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$A = 4-7$ Ξ hypernuclei based on interactions from chiral effective field theory

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In this contribution, we report on an investigation of the possible existence of bound Ξ states in systems with $A = 4-7$ baryons using the Jacobi NCSM approach in combination with chiral NN and ΞN interactions. Three shallow bound states for the NNNΞ system (with $(J^\pi, T) = (1^+, 0)$, $(0^+, 1)$ and $(1^+, 1)$) with quite similar binding energies are found. The ${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+, \frac{1}{2})$ and ${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+, \frac{3}{2})$ hypernuclei are also clearly bound with respect to the thresholds ^{4}He$+\Xi$ and ^{6}He$+\Xi$, respectively. It is also found that the binding of these systems is predominantly due the attraction of the chiral ΞN potential particularly in the ${}^{33}S_1$ channel. Furthermore, a perturbative estimate suggests that the decay widths of all the bound states could be small enough for them to be observed in experiment.

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Introduction

Experimental information on hyperon-hyperon (YY) and Ξ N scattering is practically absent because of the extremely short life time of hyperons and low-density beam fluxes. Study of double strangeness systems from both theoretical and experimental sides is therefore indispensable in order to understand baryon-baryon interactions in the $S = -2$ sector. However, over the last several decades, there have been only three $\Lambda\Lambda$ hypernuclei unambiguously determined in experiments, with the s-shell ${}^{6}_{\Lambda\Lambda}$He being so far the lightest one [1, 2]. Recently, the observation of nuclear bound states of $\Xi^- - {}^{14}$ N (${}^{15}_{\Xi}$C) [3–5] and possibly $\Xi^- - {}^{11}$ B (${}^{12}_{\Xi}$Be) [6], and the evidence from femtoscopic measurements that supports an attractive $\Xi^- p$ interaction [7, 8] raise hopes that light s-shell Ξ hypernuclei may also exist. Observation of the latter will be not only experimentally interesting but also of great importance especially for theorists in order to construct realistic YY and Ξ N potentials. On the other hand, theoretical predictions for Ξ hypernuclei, especially the light systems, can in turn provide useful guidelines for experimentalists in searching for possible bound Ξ states.

In this study, we explore the possible existence of light Ξ hypernuclei with $A = 4 - 7$ baryons based on the Jacobi no-core shell model (J-NCSM) in combination with nucleon-nucleon (NN) and ΞN interactions derived within chiral effective field theory (EFT). Chiral EFT builds on the same symmetries and symmetry breaking patterns as QCD, but is applied to study the dynamics of low-energy hadronic processes for which hadrons rather than quarks and gluons are the more relevant degrees of freedom. Together with an appropriate power counting the potentials can be organized as an expansion in powers of small external momenta. The long-ranged parts of the interactions can be established from the underlying symmetries, whereas the unresolved short-distance dynamics is parameterised in terms of so-called low energy constants (LECs) which must be determined via a fit to experiments. Thanks to the wealth of NN scattering data, the NN interaction has been very well established (up to the fifth order in the chiral expansion (N^{4}LO), see e.g. [9] and references therein). In the strangeness $S = -1, -2$ sectors, the situation is, however, less satisfactory due to the lack of appropriate YN and YY scattering data. Nevertheless, YN and YY potentials up to NLO have been derived [10–13] and reasonable predictions for Λ [14–17] and $\Lambda\Lambda$ [18] hypernuclei using chiral interactions have been reported.

