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Explicit renormalization of the nucleon-nucleon interaction in chiral EFT and non-perturbative effects

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Nucleon-nucleon interaction is studied within chiral effective field theory with a finite cutoff at next-to-leading order in the chiral expansion. The leading order interaction is resummed non-perturbatively, whereas the next-to-leading-order terms are taken into account in a perturbative manner. Explicit renormalizability of such a scheme is proven in certain important cases. In particular, it is verified whether the power-counting breaking terms originating from the integration regions characterized by large momenta can be absorbed by the renormalization of the low energy constants. The importance of non-perturbative effects is analyzed in detail.

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1. Introduction

Renormalization and power counting are the basic ingredients of any effective quantum field theory (EFT). In the nucleon-nucleon (NN) and the few-nucleon sector, chiral EFT, originally introduced by Weinberg [1, 2], has become a powerful tool to systematically study the dynamics of these systems. The non-perturbative nature of the NN interaction makes renormalization rather complicated, see, e.g. discussions in Refs. [3–6]. Weinberg power counting implies the resummation of the leading order (LO) potential (two-nucleon irreducible) diagrams, which are divergent. The divergence increases with the number of iterations of the LO potential.

In order to avoid introducing an infinite number of counter terms that would absorb those divergent contributions, one can introduce a regulator in the form of a finite (of the order of the hard scale Λ_b) cutoff Λ. Such an approach has phenomenological success. Calculation within this scheme have been extended to high orders in the chiral expansion and achieved high accuracy, see, e.g. Refs. [7–9].

A formal justification of such an approach has been addressed only recently in Ref. [10], where the iterations of the LO potential were assumed to be perturbative. In particular, it has been shown that (extended when necessary) Weinberg power counting holds for the LO nucleon-nucleon amplitude. For the next-to-leading (NLO) amplitude, the power counting can be restored by the renormalization of the LO contact interactions to all orders in the LO potential.

In this talk, we report on a generalization of the analysis of Ref. [10] and discuss the complications originating from the inclusion of the non-perturbative effects.

2. Effective Lagrangian and power counting

We start with the chiral effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots, \tag{1}$$

expressed in terms of the nucleon and pion fields. The superscripts denote the number of derivatives or the power of the pion mass M_π. The scattering amplitude obtained from this Lagrangian is expanded in terms of the small quantity Q, which is the ratio of the soft and the hard scale $Q = \frac{q}{\Lambda_b}$. The soft scale is given by external on-shell nucleon momenta p_{on}, and M_π and the hard scale Λ_b can be associated with the ρ-meson mass. The chiral order of potential (2N-irreducible) contributions to the amplitude in Weinberg's approach is given by the power counting formula

$$D = 2L + \sum_i \left(d_i + \frac{n_i}{2} - 2 \right), \tag{2}$$

where L is the number of loops, the sum runs over all vertices of the diagram, n_i is the number of nucleon lines in vertex i and d_i is the number of derivatives and the pion-mass insertions at vertex i.

Equation (2) implies that that the LO potential contains the derivativeless contact interaction and the one-pion exchange contribution, whereas the NLO potential contains two-pion-exchange contributions and subleading contact terms. In practical calculations, one can include more terms into the LO potential if they appear to be numerically large.

The 2N-reducible graphs are enhanced compared to the potential graphs due to the infrared singularity of the nucleon propagators. This makes it necessary to take into account all iterations of the LO potential V_0 at the same chiral order.

The LO ($O(Q^0)$) and NLO ($O(Q^2)$) amplitudes are given by

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]} = V_0 R = \bar{R} V_0, \tag{3}$$

$$T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \cdots = \sum_{m,n=0}^{\infty} T_2^{[m,n]} = \bar{R} V_2 R, \tag{4}$$

with the resolvents

$$R = \frac{1}{\mathbb{1} - G V_0}, \qquad \bar{R} = \frac{1}{\mathbb{1} - V_0 G}. \tag{5}$$

3. Perturbative case

In this section, we consider the situation when the series in the LO potential V_0 is convergent for both LO and NLO amplitudes, but the rate of such a convergence is slower than that of the chiral expansion and one still needs to iterate the LO potential.

The integrals in Eqs. (3), (4) converge at momenta of the order of the cutoff Λ (we will discuss here the spin-triplet channels or the spin-singlet channels with both short- and long-range LO potentials as having more singular ultraviolet behaviour). For the LO amplitude T_0, this does no lead to a power counting violation since each power of Λ is compensated by the inverse hard scale originating from the LO potential $1/\Lambda_V$. We demonstrated this explicitly in Ref. [10].

