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On the QCD contribution to vacuum energy

J. Gegelia[a,*]

[a] *Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany, High Energy Physics Institute, Tbilisi State University, 0186 Tbilisi, Georgia*

E-mail: jambul.gegelia@ruhr-uni-bochum.de

Within the framework of an effective field theory it is argued that for the cosmological constant term which is fixed by the condition of vanishing vacuum energy the graviton remains massless and there exists a self-consistent effective field theory of general relativity defined in a flat Minkowski background. Closely related issues of fine tuning of the strong interaction contribution to the vacuum energy and the compatibility of chiral symmetry with the consistency of the effective field theory of general relativity in Minkowski background are addressed.

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1. Introduction

At low energies the physics of the fundamental particles is described by an effective field theory (EFT), the Standard Model (SM) being its leading order (LO) approximation [1]. Gravitation can also be included in this framework by considering the most general effective Lagrangian of metric fields interacting with matter fields [2]. Within this approach the metric field is represented as the Minkowski background plus the graviton field. For a non-vanishing cosmological constant Λ, the free graviton propagator has a pole corresponding to a massive ghost mode [3]. As the cosmological constant term is not suppressed by any symmetry of the EFT, setting it to zero does not solve the problem, because the radiative corrections re-generate the massive ghost. On the other hand, representing Λ as a power series in $\hbar$, the coefficients can be adjusted such that the unphysical mass of the graviton is cancelled order-by-order in the loop expansion [4]. This uniquely fixes the value of the cosmological constant within an EFT in Minkowski background. Thus, to take into account Λ other than suggested by the approach of Ref. [4] it is necessary to consider an EFT in a curved metric background. To the best of our knowledge, such a systematic EFT is not available yet.

In this contribution we present the results of two-loop order calculations within EFT in Minkowski background which fix the cosmological constant from the condition that the vacuum energy (VE) has to be exactly zero [5, 6]. We compare the obtained value with the one dictated by the consistency condition of Ref. [4] with the result that at least at two-loop order they are identical. Next we discuss the implications of the condition imposed on the cosmological constant for the fine tuning problem of the quantum chromodynamics' (QCD) contribution to the VE.

The rest of this work is organized as follows: In section 2 we consider one- and two-loop contributions to the VE in an EFT of an Abelian model with spontaneously brocken gauge symmetry coupled to gravitational field. In section 3 we discuss the issue of the fine-tuning of the QCD contribution to VE and we summarize in section 4.

2. Vacuum energy in an EFT of matter and gravitational fields

The EFT of general relativity is described by the most general effective Lagrangian of gravitational and matter fields, invariant under general coordinate transformations and other underlying symmetries,

$$S = \int d^4x\sqrt{-g}\,\left\{\frac{2}{\kappa^2}(R-2\Lambda) + \mathcal{L}_{\rm gr,ho}(g) + \mathcal{L}_{\rm m}(g,\psi)\right\} = S_{\rm gr}(g) + S_{\rm m}(g,\psi),$$

where $\kappa^2 = 32\pi G$, with Newton's constant $G = 6.70881\cdot 10^{-39}$ GeV^{-2}, Λ is the cosmological constant, ψ and $g^{\mu\nu}$ represent the matter and metric fields, respectively, $g = \det g^{\mu\nu}$, R denotes the scalar curvature and $\mathcal{L}_{\rm m}(g,\psi)$ is the Lagrangian of the matter fields interacting with the metric field. It is understood that vielbein tetrad fields have to be introduced for an EFT with fermions. Coupling constants of interaction terms with higher orders of derivatives in $\mathcal{L}_{\rm gr,ho}(g)$ and $\mathcal{L}_{\rm m}(g,\psi)$ are chosen such that for energies accessible by current accelerators the contributions of these terms to physical quantities are heavily suppressed.