The lightest Ξ hypernucleus, ${}^{3}_{\Xi}$H, has been investigated in several studies [19–21]. However, the conclusion about the existence of a bound state in this system is rather interaction-model dependent. With an effective ΞN potential that mimics the phase shifts of the Nijmegen ESC08c potential [22] Garcilazo *et al.* [19] and Hiyama *et al.* [20] both obtained a deeply bound ${}^{3}_{\Xi}$H ($J^\pi = 3/2^+, T = 1/2$) state. Whereas the system is found to be unbound when ΞN potentials from chiral EFT [13] or from lattice simulations by the HAL QCD Collaboration [23] are employed [20, 21]. Recently, the $A = 4$ Ξ system has been studied by Hiyama *et al.* [20] using the Nijmegen ESC08c and HAL QCD potentials. Again, the reported results are strongly sensitive to the ΞN interaction models employed. A systematic study of these light Ξ hypernuclei using realistic NN and Ξ N interactions is therefore eagerly awaited, especially by experimentalists.

The J-NCSM is based on an expansion of the many-body wave function in a harmonic oscillator (HO) basis that depends on relative Jacobi coordinates. HO states are, however, not very well suited for calculations using interactions that induce strong correlations like nuclear and hypernuclear interactions. Hence, we apply the similarity renormalization group (SRG) evolution [24] to NN and ΞN potentials in order to soften the interactions. Such SRG evolution will generally not only modify the two-body interactions but also eventually induce three- and higher-body forces. These SRG-induced higher-body forces are omitted in this study and their effect can be estimated from studying the dependence of the binding energies on the SRG flow parameter λ.

Baryon-baryon interactions for $S = -2$

For all calculations presented here we have employed the high-order semilocal momentum-space regularized NN potential with a regulator of $\Lambda_N = 450$ MeV (N^{4}LO+(450)) [9]. The interaction in the ΞN channel is taken from Ref. [13]. The latter has been constructed in agreement with empirical constraints on the ΛΛ S-wave scattering length and with published values and upper bounds for $\Xi^- p$ elastic and inelastic cross sections [12]. Moreover, it yields a moderately attractive Ξ-nuclear potential which is consistent with experimental evidence for the existence of Ξ-hypernuclei [3, 5]. The value obtained for the Ξ single-particle potential $U_\Xi(k = 0)$ at nuclear matter saturation density, around −9 MeV [26], is noticeably smaller than the commonly cited potential depth of −14 MeV [27]. However, an application of this single-particle potential to finite Ξ nuclei by Kohno *et al.* [26, 28] yields binding energies that are quite in line with the values recently extracted in Refs. [3–5].

The considered ΞN interaction also includes coupling to other BB channels in the $S = -2$ sector (ΛΛ, ΛΣ, ΣΣ). However, calculations within the J-NCSM showed that convergence of the eigenvalue iterations (that diagonalize the many-body Hamiltonian) to the lowest lying Ξ states is rather poor when the coupling of ΞN to ΛΛ is explicitly included. For practical realization, we will therefore omit the coupling to ΛΛ. Contribution from such a ΛΛ − ΞN transition, which can anyway occur only in the $^1S_0$ partial wave with isospin $I = 0$, is then incorporated effectively by re-adjusting the strength of the $V_{\Xi N-\Xi N}$ potential [25] . For this exploratory study, we have applied this procedure to the YY-ΞN potential with a chiral cutoff of $\Lambda = 500$ MeV, that exhibits the smallest ΛΛ − ΞN coupling [13, 25]. Note that the other couplings in the $S = -2$ sector, namely ΞN − ΛΣ − ΣΣ, are unaffected by the re-adjustment and they will be fully taken into account when performing the energy calculations. We will, however, neglect YN interactions that are expected to give insignificant contributions but could potentially again induce ΛΛ components to the many-body state.

