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Detecting the pure triangle singularity effect through the decays of $\psi(2S)$

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In this paper, we discuss two processes for detecting the pure triangle singularity effect through the decays of $\psi(2S)$. The triangle singularity is proposed around 1959, but until now there is no confirmed experimental evidence. Thus, we explore the $\psi(2S) \to p\bar{p}\eta$ and $\psi(2S) \to \pi^+\pi^-K^+K^-$ processes to predict the pure triangle singularity effect by a model independent calculation. From these studies, we find that it needs to choose a proper width of the intermediate particle around several MeV, which will be the best place to find the signal of the triangle singularity effect.

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1. Introduction

In 1959, L. D. Landau proposed the concept of triangle singularity [1], whose kinematics is given in Fig. 1. In this figure, particle A is a parent particle who decays into two particles 1 and 2 first, and the directions of these two particles are back to back when looking at the rest frame of particle A. Then particle 1 further decays into particle B and particle 3, while particle 2 continues to move in its direction. At last, particle 2 and 3 can interact with each other to form a final state particle C. Once the processes $A \to 1+2$, $1 \to B+3$ and $2+3 \to C$ all really happen, which means it not only requires that particles 1, 2 and 3 are all classical particles, but also particle 2 should catch up with particle 3, then the amplitude of this loop diagram will be infinite, and such effect is called as triangle singularity. To achieve such special phenomenon, it requires that (1) the momenta of all particles are on the same line, (2) all the intermediate particles are on the mass shell and (3) speed of particle 2 is larger than that of particle 3, which are the contents of Coleman-Norton theorem [2].

From Coleman-Norton theorem, the kinematic condition decides the triangle singularity happen or not. However, the strength of the triangle singularity is determined by the three vertices of the loop, which is dynamical. Thus, such effect from the triangle loop is theoretically model independent since all the vertices can be estimated from the corresponding experimental processes.

Before Ref. [3], the triangle singularity is just discussed on the theoretical side because it requires very special kinematic condition. In 2012, BESIII collaboration discovered the isospin breaking process $\eta(1405) \to \pi^0 f_0(980)$ [4]. Then Refs. [3, 5–8] successfully explain it by considering the triangle singularity produced in the $K\bar{K}K^*$ loop. After that, in 2015, Ref. [9] explained the nature of $a_1(1420)$ with the $K^*\bar{K}K$ loop, and then in 2020 [10], the COMPASS group reanalyses the experimental data of process $\pi p \to a_1(1260) \to f_0(980)\pi$ by including the triangle singularity contribution at the first time. As a result, they conclude that the $a_1(1420)$ peak can be generated purely from the triangle singularity of the $K^*\bar{K}K$ loop without a Breit-Wigner

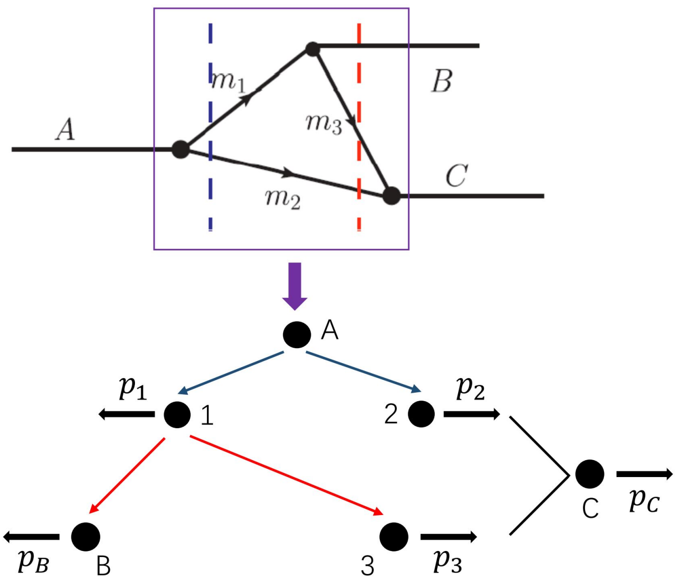

Figure 1: Kinematical mechanism of the production of a triangle singularity.

structure. From these two examples, we can see that the triangle singularity play an important role in understanding the peak structure in the invariant spectrum. Furthermore, it is conductive to study the properties of hadrons, such as $X(3872)$ [11]. Actually, there are various researches on this topic, such as [2, 3, 5–9, 11–40] (for a recent review, see Ref. [41]).