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It is most natural to impose on the EFT of general relativity in Minkowski background the condition that the energy of the vacuum has to be zero. This uniquely fixes the cosmological constant Λ as a function of other parameters of the effective Lagrangian. The question we aim to answer is if this value of Λ corresponds to a consistent perturbative EFT with no unphysical pole of the dressed graviton propagator. To address this issue we consider a specific model of a fermion, a scalar and a vector fields interacting with gravity. For Minkowski metric the LO approximation of our EFT coincides to a model with spontaneously broken Abelian gauge symmetry in unitary gauge. The action of the matter part of the considered EFT is given by

$$
\begin{aligned}
S_{\rm m} &= \int d^4x\sqrt{-g}\,\left\{\frac{1}{2}\,\bar\psi\, i e^\mu_a\gamma^a\nabla_\mu\psi - \frac{1}{2}\,\nabla_\mu\bar\psi\, i e^\mu_a\gamma^a\psi - m_F\bar\psi\psi \right.\\
&- \left.\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{M^2}{2}\,g^{\mu\nu}A_\mu A_\nu + \frac{g^{\mu\nu}}{2}\,\partial_\mu H\partial_\nu H - \frac{m^2}{2}\,H^2 + \mathcal{L}_{\rm MI} + \mathcal{L}_{\rm HO}\right\}, \qquad (1)
\end{aligned}
$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ, H and ψ are the vector, scalar and fermion fields, respectively, $\mathcal{L}_{\rm MI}$ denotes the renormalizable interactions between matter fields and $\mathcal{L}_{\rm HO}$ are interactions of higher orders. The covariant derivative acting on the fermion field has the form

$$
\begin{aligned}
\nabla_\mu\psi &= \partial_\mu\psi - \omega^{ab}_\mu\sigma_{ab}\psi,\\
\nabla_\mu\bar\psi &= \partial_\mu\bar\psi + \bar\psi\,\sigma_{ab}\,\omega^{ab}_\mu,
\end{aligned}
$$

where $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ and

$$
\begin{aligned}
\omega^{ab}_\mu &= -g^{\nu\lambda}e^a_\lambda\left(\partial_\mu e^b_\nu - e^b_\sigma\Gamma^\sigma_{\mu\nu}\right),\\
\Gamma^\lambda_{\alpha\beta} &= \frac{1}{2}\,g^{\lambda\sigma}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right)\,.
\end{aligned}
$$

The vielbein and metric fields satisfy the following relations:

$$
\begin{aligned}
e^a_\mu e^b_\nu\eta_{ab} = g_{\mu\nu}, &\quad e^\mu_a e^\nu_b\eta^{ab} = g^{\mu\nu},\\
e^a_\mu e^b_\nu g^{\mu\nu} = g^{ab}, &\quad e^\mu_a e^\nu_b g_{\mu\nu} = g_{ab}.
\end{aligned}
$$

The energy-momentum tensor (EMT) of the matter fields corresponding to the action of Eq. (1) has the form:

$$
\begin{aligned}
T^{\mu\nu}_m &= -g^{\mu\alpha}g^{\nu\rho}g^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + M^2\,g^{\mu\alpha}g^{\nu\beta}A_\alpha A_\beta + \partial_\mu H\partial_\nu H\\
&- g^{\mu\nu}\left\{-\frac{1}{4}g^{\alpha\rho}g^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + \frac{M^2}{2}\,g^{\alpha\beta}A_\alpha A_\beta + \frac{g^{\alpha\beta}}{2}\,\partial_\alpha H\partial_\beta H - \frac{m^2}{2}\,H^2\right\}\\
&+ \frac{i}{4}\left(\bar\psi\, e_{a\mu}\gamma^a\nabla_\nu\psi + \bar\psi\, e_{a\nu}\gamma^a\nabla_\mu\psi - \nabla_\mu\bar\psi\, e_{a\nu}\gamma^a\psi - \nabla_\nu\bar\psi\, e_{a\mu}\gamma^a\psi\right) + T^{\mu\nu}_{\rm HO} + T^{\mu\nu}_{\rm MI}\,,
\end{aligned}
$$

where $T^{\mu\nu}_{\rm MI}$ and $T^{\mu\nu}_{\rm HO}$ correspond to $\mathcal{L}_{\rm MI}$ and $\mathcal{L}_{\rm HO}$, respectively. For the EMT of the gravitational field we use the expression by Landau and Lifschits [7]:

$$
\begin{aligned}
T^{\mu\nu}_{\rm gr}(g) &= \frac{4}{\kappa^2}\,\Lambda\, g^{\mu\nu} + T^{\mu\nu}_{LL}(g)\,, \\
(-g)T^{\mu\nu}_{LL}(g) &= \frac{2}{\kappa^2}\Bigg(\frac{1}{8}\, g^{\lambda\sigma} g^{\mu\nu} g_{\alpha\gamma} g_{\beta\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda} - \frac{1}{4}\, g^{\mu\lambda} g^{\nu\sigma} g_{\alpha,\gamma} g_{\beta\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda} \\
&\quad - \frac{1}{4}\, g^{\lambda\sigma} g^{\mu\nu} g_{\beta\alpha} g_{\gamma\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda} + \frac{1}{2}\, g^{\mu\lambda} g^{\nu\sigma} g_{\beta\alpha} g_{\gamma\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda} \\
&\quad + g^{\beta\alpha} g_{\lambda\sigma}\, \mathfrak{g}^{\nu\sigma}{}_{,\alpha}\ \mathfrak{g}^{\mu\lambda}{}_{,\beta} + \frac{1}{2}\, g^{\mu\nu} g_{\lambda\sigma}\, \mathfrak{g}^{\lambda\beta}{}_{,\alpha}\ \mathfrak{g}^{\alpha\sigma}{}_{,\beta} - g^{\mu\lambda} g_{\sigma\beta}\, \mathfrak{g}^{\nu\beta}{}_{,\alpha}\ \mathfrak{g}^{\sigma\alpha}{}_{,\lambda} \\
&\quad - g^{\nu\lambda} g_{\sigma\beta}\, \mathfrak{g}^{\mu\beta}{}_{,\alpha}\ \mathfrak{g}^{\sigma\alpha}{}_{,\lambda} + \mathfrak{g}^{\lambda\sigma}{}_{,\sigma}\ \mathfrak{g}^{\mu\nu}{}_{,\lambda} - \mathfrak{g}^{\mu\lambda}{}_{,\lambda}\ \mathfrak{g}^{\nu\sigma}{}_{,\sigma}\Bigg)\,,
\end{aligned}
$$

with $\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$ and $\mathfrak{g}^{\mu\nu}{}_{,\lambda} = \partial \mathfrak{g}^{\mu\nu}/\partial x^\lambda$. The full EMT of matter and gravitational fields $T^{\mu\nu} = T^{\mu\nu}_{\rm m}(g,\psi) + T^{\mu\nu}_{\rm gr}(g)$ defines the conserved full four-momentum

$$
P^\mu = \int (-g)\, T^{\mu\nu} dS_\nu\,, \tag{2}
$$

where the integration extends over any hypersurface containing the whole three-dimensional space.

It is clear from Eq. (2) that the condition of vanishing VE is satisfied if the vacuum expectation value of EMT$\times(-g)$ is zero. This quantity can be expressed via a path integral as follows:

$$
\langle 0|(-g)T^{\mu\nu}|0\rangle = \int \mathcal{D}g\mathcal{D}\psi(-g)\,\left[T^{\mu\nu}_{\rm gr}(g) + T^{\mu\nu}_{\rm m}(g,\psi)\right] e^{i\int d^4x\sqrt{-g}\,[\mathcal{L}(g,\psi)+\mathcal{L}_{\rm GF}]}, \tag{3}
$$

where $\mathcal{L}_{\rm GF}$ is the gauge fixing term and the Faddeev-Popov determinant is included in the integration measure. By imposing the condition of vanishing VE we uniquely fix all coefficients Λ_i in the power series expansion of the cosmological constant in terms of $\hbar$:

$$
\Lambda = \sum_{i=0}^{\infty} \hbar^i \Lambda_i\,. \tag{4}
$$

Perturbative calculations of the vacuum expectation value of Eq. (3) are performed by splitting the metric and vielbein fields as sums of the Minkowskian background and the quantum fluctuations