In Fig. 1 we compare the ΞN phase shifts of the four S-wave states, $^{11}S_0$, $^{31}S_0$, $^{13}S_1$ and $^{33}S_1$, computed with the chiral NLO(500) potential with those predicted by the Nijmegen ESC08c [22] and the HAL QCD [23] potentials. The latter two potentials have been employed in $A = 3, 4$ Ξ hypernuclear calculations by Hiyama *et al.* [20] recently. As expected, the original NLO(500) interaction (cf. the dotted line) and the re-adjusted potential differ only slightly in the $^{11}S_0$ phase shifts. Overall, the NLO(500) and HAL QCD ΞN phase shifts exhibit a rather similar trend, but differ substantially from those of the Nijmegen ESC08c potential. While the ESC08c is strongly attractive in the $^{33}S_1$ channel (which even leads to a deuteron-like ΞN bound state that is not observed in experiment), the chiral NLO(500) (HAL QCD) interaction is only moderately (weakly) attractive in this channel. Moreover, the $^{11}S_0$ ΞN interaction is quite attractive in the HAL QCD and NLO(500) potentials, but it is actually repulsive in the ESC08c model. Furthermore, despite

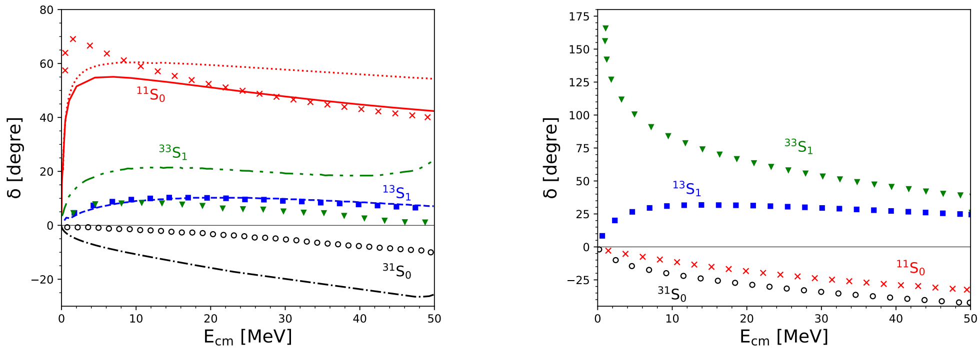

Figure 1: ΞN phase shifts predicted by the NLO(500) and HAL QCD potentials (left panel) compared to those of the Nijmegen ESC08c model (right panel). The NLO(500) results are shown by lines: $^{11}S_0$ (dotted, red), $^{31}S_0$ (dash-dotted, black), $^{13}S_1$ (dashed, blue) and $^{33}S_1$ (dash-double-dotted, green). The solid line indicates the $^{11}S_0$ phase shift of the re-adjusted NLO(500) potential, see text. The HAL QCD and ESC08c results (values are taken from [20]) for $^{11}S_0$, $^{31}S_0$, $^{13}S_0$ and $^{33}S_0$ are indicated by crosses, circles, squares, and triangles, respectively. Note the different scales in the left and right panels.

some similarities between the results by the NLO(500) and HAL QCD potentials, there are visible differences in their predictions for all ΞN partial waves except the $^{13}S_1$ where there is no channel coupling. As we discuss later, such variations may lead to qualitative differences in the predictions of the two interactions for light Ξ systems.

Results and discussion

As we mentioned earlier, in order to speed up the convergence of the J-NCSM calculations, we evolve the NN and YY interactions using the similarity renormalization group (SRG) [24]. The NN interaction is SRG evolved to a flow parameter of $\lambda_{NN} = 1.6$ fm^{-1}. This value of λ_{NN} has been used in Refs. [17, 18], and the resulting NN potential predicts rather accurate binding energies in ordinary nuclei even if three-nucleon forces are neglected for this parameter. We also include the electromagnetic NN interaction [29] as well as the Coulomb point-like interaction between Ξ^- and a proton explicitly. However, for simplicity, these interactions will only be included after the SRG evolution. The $S = -2$ potential is, on the other hand, SRG-evolved to a wide range of SRG λ_{YY} flow parameter, namely $1.4 \leq \lambda_{YY} \leq 3.0$ fm^{-1}. The variations of the binding energies with respect to λ_{YY} then allow one to quantify the possible contribution of the omitted SRG-induced three- and more-body forces. The λ_{YY} dependence for the separation energy $B_\Xi(^4_\Xi\mathrm{H}(1^+,0))$ is illustrated in Fig. 2. One sees that the overall variation of $B_\Xi(^4_\Xi\mathrm{H}(1^+,0))$ is moderate, about 190 ± 30 keV. It is however clearly larger than the dependence of $B_{\Lambda\Lambda}$ on the SRG-flow parameter, which was found to be of the order of 100 keV [18]. This could probably be related to the fact that π exchange, which is expected to be suppressed in the $\Lambda\Lambda$ interaction due to isospin conservation, contributes to the ΞN interaction already at leading order. Long-range interactions are usually affected earlier during the SRG evolution. Nonetheless, the variation of B_Ξ is much smaller than the one observed for single Λ hypernuclei (see e.g. [17]). For the $A = 5, 7$ Ξ hypernuclei, we