However, we should emphasise here that this pure triangle singularity phenomenon has not be fully confirmed by the experimental data. The experimental data of Refs. [4, 10] just show the possibility of the existence of the triangle singularity, but they still have other interpretations such as the existence of a new resonance. Thus, we need a very precise theoretical calculation to predict the effect of the triangle singularity in some processes, and then it will be confirmed by the experimental data, and by this way, the triangle singularity will be confirmed.

There are several typical reasons to make the precise prediction and detection very difficult. The threshold effect is the first reason to hind the discovery of triangle singularity. The threshold enhancement will be mixed with the triangle singularity, and it is very complicated to distinguish them [14]. Thus, a detectable pure triangle singularity must be far away from the threshold enhancement. Second, the key to make precise prediction is that the three vertices should be all known. In Refs. [2, 3, 5–9, 11–40], most of them can only give the line shape since not all the involved vertices can be determined. For example, because of the lack of the experimental data to constraint the $\Lambda_b \chi_{c1} \Lambda^*$ vertex, only the line shape can be calculated for the $\Lambda_b \to J/\psi pK$ process by the $\chi_{c1} p \Lambda^*$ loop mechanism [18], where the peak of $P_c(4450)$ can be interpreted as a triangle singularity effect. At last, the widths of internal particles should also be considered. Before this study, we thought that the narrow internal state should be the better choice since it can produce a sharp peak. However, too narrow peak is not proper for detection because it will lead to a high energy resolution and a huge event statistics since the small total decay width. For this point, we will discuss later.

In this paper, we make two proposals for a precise prediction of the triangle singularity theoretically. We strongly recommend the experiments, especially BESIII and STCF (in the future), to do precise analysis on the $\psi(2S) \to p\bar{p}\eta$ and $\psi(2S) \to \pi^+\pi^-K^+K^-$ decays.

This paper is organized as follows. After the introduction, a briefly formalism, numerical results and corresponding discussion of two processes is introduced in Sec. 2 and Sec. 3, respectively. Finally a summary is presented.

2. For $\psi(2S) \to p\bar{p}\eta$

2.1 The mechanism

In the $\psi(2S) \to p\bar{p}\eta$ reaction, we find one triangle loop mechanism as shown in Fig.6 (a). The triangle loop caused by $J/\psi p\eta$ can satisfy the condition of the triangle singularity. Furthermore, the three vertices $L_i (i = 1, 2, 3)$ are all well known. For example, L_1 and L_2 can be extracted from the decay widths of $\psi(2S) \to J/\psi\eta$ and $J/\psi \to p\bar{p}$, respectively. In addition, L_3 vertex is $p\eta \to p\eta$, which can be extracted from the experimental data of the reaction $\pi^- p \to \eta n$ and the branching ratios of $N^*(1535) \to \pi N/\eta N$ and $N^*(1650) \to \pi N/\eta N$, where we only make a reasonable assumption that resonances $N^*(1535)$ and $N^*(1650)$ are dominate. Thus, all the three vertices are fixed. Then, in this process, the position of the triangle singularity located on the invariant mass of $p\eta$ is around 1.563 GeV, which is higher than the threshold around 80 MeV. Thus, the threshold effect will be distinguished clearly. In summary, we can guarantee the accuracy of this triangle singularity effect at theoretical side.

