller (in fact, consistent with zero) than that from the pure singlet state $|\eta_0\rangle$. This is an interesting result as one would naively expect the amount of mixing in the π^0-η system to be larger than the one found in the π^0-η' sector, based on mass arguments. This can be explained, though, by the fact that the $U(1)_A$ anomaly mediates $\eta_0 \leftrightarrow \pi_3$ transitions and, therefore, provides an additional contribution to the associated mixing. Note that Escribano et al. [12] made use of the large-N_c limit in their calculations, which effectively rids the theory of the chiral anomaly; hence, the effect mentioned above does not surface in their estimations for the mixing parameters. On the other hand, Kroll obtained in Ref. [9] first order theoretical results for the mixing parameters, neglecting, thus, any high-order symmetry breaking corrections; this is a sound approximation for the η-η' system but might potentially compromise the results for the π^0-η and π^0-η' sectors where the mixing parameters are very small.

Another fit is carried out fixing $\epsilon_{12} = \epsilon_{13} = 0$ and leaving all the other parameters free. The quality of the fit is significantly decreased with $\chi^2_{\text{min}}/\text{d.o.f.} \simeq 21.3/4 \simeq 5.3$, highlighting the fact that a certain amount of mixing between the neutral π^0 with the η and η' mesons different from zero is required to correctly describe the data.

Fixing the parameters $z_+ = 1$ and $z_{\text{K}^0} = z_{\text{K}^+}$, which accounts for *turning off* the secondary mechanism of isospin-symmetry breaking, and performing a fit with all the other parameters left

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free, we find

$$
\begin{aligned}
g &= 0.69 \pm 0.01\ \text{GeV}^{-1}\ , & m_s/\overline{m} &= 1.17 \pm 0.06\ , \\
\phi_{23} &= (41.5 \pm 0.5)^\circ\ , & \phi_V &= (4.0 \pm 0.2)^\circ\ , \\
\epsilon_{12} &= (2.4 \pm 1.0)\ \%\ , & \epsilon_{13} &= (2.5 \pm 0.9)\ \%\ , \\
z_{\rm NS} &= 0.89 \pm 0.03\ , & z_{\rm S} &= 0.77 \pm 0.04\ , \\
& & z_{\rm K} = 0.90 \pm 0.03\ , &
\end{aligned}
\tag{13}
$$

where the quality of the fit is better, i.e. $\chi^2_{\rm min}/\text{d.o.f.} \simeq 5.6/3 \simeq 1.9$. The z's in Eq. (13) are different from unity, signalling that the secondary mechanism of flavour $SU(3)$-symmetry breaking is still required for the correct description of the experimental data. This statement can be tested by performing a fit where all the z's are fixed to one and it is found that the quality of the fit is substantially decreased, i.e. $\chi^2_{\rm min}/\text{d.o.f.} \simeq 41.8/6 \simeq 7.0$.

The estimates for ϵ_{12} and ϵ_{13} in Eq. (13) are, again, not compatible with zero with a confidence level of 2.4σ and 2.8σ, respectively. In general, the estimations from Eq. (13) are very approximate to the ones shown in Eq. (12). It is interesting to see that reducing the number of free parameters in the last fit leads to a substantial increase in the quality of the fit. This is related to the fact that, despite the residual $\chi^2_{\rm min}$ being smaller when ten free parameters are employed, this reduction does not compensate for the loss of one degree of freedom. Accordingly, it appears that the introduction of the secondary mechanism of isospin-symmetry breaking is not required to reproduce the experimental data. For this reason, the degrees of freedom z_+, $z_{\rm K^0}$ and $z_{\rm K^+}$ will be fixed to $z_+ = 1$ and $z_{\rm K^0} = z_{\rm K^+}$ for any subsequent fits.

Two more statistical fits using the estimated values for ϵ_{12} and ϵ_{13} from Kroll [9] and Escribano et al. [12] can be performed. Starting with Kroll's estimations $\epsilon_{12} = (1.6 \pm 0.2)$ % and $\epsilon_{13} = (-0.8 \pm 0.1)$ % we obtain the values shown in Table 2 under the column Fit 3. Likewise, using Escribano et al.'s $\epsilon_{12} = (7.5 \pm 0.2) \times 10^{-3}$ and $\epsilon_{13} = (-6.3 \pm 0.2) \times 10^{-3}$ and performing the fit once more, the results displayed in Table 2 under the column Fit 4 are found. This shows that the theoretical estimations for the mixing parameters ϵ_{12} and ϵ_{13} provided by Kroll [9] and Escribano et al. [12] do not appear to agree with the most recent experimental data [13]. It must be stressed, though, that the phenomenological model presented in this letter is based on the relatively simple standard quark model with a quantum mechanical extension, whilst Refs. [9] and [12] used more sophisticated theoretical approaches. Having said this, those estimations had limited numerical input from experiment due to their intrinsic theoretical nature.