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + \kappa h_{\mu\nu}, \\
g^{\mu\nu} &= \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu_\lambda h^{\lambda\nu} - \kappa^3 h^\mu_\lambda h^\lambda_\sigma h^{\sigma\nu} + \cdots\,, \\
e^a_\mu &= \delta^a_\mu + \frac{\kappa}{2}\, h^a_\mu - \frac{\kappa^2}{8}\, h_{\mu\rho} h^{a\rho} + \cdots\,, \\
e^\mu_a &= \delta^\mu_a - \frac{\kappa}{2}\, h^\mu_a + \frac{3\kappa^2}{8}\, h_{a\rho} h^{\mu\rho} + \ldots\,,
\end{aligned}
$$

and applying standard quantum field theoretical techniques to obtain the Feynman rules. An infinite number of diagrams contribute to the vacuum expectation value of EMT$\times(-g)$ already at tree order, however, all of them vanish if we take $\Lambda_0 = 0$. This also removes the tree order mass term from the graviton propagator, corresponding to a ghost degree of freedom.

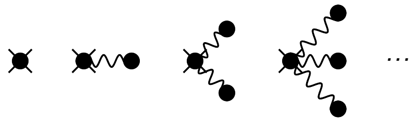

Figure 1: Tree order diagrams contributing to the vacuum expectation value of EMT$\times(-g)$. Filled circle corresponds to the cosmological constant term. The cross stands for EMT$\times(-g)$, wiggly lines represent graviton propagators.

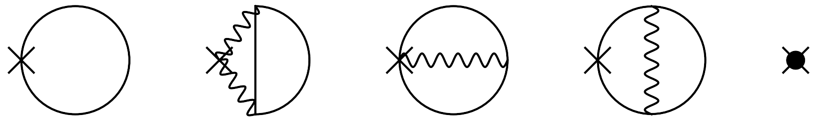

Figure 2: Diagrams contributing to the vacuum expectation value of EMT $\times$ $(-g)$. Filled circle represents the contribution of the cosmological constant term. The cross stands for EMT$\times(-g)$, wiggly and solid lines correspond to graviton and matter field propagators, respectively.

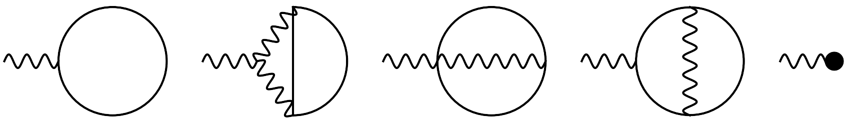

Figure 3: Diagrams contributing to the graviton tadpole. The filled circle represents the contribution of the cosmological constant term. Wiggly and solid lines correspond to the graviton and the matter field propagators, respectively.

To obtain one-loop contributions to the vacuum expectation value of EMT$\times(-g)$, we calculated the corresponding diagrams and by demanding that Λ_1 cancels this contribution obtained

$$\Lambda_1 = \frac{\kappa^2 \mu^{4-d} m_F^d \Gamma\left(1-\frac{d}{2}\right)}{2^d \pi^{\frac{d}{2}} d} - \frac{\kappa^2 \Gamma\left(1-\frac{d}{2}\right)\left(m^d + (d-1)M^d\right)}{2^{d+6}\pi^{\frac{d}{2}+4}\, d},$$

with μ the scale of the dimensional regularization and d the dimension of the spacetime. As a consequence of the definition of EMT of the matter fields the same Λ_1 cancels the one-loop contribution to the vacuum expectation value of $h_{\mu\nu}$, and consequently, also the graviton self-energy at zero momentum, i.e. graviton mass, as implied by corresponding Ward identity [4].

Next we calculated two-loop diagrams contributing to the vacuum expectation values of EMT$\times(-g)$ and $h_{\mu\nu}$. As a result of very non-trivial cancellations at two-loop order we found that the same value

$$\Lambda_2 = -\frac{d^3 \kappa^4 \mu^{8-2d} m_F^{2d-2} \csc\left(\frac{\pi d}{2}\right)\Gamma\left(-\frac{d}{2}\right)}{2^{2d+7}\pi^{d-1}(d-2)\Gamma\left(\frac{d}{2}\right)} - \frac{d(d+1)\kappa^4 M^{2d-2}\csc\left(\frac{\pi d}{2}\right)\Gamma\left(1-\frac{d}{2}\right)}{2^{2(d+3)}\pi^{d-1}\Gamma\left(\frac{d}{2}\right)}$$

cancels both quantities. To check the reliability of the obtained results we also calculated the two-loop contributions to the graviton self-energy and verified that the same value of Λ_2 ensures that the graviton remains massless in agreement with the Ward identity.