On the other hand, the unrenormalized NLO amplitude from Eq. (4) violates the power counting of Eq. (2) and contains terms of order $O(Q^0)$ in the channels that couple to S-waves. In Ref. [10], we showed that such power-counting breaking contributions can be absorbed by a renormalization of the LO contact interactions as one would expect from the principles of quantum field theory.

In order to renormalize the NLO amplitude, we perform subtractions at zero external momentum in the spirit of the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) renormalization procedure [11–13]. We introduce the subtraction operation $\mathbb{T}$ that replaces an operator X with matrix elements $X_{l'l}(p', p; p_{\rm on})$ with its value at $p = p' = p_{\rm on} = 0$:

$$\mathbb{T}(X) = X_{00}(0,0,0) V_{\rm ct}\,, \tag{6}$$

where $V_{\rm ct}$ is the contact operator

$$V_{\rm ct} = |\chi\rangle\langle\chi|, \qquad \langle p, lsj|\chi\rangle = \delta_{l,0}. \tag{7}$$

The subscripts of X refer to the orbital angular momentum in the lsj basis, p and p' are the initial and final off-shell momenta and $p_{\rm on}$ is the on-shell momentum. Analogously, we define the subtraction operation $\mathbb{T}^{m_i,n_i}$ for a subdiagram of $T_2^{[m,n]}$.

The BPHZ $\mathbb{R}$-operation that renormalizes $T_2^{[m,n]}$ is given by

$$\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}, \tag{8}$$

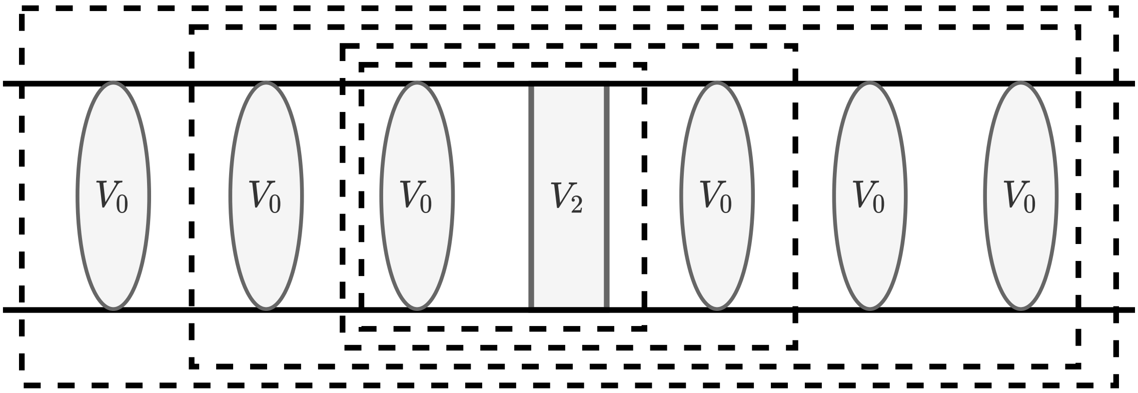

Figure 1: An example of a forest for the amplitude $T_2^{[3,3]}$.

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where $\mathcal{F}^{m,n}$ represents the set of all forests, i.e, the set of all possible distinct sequences of nested subdiagrams (m_i, n_i):

$$U_k = ((m_{k;1}, n_{k;1}), (m_{k;2}, n_{k;2}), \dots),$$
$$m \geq m_{k;i+1} \geq m_{k;i} \geq 0, \; n \geq n_{k;i+1} \geq n_{k;i} \geq 0. \tag{9}$$

An example of a forest that contributes to Eq. (8) for $m = n = 3$ is shown in Fig. 1. One can show that the on-shell renormalized amplitude $\mathbb{R}(T_2^{[m,n]})$ satisfies the following inequality:

$$\left|\mathbb{R}(T_2^{[m,n]})(p_{\rm on})\right| \leq \frac{8\pi^2 \mathcal{M}_{T_2}}{m_N \Lambda_V} \left(\tilde{\mathcal{M}}_{T_2} \frac{\Lambda}{\Lambda_V}\right)^{m+n} \frac{p_{\rm on}^2}{\Lambda_b^2} \log \frac{\Lambda}{M_\pi}, \tag{10}$$

where $\mathcal{M}_{T_2}$ and $\tilde{\mathcal{M}}_{T_2}$ are constants of order one, and $\frac{8\pi^2}{m_N \Lambda_V}$ is a normalization factor. Therefore, $\mathbb{R}(T_2^{[m,n]})$ is of order $O(Q^2)$ (strictly speaking $o(Q)$ due to logarithmic corrections) in accordance with the original power counting.