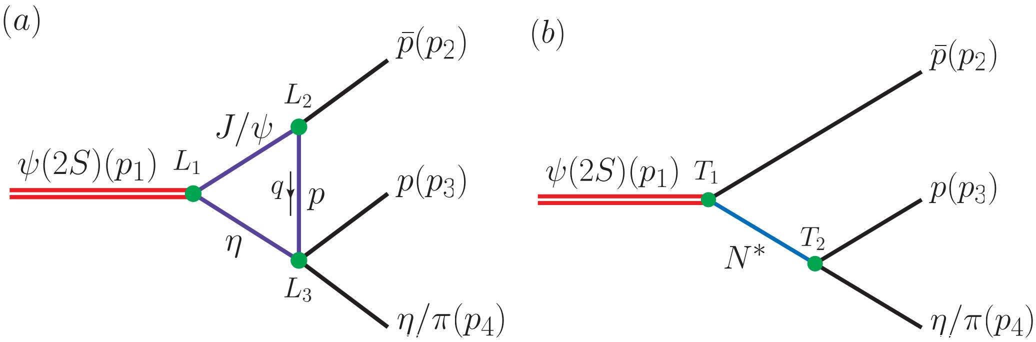

Figure 2: The Feynman diagrams describing the process $\psi(2S) \to p\bar{p}\eta/p\bar{p}\pi$. (a): loop diagram where triangle singularity happens; (b): tree diagram that called "background".

Furthermore, to estimate the background, we include the tree diagram as shown in Fig.6 (b), where N^* are the two closet states, $N^*(1535)$ and $N^*(1650)$. On the other hand, there is another tree diagram for $\psi(2S) \to J/\psi\eta \to p\bar{p}\eta$, however, the contribution of it can be fully removed by the cut "$m_{p\bar{p}} < 3.067$ GeV" where $m_{p\bar{p}}$ is the invariant mass of final proton and anti-proton following Ref. [42]. The detailed formulae for the amplitudes can be found in Ref. [43].

2.2 The numerical results and discussion

In Fig. 3, we show the pure triangle singularity contribution. To confirm that the effect of this triangle singularity is model independent, we use two values of form factor parameter α to perform the calculation, the corresponding results are shown as green solid and blue dashed lines here. It is obvious that the peak is independent on the value of α, and the reason is that the triangle singularity happens when the internal particles are all on the mass shell, thus, the form factor is close to 1 for any value of α. Furthermore, we also check the contributions of $N^*(1535)$ and $N^*(1650)$, and we can find that the main contribution is from $N^*(1535)$. Also, the position of the triangle singularity is just higher than the center mass of $N^*(1535)$ around 30 MeV, but 100 MeV away from $N^*(1650)$.

Then in Fig. 4, we show the full calculation including the tree diagram. Typically, the black solid and red dotted lines are for the results without and with $m_{p\bar{p}} < 3.067$ GeV cut. Then we can find a small cusp at the right shoulder of the peak of $N^*(1535)$, and it is just the signal of the predicted triangle singularity.

The small cusp has a very narrow width around 5 MeV and the enhancement is around 10%. It needs at least 6 billion events of $\psi(2S)$, which will generate around 50 events for this enhancement by including 30% detect efficiency. In addition, to show this cusp, it asks for the energy resolution around at least 2-3 MeV. However, the recent events collected by the BESIII collaboration is around 3 billion and the highest resolution is around 4 MeV [44], both of them can not achieve the requirements of the detection of our predicted signal. We expect the future experiment, i.e., Super Tau-Charm Facility (STCF), to find this signal.

In this study, we find the signal of triangle singularity is so small, and we try to understand it in detail. As a result, we find that the main reason is that the width of J/ψ is too small. To