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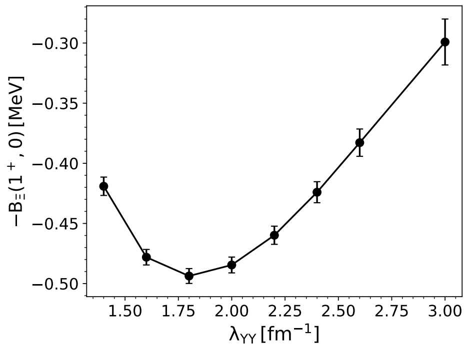

(a)

Figure 2: Dependence of $B_\Xi({}^{4}_{\Xi}\mathrm{H}(1^+,0))$ on the flow parameter λ_{YY}.

	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	0.48 ± 0.01	0.74
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	0.71 ± 0.08	0.2
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.64 ± 0.11	0.01
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	-	-
${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$	2.16 ± 0.10	0.19
${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$	3.50 ± 0.39	0.2

Table 1: Ξ separation energies B_Ξ and estimated decay widths Γ for $A = 4-7$ Ξ hypernuclei. All calculations are based on the YY-ΞN interaction NLO(500) and the NN interaction SMS N^{4}LO+(450). Both potentials are SRG-evolved to a flow parameter of $\lambda_{\mathrm{NN}} = \lambda_{YY} = 1.6$ fm^{-1}. The values of B_Ξ in NNNΞ, ${}^{5}_{\Xi}$H and ${}^{7}_{\Xi}$H are measured with respect to the binding energies of the core nuclei ^{3}H, ^{4}He and ^{6}He, respectively.

observed similarly large absolute variations, but still they are relatively small as compared to the estimated Ξ separation energies. Therefore, in all cases, the SRG dependence is small enough that it should not affect our conclusions on the possible existence of the bound states found. In the following discussion, we will therefore focus on results computed for a specific SRG flow parameter, namely $\lambda_{YY} = 1.6$ fm^{-1}. Those results, for the separation energies B_Ξ of $A = 4-7$ Ξ hypernuclei together with its perturbatively estimated decay width Γ, are listed in Table 1. The $\Xi^- p$ Coulomb interaction is estimated to contribute about 200, 600, and 400 keV to the binding energies of NNNΞ, ${}^{5}_{\Xi}$H and ${}^{7}_{\Xi}$H, respectively. Thus, the binding of all the studied systems is predominantly due to the strong ΞN interaction. In general, we observed three weakly bound states $(1^+, 0)$, $(0^+, 1)$ and $(1^+, 1)$ in NNNΞ, with quite similar B_Ξ's but substantially different decay widths. Interestingly, our result for B_Ξ(NNN$\Xi(1^+, 0)$) is close to the value of 0.36 ± 0.16 MeV obtained with the HAL QCD potential [20]. The other two states, $(0^+, 1)$ and $(1^+, 1)$ are, however, not bound with that interaction. Furthermore, there are substantial differences between our separation energies B_Ξ(NNNΞ) and the results for the ESC08c potential [22] reported in [20]. As mentioned earlier, this version of the Niimegen potential predicts an unrealistically large strength for the ΞN interaction that even results in a bound state in the ΞN system.