4. Non-perturbative effects

In this section, we extend the renormalization to the non-perturbative case when the series in Eqs. (3) and (4) do not converge. In this situation, we apply the Fredholm method. The resolvent R of the partial-wave Lippmann-Schwinger equation can be represented as [14]

$$R = \mathbb{1} + \frac{Y}{D}, \qquad \bar{R} = \mathbb{1} + \frac{\bar{Y}}{D}, \tag{11}$$

where the Fredholm determinant is a function of only the on-shell momentum: $D = D(p_{\rm on})$. Analogous representations for T_0 and T_2 are given by

$$T_{0;l'l}(p', p; p_{\rm on}) = \frac{N_{0;l'l}(p', p; p_{\rm on})}{D(p_{\rm on})}, \qquad T_{2;l'l}(p', p; p_{\rm on}) = \frac{N_{2;l'l}(p', p; p_{\rm on})}{D(p_{\rm on})^2}. \tag{12}$$

Although the whole expressions in Eqs. (11) and (12) are non-perturbative, each individual quantity converges as a series in V_0:

$$Y = \sum_{n=1}^{\infty} Y_n, \qquad \bar{Y} = \sum_{n=1}^{\infty} \bar{Y}_n, \qquad D = \sum_{n=0}^{\infty} D_n, \qquad N_0 = \sum_{n=1}^{\infty} (N_0)_n, \qquad N_2 = \sum_{n=0}^{\infty} (N_2)_n, \tag{13}$$

which allows us to analyze the non-perturbative case similarly to the perturbative one.

The quantities N_0 and D can be shown to be bounded as

$$|N_0| \leq \frac{8\pi^2 \mathcal{M}_{N_0}}{m_N \Lambda_V}, \qquad |D| \leq \mathcal{M}_D, \tag{14}$$

where $\mathcal{M}_{N_0}$ and $\mathcal{M}_D$ are constants of order one. On the other hand, we assume that $|D|$ is also bounded from below as

$$|D| \geq \tilde{\mathcal{M}}_D \left(\frac{p_{\rm on}}{M_\pi} + \kappa \right), \tag{15}$$

where κ can be small compared to one, which enables us to consider also the situation when the amplitude is enhanced close to threshold due to the presence of a (quasi) bound state. In particular, the nucleon-nucleon scattering amplitude in the channels ${}^1S_0$ and ${}^3S_1 - {}^3D_1$ can be analyzed under this assumption.

In order to renormalize the amplitude T_2 in the non-perturbative case, it is convenient to introduce the quantities ψ and $\bar{\psi}$ that correspond to the scattering wave functions at the origin in coordinate space:

$$\begin{aligned} &|\bar{\psi}\rangle = \bar{R}(p_{\rm on})|\chi\rangle, \qquad \langle\psi| = \langle\chi|R(p_{\rm on}), \\ &\bar{\psi}_l(p) = \psi_l(p) = \langle\psi|p, lsj\rangle = \langle p, lsj|\bar{\psi}\rangle. \end{aligned} \tag{16}$$

or more explicitly:

$$\psi_l(p_{\rm on}) = \delta_{l,0} + \int \frac{p^2 dp}{(2\pi)^3} G(p; p_{\rm on}) T_{0;l,0}(p, p_{\rm on}; p_{\rm on}). \tag{17}$$

The renormalization of the NLO amplitude in P-waves and higher works automatically, i.e. T_2 does not violate power counting. For the S-waves, Eq. (8) can be resummed explicitly, and we obtain for the on-shell amplitude:

$$\mathbb{R}(T_{2;l',l})(p_{\rm on}) = T_{2;l',l}(p_{\rm on}) - T_{2;0,0}(0) \frac{\psi_{l'}(p_{\rm on})\psi_l(p_{\rm on})}{\psi_0(0)^2}\,. \tag{18}$$

In Eq. (18), we choose the renormalization condition

$$\mathbb{R}(T_2)(0) = 0\,. \tag{19}$$

Introducing the quantity ν,

$$\psi_l(p_{\rm on}) = \frac{\nu_l(p_{\rm on})}{D(p_{\rm on})}, \tag{20}$$

we can rewrite Eq. (18) as follows:

$$\mathbb{R}(T_{2;l'l})(p_{\rm on}) = \frac{1}{D(p_{\rm on})^2}\left(N_{2;l'l}(p_{\rm on}) - N_{2;0,0}(0)\frac{\nu_{l'}(p_{\rm on})\nu_l(p_{\rm on})}{\nu_0(0)^2}\right) =: \frac{\mathbb{R}(N_{2;l'l})(p_{\rm on})}{D(p_{\rm on})^2}. \tag{21}$$