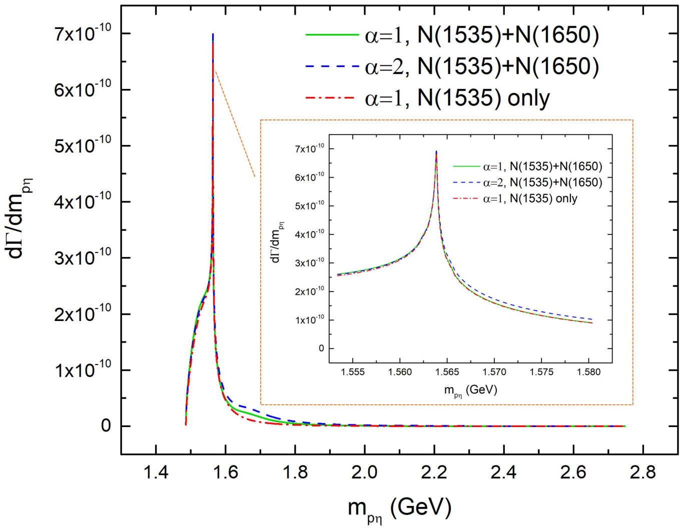

Figure 3: The $p\eta$ invariant mass spectrum of $\mathcal{M}^{\text{Loop}}$. The green solid line represents that α in the form factor is 1 and both $N(1535)$ and $N(1650)$ are included. The blue dashed line represents that α is 2 and both $N(1535)$ and $N(1650)$ are included. The red dash-dot line represents that α is 1 and only $N(1535)$ is considered.

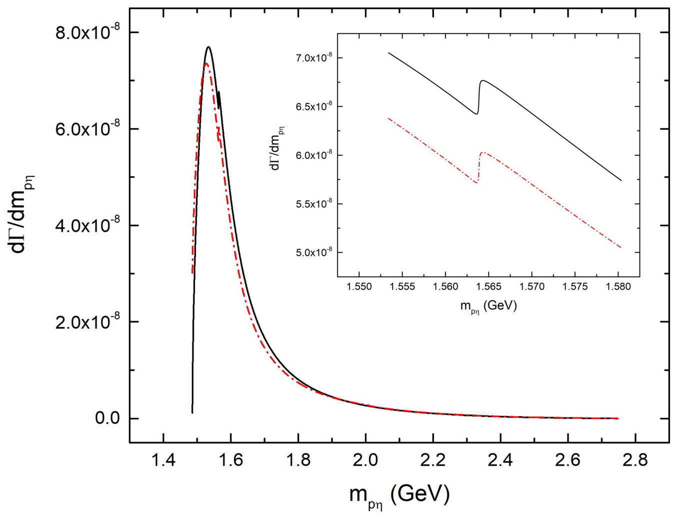

Figure 4: The $p\eta$ invariant mass distributions of the $\psi(2S) \to p\bar{p}\eta$ process. For black solid line, we consider the interface between $\mathcal{M}^{\text{Loop}}$ and $\mathcal{M}^{\text{Tree}}$ with the relative phase angle being 0, and here both $N(1535)$ and $N(1650)$ exchange are included and α is set to 1. For the red dashed-dotted line, there includes one more tree diagram $\psi(2S) \to \eta(J/\psi \to \bar{p}p)$ with the relative phase angle being 0 and having a $m_{p\bar{p}} < 3.067$ GeV cut following Ref. [42].

make this point clear, we just increase the total width of J/ψ from 92.9 keV to 929 keV but fix the branching ratio of J/ψ to $p\bar{p}$, then the peaks caused by the pure triangle singularity with these two different assumptions of the width of J/ψ is presented in Fig. 5. From Fig. 5 we can see that when we enlarge the width of J/ψ, the peak of the triangle singularity not only becomes wider, but also becomes higher. The detailed reason for broadening this peak is discussed in Ref. [8], while for the strength of triangle singularity is explained in Ref. [43]. Thus, we will note that it is not proper to find too narrow intermediate states to measure the triangle singularity effect. By our estimation, the best choice is that the width of particle 1 is around several MeV.