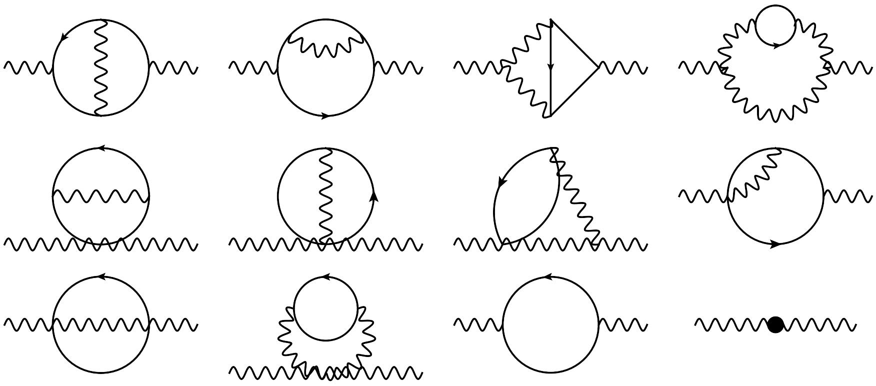

Figure 4: Diagrams contributing to the graviton self-energy. The filled circle corresponds to the contribution of the cosmological constant term. Wiggly and solid lines represent the graviton and the matter field propagators, respectively.

3. QCD contribution to the vacuum energy

The QCD contribution to VE which occurs due to the explicit breaking of chiral symmetry can be calculated with great accuracy. Consider two-flavour QCD Lagrangian of massless up and down quarks with external scalar and pseudoscalar sources $s(x)$ and $p(x)$:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\,\bar{\psi}\gamma_\alpha D^\alpha \psi - \bar{\psi}(s - i\,\gamma_5 p)\psi\,,$$

where $\psi = (\psi_u, \psi_d)^T$ is a doublet of up and down quark fields. This Lagrangian is invariant under $SU(2)_L \times SU(2)_R$ chiral symmetry transformations

$$\frac{1}{2}(1-\gamma_5)\psi \to L\,\frac{1}{2}(1-\gamma_5)\psi\,, \quad \frac{1}{2}(1+\gamma_5)\psi \to R\,\frac{1}{2}(1+\gamma_5)\psi\,, \quad (s+i\,p) \to L\,(s+i\,p)R^\dagger\,,$$

with L and R elements of $SU(2)_L$ and $SU(2)_R$, respectively. Chiral symmetry of massless QCD is spontaneously breaken with pions appearing as Goldstone bosons. The low-energy effective Lagrangian of pions at lowest order has the form

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{F_\pi^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\,, \tag{5}$$

where the field U is given in terms of the pion fields π^a ($a = 1,2,3$) as $U = \exp\left(\frac{i\,\tau^a \pi^a}{F_\pi}\right)$, and $\chi = 2B_0(s+ip)$. Here B_0 is related to the vacuum expectation value of the quark condensate and F_π is the pion decay constant. This Lagrangian is invariant under chiral transformations

$$U \to LUR^\dagger\,, \quad (s+i\,p) \to L(s+i\,p)R^\dagger.$$

The effective field theory corresponding to two-flavour QCD without external sources is obtained by substituting

$$s = \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix}, \quad p = 0\,.$$

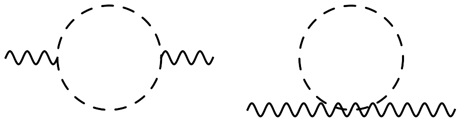

Figure 5: One-loop diagrams with pions contributing to the self-energy of the graviton. Wiggly and dashed lines represent gravitons and pions, respectively.

As the quark masses explicitly break the chiral symmetry, the pions obtain a small mass to leading order in the chiral expansion

$$M_\pi^2 = B_0(m_u + m_d) + O(m_q^2)\,,$$

where m_q stands for the light quark masses.