For the ${}^5_\Xi$H system, we obtained the result of $B_\Xi({}^5_\Xi\text{H}) = 2.16 \pm 0.1$ MeV and $\Gamma({}^5_\Xi\text{H}) = 0.19$ MeV for the separation energy and decay width, respectively. Surprisingly, these values agree roughly with the estimations by Myint and Akaishi [30] of 1.7 MeV and 0.2 MeV, respectively. However, in contrast to our finding that ${}^5_\Xi$H is bound primarily due to the strong ΞN interaction, the authors in [30] estimate that most of the 1.7 MeV binding energy in ${}^5_\Xi$H comes from the ^{4}He-Ξ^- Coulomb interaction. A quite similar separation energy, $B_\Xi({}^5_\Xi\text{H}) = 2.0$ MeV, is also reported in the work by Friedman and Gal, where an optical potential is employed [31]. But also, here, the agreement could be more or less accidental, given that the Ξ-nuclear interaction used as starting point in that work is with $U_\Xi \lesssim -20$ MeV significantly more attractive than the value of around $U_\Xi \approx -9$ MeV [26] predicted by the chiral ΞN potential.

The prediction of the chiral ΞN interaction for ${}^7_\Xi$H $(\frac{1}{2}^+, \frac{3}{2})$, $B_\Xi({}^7_\Xi\text{H}) = 3.50 \pm 0.39$ MeV, is only slightly larger than the binding energy of 3.15 MeV reported by Fujioka *et al.* [32] when the HAL QCD potentials is employed in combination with a four-body ($\alpha nn\Xi$) cluster model [33]. An earlier study utilizing older $S = -2$ potentials from the Nijmegen group yielded a somewhat smaller binding energy, $B_\Xi = 1.8$ MeV [33]. Note that also for this system the decay width predicted by us, $\Gamma = 0.2$ MeV, is rather small.

Finally, in order to shed light on the relation between the properties of the employed chiral ΞN potential and the reported binding of the $A = 4-7$ Ξ systems, we provide in Table 2 the contributions of different ΞN partial waves to the expectation value of the $S = -2$ potential $\langle V^{S=-2}\rangle$. These results have been computed using model spaces up to $\mathcal{N} = 28$ and a HO frequency of $\omega = 10$ MeV for NNNΞ, $\mathcal{N} = 14$ at $\omega = 16$ MeV for ${}^5_\Xi$H and $\mathcal{N} = 10$ at $\omega = 16$ MeV for ${}^7_\Xi$H. Here the second largest model space is chosen for each system in order to save computational resources. And, ω is the optimal HO frequency for the corresponding model space. For completeness, we also provide the energy expectation values in the last column of Table 2. It is clear that in all the considered states except NNN$\Xi(0^+, 0)$ the attractive ΞN interaction in the ${}^{33}S_1$ channel, which accounts for more than 50% of the total expectation value $\langle V^{s=-2}\rangle$, plays the most important role in binding these systems. The attraction in the ${}^{11}S_0$ channel is essential too, specifically for the binding in NNN$\Xi(1^+, 0)$ and $(0^+, 1)$, amounting to more than 30% of $\langle V^{S=-2}\rangle$. Its contribution, however, becomes less significant in other states. The ΞN repulsion in ${}^{31}S_0$ on the other hand contributes predominantly to the expectation value $\langle V^{S=-2}\rangle$ of NNN$\Xi(0^+, 0)$ (naturally with opposite sign), which causes the system to be unbound. The expectation value $\langle V^{S=-2}({}^{31}S_0)\rangle$ is likewise sizable for ${}^5_\Xi$H and ${}^7_\Xi$H, but its effect is largely canceled by the attraction in the ${}^{11}S_0$ channel. Finally, contributions of different ΞN partial waves to the binding of the s-shell Ξ hypernuclei can also be understood approximately in terms of an effective $\tilde{V}_{\Xi N}$ potential. Analytical expressions for $\tilde{V}_{\Xi N}$ for different states in the $A = 4, 5$ Ξ systems together with a discussion can be found in Ref. [25].