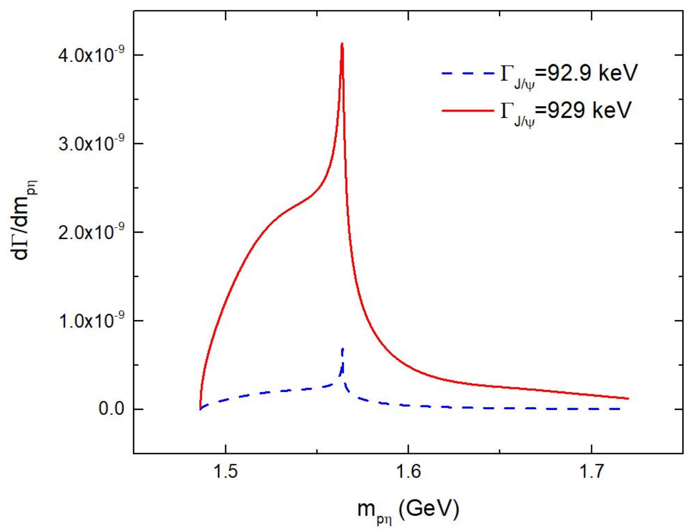

Figure 5: The peaks caused by the triangle singularity with different assumptions of the width of J/ψ.

3. For $\psi(2S) \to \pi^+\pi^-K^-K^+$

In this reaction, the triangle singularity mechanism and tree diagram are shown in Fig. 6 (a) and (b), respectively. The detailed formulae for the amplitudes can be find in Ref. [45]. The main idea to study this process is that the position of triangle singularity in K^+K^- invariant mass spectrum is moving since the broad width of ρ meson. Then we can get the pure triangle singularity in the Daliz plot of $\pi^+\pi^-$ and K^+K^- invariant masses as shown in the left plot of Fig. 7, while this signal will be suppressed after including the tree diagram as shown in the right side of Fig. 7 because of the narrow width of J/ψ as discussed before. In Fig. 7, we can find that the bright line standing for the triangle singularity is crossing the range of invariant mass of K^+K^- from 1.16 GeV to 1.19 GeV, and it means that the triangle singularity is moving, whose position is rely on the invariant mass of $\pi^+\pi^-$ ($= m_\rho$). To show this effect more clear, we can choose the center point and integration interval of the $\pi^+\pi^-$ invariant mass as m_ρ and $[m_\rho - \Gamma_\rho, m_\rho + \Gamma_\rho]$ properly, then the invariant mass spectra of $m_{K^+K^-}$ are shown in Fig. 8 with different m_ρ and a fixed $\Gamma_\rho = 16$ MeV. Clearly, the peak for the triangle singularity is moving, which shows a totally different behavior from a fixed resonance. Thus, if such movement of the peak can be observed, the triangle singularity will be confirmed. Again, as discuss before, such signal is really hard to detect when on the detector of the BESIII collaboration, but it is possible in the future STCF.

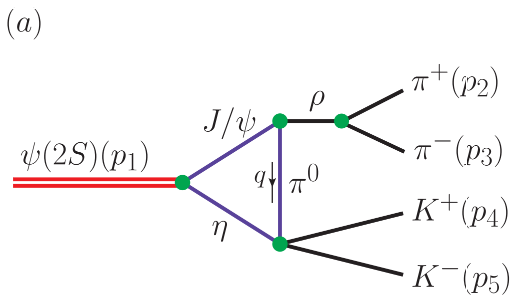

Figure 6: The Feynman diagrams describing the process $\psi(2S) \to \pi^+\pi^-K^+K^-$. (a): loop diagram where triangle singularity happens; (b): tree diagram called "background".