The effective Lagrangian of Eq. (5) generates a tree-order contribution to the vacuum energy

$$\Lambda_m = -\langle 0|\mathcal{L}_2|0\rangle = -F_\pi^2 B_0(m_u + m_d) = -F_\pi^2 M_\pi^2\,.$$

There is no other term in the effective Lagrangian which could compensate this contribution, e.g. $\mathrm{Tr}(\chi + \chi^\dagger)$ would contribute to VE but it violates the chiral symmetry. To cancel the large QCD contribution to VE it seems unavoidable that one needs to adjust numerically Λ in EFT of pions interacting with gravitation where it is one of the parameters [8]. The numerical value of the QCD contribution to the vacuum energy reads

$$\Lambda_m = 1.5\times 10^8\ \mathrm{MeV}^4 = 0.63\times 10^{43}\Lambda_{\rm exp},$$

where $\Lambda_{\rm exp} = 2.4\times 10^{-47}\ \mathrm{GeV}^4$ is the observed value.

It is observed in Ref. [8]: "Because of the large multiplier, if one holds all the other parameters of the Standard Model fixed, a change of the up quark mass in its forty-first digit would produce a change in Λ outside the anthropically allowed range. ... because the calculation is so well controlled, it illustrates the degree of fine-tuning required as well as the futility of thinking that some feature of the Standard Model could lead to a vanishing contribution to Λ." As shown below, it turns out that this problem has a solution not involving any numerical fine tuning.

It is straightforward to construct an EFT action of pions including the interaction with the gravitational field:

$$S_{\rm gr}(g) + \int d^4x\sqrt{-g}\left[-F_\pi^2 M_\pi^2 + \frac{1}{2}\, g^{\mu\nu}\partial_\mu\pi^a\partial_\nu\pi^a - \frac{M_\pi^2}{2}\,\pi^a\pi^a + O(\pi^4)\right]. \tag{6}$$

To cancel the tree order contribution of pions to the VE we need to take

$$\Lambda_0 = F_\pi^2 M_\pi^2$$

in Eq. (4). This value of Λ_0 exactly cancels also the graviton mass at tree order. At one-loop order there are two diagrams, contributing to the graviton self-energy, generated by the effective Lagrangian of Eq. (6), as shown in Fig. 5.

By demanding that the order $\hbar$ term exactly cancels the contribution of these two one-loop diagrams for $p^2 = 0$ we obtain

$$\Lambda_1 = \frac{3\kappa^2\left(2M_\pi^2\, A_0\left(M_\pi^2\right) + M_\pi^4\right)}{512\pi^2}\,,\; A_0(M^2) = \frac{-i\mu^{4-d}}{\pi^2(2\pi)^{d-4}}\int \frac{d^d k}{k^2 - M^2 + i0^+}.$$

Thus, Λ has to be a fixed function of the light quark masses (pion masses) for any values of the parameters of the effective Lagrangian. This condition invalidates arguments supporting the need of the numerical fine-tuning. Notice, however, that chiral invariance of the effective Lagrangian of pions does not allow a quark mass dependent cosmological constant term. Thus, the consistency condition of the EFT of general relativity in Minkowski background is not compatible with the chiral symmetry of QCD with external sources.

The solution to this problem is that the chiral symmetry of the QCD Lagrangian with external sources is not an exact symmetry of the full theory when the gravity is also taken into account. In EFT of general relativity without ghost in Minkowski background the cosmological constant term is a fixed function of other parameters, i.e. $\Lambda \equiv \Lambda(m_u, m_d, g, e, \ldots)$. The effective Lagrangian $\mathcal{L}_\mathrm{m}(g,\psi)$ at LO coincides with the Lagrangian of the SM taken in a non-flat metric field. To obtain the LO Lagrangian of the strong interaction alone we drop "non-renormalizable" terms and “switch off” EW interactions. To “switch off" gravity we approximate the metric field $g^{\mu\nu}$ by the constant Minkowski metric. The resulting Lagrangian for two flavors of quarks has the form:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar\psi\left(i\,\gamma_\alpha D^\alpha - \mathcal{M}\right)\psi + L_0(m_u, m_d, g)\,,$$

where $L_0(m_u, m_d, g) = -4\Lambda(m_u, m_d, g, 0, 0, \cdots)/\kappa^2$.