A final fit is carried out where the experimental points associated to the neutral and charged $K^* \to K\gamma$ transitions are not considered[4]. Accordingly, the free parameters $z_{\rm K}$, or $z'_{\rm K^0}$ and $z'_{\rm K^+}$, are not included in this fit, and the parameters $m_s/\overline{m}$ and $z_{\rm S}$ are considered jointly again. The estimated values from the fit are displayed again in Table 2 under the column Fit 5. The estimates for ϵ_{12} and ϵ_{13} are again incompatible with zero at a confidence level of 2.4σ and 2.8σ, respectively.

A summary of all the fitted parameters is shown in Table 2. The robustness of the fitted values for the parameters g, ϵ_{12}, ϵ_{13}, ϕ_{23} and ϕ_V across Fits 1, 2 and 5 is remarkable. In addition, the consistency of the z parameters across all the fits is also very good.

[4]Note that, traditionally, strange decay width measurements have suffered from larger uncertainties than the other radiative decays.

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Table 2: Summary of fitted values for the parameters corresponding to the different fits discussed.

Parameter	Fit 1	Fit 2	Fit 3	Fit 4	Fit 5
g (GeV^{-1})	0.69 ± 0.01	0.69 ± 0.01	0.69 ± 0.01	0.70 ± 0.01	0.69 ± 0.01
ϵ_{12}	(2.3 ± 1.0) %	(2.4 ± 1.0) %	-	-	(2.4 ± 1.0) %
ϵ_{13}	(2.5 ± 0.9) %	(2.5 ± 0.9) %	-	-	(2.5 ± 0.9) %
ϕ_{23} (°)	41.5 ± 0.5	41.5 ± 0.05	41.4 ± 0.5	41.4 ± 0.5	41.5 ± 0.5
ϕ_V (°)	4.0 ± 0.2	4.0 ± 0.2	3.1 ± 0.1	3.2 ± 0.1	4.0 ± 0.2
$m_s/\overline{m}$	-	1.17 ± 0.06	1.17 ± 0.06	1.17 ± 0.06	-
$z_S\overline{m}/m_s$	0.65 ± 0.01	-	-	-	0.65 ± 0.01
z_{NS}	0.89 ± 0.03	0.89 ± 0.03	0.86 ± 0.02	0.85 ± 0.02	0.89 ± 0.03
z_+	0.95 ± 0.05	-	-	-	-
z_S	-	0.77 ± 0.04	0.77 ± 0.04	0.77 ± 0.04	-
z_K	-	0.90 ± 0.03	0.90 ± 0.03	0.90 ± 0.03	-
z'_{K^0}	1.01 ± 0.04	-	-	-	-
z'_{K^+}	0.76 ± 0.04	-	-	-	-
χ^2_{min}/d.o.f.	2.3	1.9	4.4	4.8	1.9

5. Conclusions

The phenomenological model based on the standard quark model with two sources of flavour $SU(3)$-symmetry breaking proposed by Bramon et al. in Ref. [6] has been tested using the most up-to-date $VP\gamma$ experimental data [13] in section 3. It has been shown that the quality of the most recent empirical data is sufficiently good to see that the model struggles to accurately reproduce experiment. Consequently, the objective of the present work has been to enhance this phenomenological model to reconcile it with experiment. This has been achieved by introducing isospin symmetry-breaking effects into the model.

The main result drawn from the present investigation is that the quality of the most up-to-date experimental data [13] enables a small amount of isospin-symmetry breaking that is inconsistent with zero, with a confidence level of approximately 2.5σ, using the enhanced phenomenological model. The quality of the performed fits is good, with e.g. χ^2_{min}/d.o.f. $\simeq 1.9$. In addition, the estimations for the fit parameters appear to be very robust across the fits that have been performed. The fitted values for $g = 0.69 \pm 0.01$ GeV^{-1}, $\phi_{23} = (41.5 \pm 0.5)^\circ$, $\phi_V = (4.0 \pm 0.2)^\circ$ and $m_s/\overline{m} = 1.17 \pm 0.06$ are in good agreement with those from other analysis available in the published literature (e.g. [5, 14]). Contrary to this, our estimates for the parameters controlling the mixing in the π^0-η and π^0-η' sectors, i.e. $\epsilon_{12} = (2.4 \pm 1.0)$ % and $\epsilon_{13} = (2.5 \pm 0.9)$ % (using the mathematical definition from Eq. (9)) or $\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)$ % and $\epsilon' = \epsilon_{\pi\eta'} = (3.5 \pm 0.9)$ % (once translated into Kroll's [9] and Escribano et al.'s [12] definitions), are not in accordance with the estimations that were provided by these authors in Ref. [9] and [12].

To conclude, it is worth highlighting that all the results from the present investigation appear to indicate that a phenomenological model including simple quark model concepts, with a quantum mechanical extension implementing a second source of flavour symmetry breaking, is still sufficient

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to describe to a large degree of accuracy the radiative decays, and the rich and complex mixing phenomenology in the pseudoscalar meson sector.

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