One can prove that $\mathbb{R}(T_{2;l'l})(p_{\rm on})$ satisfies the power counting of Eq. (2) and is of order $O(Q^2)$ if the renormalized quantity $\mathbb{R}(N_2)$ can be represented as convergent series in V_0,

$$\mathbb{R}(N_2) = \sum_{m,n=0}^{\infty} [\mathbb{R}(N_2)(p_{\rm on})]_{m,n} < \infty, \tag{22}$$

$$[\mathbb{R}(N_2)]_{m,n} = \sum_{m_1=0}^{m}\sum_{n_1=0}^{n} D_{m_1} D_{n_1} [\mathbb{R}(T_2)]_{m-m_1,n-n_1}, \tag{23}$$

and the convergence rate is sufficiently high. The function $\nu(p_{\rm on})$ is a convergent series in V_0. However, it appears in the denominator in the definition of $\mathbb{R}(N_2)$. Therefore, a sufficient condition for the series in Eq. (22) to converge is the absence of zeros of the function $\nu(\lambda, p_{\rm on})$ of a complex variable λ inside the circle $|\lambda| \le 1$

$$\nu(\lambda, p_{\rm on}) \neq 0, \qquad |\lambda| \le 1. \tag{24}$$

The function $\nu(\lambda, p_{\rm on})$ is obtained from $\nu(p_{\rm on})$ by rescaling $V_0 \to \lambda V_0$.

Condition (22) (or (24)) is the constraint on the short range part of the LO potential. If it holds, the renormalization condition in Eq. (19) leads to the following bound on $\mathbb{R}(N_2)$ (assuming $\Lambda \sim \Lambda_V$):

$$|\mathbb{R}(N_2)(p_{\rm on})| \le \frac{8\pi^2 \mathcal{M}_{N_2}}{m_N \Lambda_V}\left(\frac{p_{\rm on}^2}{\Lambda_b^2} + \kappa^2\right)\Phi_{\log}, \tag{25}$$

where $\Phi_{\log}$ is a factor that can contain, e.g., $\log\frac{M_\pi}{\Lambda}$ terms that do not modify the power counting. Equation (25) together with Eq. (15) yields

$$|\mathbb{R}(T_2)(p_{\rm on})| \le \frac{8\pi^2 \mathcal{M}_{T_2;\rm NP}}{m_N \Lambda_V}\frac{M_\pi^2}{\Lambda_b^2}\Phi_{\log}, \tag{26}$$

where $\mathcal{M}_{N_2}$ and $\mathcal{M}_{T_2;\rm NP}$ are constants of order one. The bound in Eq. (26) satisfies the desired power counting.

Note that in some cases, the condition in Eq. (24) is obeyed automatically. For example, if the LO potential V_0 in the ${}^1S_0$ channel is fully local, $\psi(p_{\rm on})$ coincides with the inverse of the Jost function $f(p_{\rm on})$ and the inverse of the Fredholm determinant [14],

$$\psi(p_{\rm on}) = f(p_{\rm on})^{-1} = D(p_{\rm on})^{-1}, \tag{27}$$

and therefore,

$$\nu_0(p_{\rm on}) = \nu_0(0) = 1. \tag{28}$$

5. Cutoff dependence

The formalism developed in Ref. [10] allows one to treat a regulator of the LO potential explicitly as a higher order effect. The difference between the regulated and the unregulated LO potential is considered as a perturbation of order $O(Q^2)$

$$\delta_\Lambda V_0 = V_0^{\Lambda=\infty} - V_0^\Lambda = O(Q^2). \tag{29}$$

The corresponding unrenormalized NLO amplitude is of order:

$$\delta T_2^\Lambda = (1 + T_0 G)\delta V_0^\Lambda (1 + G T_0) \sim O(Q^0). \tag{30}$$

After renormalization, the power counting is expected to be restored

$$\mathbb{R}\left(\delta T_2^\Lambda\right) \sim O(Q^2). \tag{31}$$

However, as was discussed in the previous section, this is not guaranteed in general. Additional constraints on the LO potential, see Eqs. (22) and (24), must be imposed.

6. Summary

We have studied the nucleon-nucleon interaction within Chiral effective field theory with a finite cutoff. The renormalizability of the considered scheme and the power-counting restoration at next-to-leading order is achieved by applying the BPHZ subtraction scheme if the LO potential can be treated perturbatively.

In the non-perturbative case, i.e. when the series in the LO potential does not converge, additional constraints have to be imposed on the short range part of the LO interaction. This has consequences also for the cutoff dependence of the scattering amplitude.

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