This term does not contradict to any *physical symmetries* with observable consequences, however, it does not contribute to physical quantities if gravity is not taken into account. The above Lagrangian gives the following contribution to the VE

$$\Lambda_m = \langle 0|m_u\bar\psi_u\psi_u + m_d\bar\psi_d\psi_d|0\rangle - \langle 0|L_0(m_u, m_d, g)|0\rangle = -F_\pi^2 M_\pi^2 - L_0(m_u, m_d, g).$$

By taking $L_0(m_u, m_d, g) = -F_\pi^2 M_\pi^2 + O(M_\pi^4) = -F_\pi^2 B_0(m_u + m_d) + O(m_q^2)$ we obtain for the contribution to the VE $\Lambda_m = 0 + O(m_q^2)$. Further, by adjusting the higher order terms in light quark masses m_q in $L_0(m_u, m_d, g)$ we can achieve that the QCD contribution to the vacuum energy Λ_m exactly vanishes for any values of m_q.

Notice that the L_0 term of the Lagrangian does not affect the construction of the low-energy EFT of the strong interaction. One starts with the following underlying Lagrangian with external sources

$$\mathcal{L}_\mathrm{ext} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\,\bar\psi\gamma_\alpha D^\alpha\psi + \bar\psi\gamma^\mu(v_\mu + a_\mu\gamma_5)\psi - \bar\psi(s - i\,\gamma_5 p)\psi + L_0(m_u, m_d, g), \quad (7)$$

where $v_\mu(x)$, $a_\mu(x)$, $s(x)$ and $p(x)$ are Hermitian, colour neutral matrices in flavour space and $s(x) = \mathcal{M} + \cdots$ incorporates the quark mass term, and proceeds in exact analogy to Ref. [9]. Generating functional of Green’s functions of scalar, pseudo-scalar, vector and axial vector currents has the form:

$$\langle 0_\mathrm{out}|0_\mathrm{in}\rangle_{v,a,s,p} = e^{iZ[v,a,s,p]} = \frac{\int \mathcal{D}A\mathcal{D}q\; e^{i\int d^4x\,\mathcal{L}_\mathrm{ext}(x)}}{\int \mathcal{D}A\mathcal{D}q\; e^{i\int d^4x\,\mathcal{L}(x)}}.$$

It does not depend on L_0 and therefore the construction of the low-energy EFT, namely chiral perturbation theory, proceeds exactly the same way as for the Lagrangian of Eq. (7) without the L_0 term. The Lagrangian of Eq. (7) is invariant under local chiral transformations

$$\psi(x) \to \left[\frac{1}{2}(1+\gamma_5)R(x) + \frac{1}{2}(1-\gamma_5)L(x)\right]\psi(x)\,,$$

provided that the external sources are transformed the following way:

$$\begin{aligned} v'_\mu + a'_\mu &= R(v_\mu + a_\mu)R^\dagger + iR\partial_\mu R^\dagger, \\ v'_\mu - a'_\mu &= L(v_\mu - a_\mu)L^\dagger + iL\partial_\mu L^\dagger, \\ s' + ip' &= R(s+ip)L. \end{aligned}$$

Thus none of the physically relevant symmetries of QCD are affected by adding to the QCD Lagrangian of the term $L_0(m_u, m_d, g)$, which is nothing else then (a contribution to) the cosmological constant.

4. Summary

If is natural to assume that if there is any physical reason for choosing a fixed value of the cosmological constant Λ then it must be the condition of vanishing of the VE. In the framework of an EFT of an Abelian model with spontaneously broken gauge symmetry, coupled to gravity, we have performed a two-loop order calculation of the VE. We obtained that the value of Λ canceling the contributions to VE also eliminates the vacuum expectation value of the graviton field and the massive ghost, thus leading to a consistent EFT of general relativity in Minkowski background. Performing a similar analysis for QCD we found that the issue of numerical fine-tuning of the QCD contribution to VE is solved through the condition that $\Lambda(m_q, g)$ as a function of QCD parameters exactly cancels the QCD contribution to VE for any values of these parameters. While this solution seems to be incompatible with the chiral symmetry of QCD with external sources, we observe that there is no contradiction with any symmetries of QCD with observable physical consequences.

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