	$V^{S=-2}$					E
	$^{11}S_0$	$^{31}S_0$	$^{13}S_1$	$^{33}S_1$	total	
$^{4}_{\Xi}$H$(1^+,0)$	-1.95	0.02	-0.7	-2.31	-5.21	-8.97
$^{4}_{\Xi}$n$(0^+,1)$	-0.6	0.25	-0.004	-0.74	-1.37	-9.07
$^{4}_{\Xi}$n$(1^+,1)$	-0.02	0.16	-0.13	-1.14	-1.30	-9.0
$^{4}_{\Xi}$H$(0^+,0)$	-0.002	0.08	-0.01	-0.006	-0.11	-6.94
$^{5}_{\Xi}$H$(1/2^+,1/2)$	-0.96	0.94	-0.58	-3.63	-4.88	-31.43
$^{7}_{\Xi}$H$(1/2^+,3/2)$	-1.23	1.79	-0.79	-6.74	-8.04	-33.22

Table 2: Contributions of different partial waves to $\langle V^{S=-2}\rangle$ (first five columns), and the total binding energy (last column) for the $A = 4 - 7\ \Xi$ hypernuclei. The results are extracted at $\mathcal{N} = 28$, $\omega = 10$ MeV for NNNΞ, at $\mathcal{N} = 14$, $\omega = 16$ MeV for $^{5}_{\Xi}$H and at $\mathcal{N} = 10$, $\omega = 16$ MeV for $^{7}_{\Xi}$H. All energies are given in MeV. Same interactions as in Table 1. Note that the calculated binding energy of ^{3}He(^{3}H) is -7.79 (-8.50) MeV.

Conclusion

In this work, we employed the Jacobi NCSM in combination with the chiral NLO(500) ΞN potential to explore $A = 4-7\ \Xi$ hypernuclei. Particle conversions in the $S = -2$ sector like $\Lambda\Sigma - \Xi N - \Sigma\Sigma$ are fully taken into account, whereas the transition $\Lambda\Lambda - \Xi N$ is omitted for technical reasons. Instead, the contribution from the $\Lambda\Lambda - \Xi N$ coupling is incorporated effectively into the strength of the $V_{\Xi N}$ potential. Furthermore, to speed up the convergence, the ΞN potential is SRG-evolved to a wide range of SRG flow parameters. We observed a moderate effect of SRG evolution on the Ξ separation energies. It is larger than that observed for ΛΛ hypernuclei, but much smaller than the SRG-dependence of the Λ separation energies in the $S = -1$ sector.

We found three loosely bound states $(1^+, 0)$, $(0^+, 1)$ and $(1^+, 1)$ for the NNNΞ system, while the $^{5}_{\Xi}$H and $^{7}_{\Xi}$H hypernuclei are more tightly bound. The binding of these Ξ hypernuclei is primarily due to the attraction of the chiral ΞN potential in the $^{33}S_1$ channel. On the other hand, the repulsive nature of the $^{31}S_0$ partial wave prevents the binding of the NNNΞ$(0^+, 0)$ state. All the investigated Ξ bound states are expected to have very small decay widths so that they could be observed in experiments. An experimental search for the $^{7}_{\Xi}$H hypernucleus is planned at J-PARC [34]. The other lighter systems, such as NNNΞ, could potentially be detected in heavy ion collisions. Establishing the existence of those light Ξ hypernuclei, and their binding energies, will be an important step towards a better understanding of BB interactions.

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