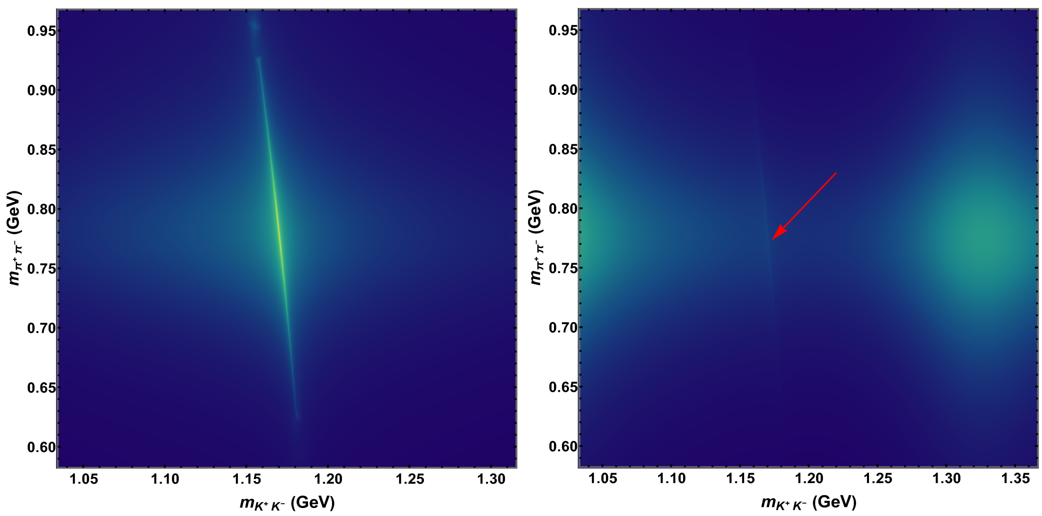

Figure 7: The left and right Dalitz plots of the $\psi(2S) \to \pi^+\pi^-K^+K^-$ process are calculated by including the contribution of Fig. 6 (a) only and both $(a)(b)$, respectively. The $m_{\pi^+\pi^-}$ and $m_{K^+K^-}$ are limited in [0.59 GeV, 0.96 GeV] and [1.04 GeV, 1.31 GeV] respectively, and the thin band in the middle is the contribution of the triangle singularity. In the right plots, the two bright spots located around 1 and 1.35 GeV correspond to the contributions of $a_0(980)$ and $a_2(1320)$ respectively. The subtle thin band pointed by the red arrow in the middle is the contribution of the triangle singularity.

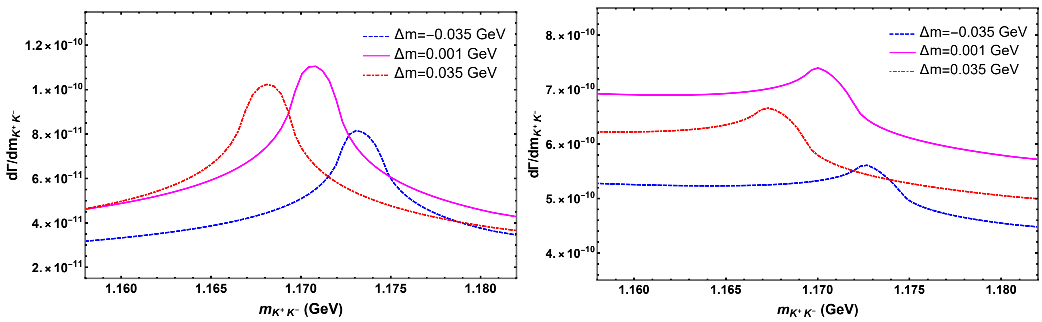

Figure 8: The K^+K^- invariant mass spectra of the $\psi(2S) \to \pi^+\pi^-K^+K^-$ process, where the Δm means that the integration on $m_{\pi^+\pi^-}$ is carried out within the interval $[m_\rho + \Delta m - \Delta\Gamma, m_\rho + \Delta m + \Delta\Gamma]$ with $\Delta\Gamma = 16$ MeV. In left and right sides, we include the contributions of the loop diagram only and in addition with the tree diagrams, respectively.

4. Summary and Outlook

In this paper, we first discuss the difficulties to do a precise calculation and measurement of the triangle singularity. There are mainly three reasons, the threshold effect, the unknown vertices and the too narrow or too large widths of the intermediate particles in the triangle loop. Then we construct two possible processes and make precise predictions at theoretical side. For the second process with four body final state, we show a special phenomena as the movement of the triangle singularity. Also, through this study, we investigate how the width of particle 1 as shown in Fig. 1 influence the strength of the signal. Although the predicted signals are too weak to detect now, the future STCF experiment may provide an opportunity to catch these signals.

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