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Large-N_c constraints for beyond the standard model few-nucleon currents in effective field theory

Thomas R. Richardson

Duke University,
Durham, NC, USA 27708 Street number, City, Country

E-mail: trr20@duke.edu

Low energy experiments that search for Beyond the Standard Model (BSM) physics often rely on nuclear targets. Therefore, it is imperative that we obtain a clear theoretical picture of the nuclear physics involved. Effective field theory provides a model-independent framework to capture the nuclear physics in terms of few-nucleon currents. However, every operator in an effective theory is accompanied by an undetermined low energy coefficient that must be determined from data or a nonperturbative quantum chromodynamics calculation such as a lattice calculation. For many processes, these determinations are not yet possible; thus, other theoretical constraints are necessary in order to guide the interpretation of experimental bounds. Here, we review recent constraints obtained from the large-N_c limit of QCD, where N_c is the number of colors, for BSM few-nucleon currents relevant for neutrinoless double beta decay and dark matter direct detection.

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1. Introduction

Searches for Beyond the Standard Model (BSM) physics often rely on nuclear targets. Two particular examples are the searches for neutrinoless double beta decay ($0\nu\beta\beta$) [1–14] and the direct detection of dark matter [15–19]. In order to interpret the data from such experiments, it is imperative to obtain a clear theoretical picture of the nuclear currents relevant for these processes.

Effective field theory (EFT) techniques can be used to develop a complete set of current operators in a nearly model-independent manner. For nuclear physics, the most successful EFTs are chiral EFT (ChEFT) [20–32] and pionless EFT (EFT$_{\not\pi}$) [33–36], which are low energy theories that respect the symmetries of quantum chromodynamics (QCD). The model dependencies in the nuclear EFT framework are only with respect to the content of the BSM particles, e.g., spin 1/2 weakly interacting massive particles (WIMPs) as dark matter candidates. Otherwise, these EFTs yield model-independent results for nuclear currents and relevant observables.

Despite the ability to write down generic interactions between nuclear currents and external probes in the EFT framework, every operator in an EFT has an undetermined low energy coefficient (LEC). The LECs have to be determined either from data or from matching the EFT to a higher scale theory. In the absence of conclusive data, the former is not possible. In the case of nuclear EFTs, the latter requires nonperturbative calculations such as lattice QCD calculations. While this avenue is promising, there is still some time before results for many processes will be available.

Without the determinations of the LECs from experiment or lattice calculations, we are in need of theoretical constrains from other sources. Even when experiment and lattice calculations are feasible, other theoretical constraints would be beneficial in order to discern where computational and financial resources are best spent. Here, we use the spin-flavor symmetry of baryons [37–42] that arises in the large-N_c limit of QCD [43–45], N_c being the number of colors, to constrain the relative sizes of the EFT couplings. Because the large-N_c limit furnishes an expansion in powers of $1/N_c$, these constraints are not only useful for constraining possible BSM couplings to few-nucleon systems, but they also yield insight regarding the QCD structure of baryons.

Outline.

2. The large-N_c expansion and nuclear effective field theory

In the large-N_c limit of QCD [43–45], the baryon states transform under a contracted SU(2F) spin-flavor group [37–40]. For two quark flavors, large-N_c baryons transform as a multiplet of SU(4). This symmetry furnishes an expansion in powers of $1/N_c$ for the baryon matrix elements of QCD operators [41, 42], i.e.,

$$\langle B'|\, O_{\text{QCD}}^{(m)}\, |B\rangle = \langle B'|\, N_c^m \sum_{n,s,t} c_n \left(\frac{\hat{J}^i}{N_c}\right)^s \left(\frac{\hat{I}^a}{N_c}\right)^t \left(\frac{\hat{G}^{ia}}{N_c}\right)^{n-s-t} |B\rangle \,, \tag{1}$$

where

$$\hat{J}^i = q^\dagger \frac{\sigma^i}{2} q, \qquad \hat{I}^a = q^\dagger \frac{\tau^a}{2} q, \qquad \hat{G}^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q \,. \tag{2}$$

The matrices σ^i (τ^a) are the usual Pauli matrices in spin (isospin) space. The q field is a colorless, bosonic quark field, which can be understood from the fact that a large-N_c baryon must be composed of N_c quarks antisymmeterized in their color indices and are therefore completely symmetric in their combined spin and isospin indices. The matrix elements of the one-body operators scale as [46]

$$\langle B'|\frac{\hat{J}^i}{N_c}|B\rangle\,,\ \langle B'|\frac{\hat{I}^a}{N_c}|B\rangle \lesssim O\left(\frac{1}{N_c}\right), \qquad \langle B'|\frac{\hat{G}^{ia}}{N_c}|B\rangle \lesssim O(1)\,. \tag{3}$$

In Eq. (1), the indices on the right-hand side are contracted in such a way to reproduce the transformation properties of the left-hand side. Moreover, operator reduction rules imply that any operator other than $\hat{J}^2$ in which two indices are contracted are redundant and may be eliminated [42].

The large-N_c expansion has been used to infer the structure of several single-baryon matrix elements as well as the two-nucleon matrix elements of the Hamiltonian in the symmetry conserving [46–50] and symmetry violating [51–55] sectors. Specifically, the Hamiltonian takes on a Hartree form [41, 42, 56, 57]

$$H = N_c \sum_{n,s,t} v_{stn} \left(\frac{\hat{S}^i}{N_c}\right)^s \left(\frac{\hat{I}^a}{N_c}\right)^t \left(\frac{\hat{G}^{ia}}{N_c}\right)^{n-s-t}\,, \tag{4}$$

where the coefficients v_{stn} are at most $O(1)$ and are momentum dependent. Additionally, the matrix two-baryon matrix elements of the Hamiltonian factorize [46]

$$\left\langle N_\alpha N_\beta\middle| O_1 O_2 \middle|N_\gamma N_\delta\right\rangle \to \langle N_\alpha| O_1 \left|N_\gamma\right\rangle \left\langle N_\beta\middle| O_2 \left|N_\delta\right\rangle + \text{ crossed}\,, \tag{5}$$

where α, β, γ and δ denote the spin, isospin, and momenta of the fields. Therefore, the scaling of two-baryon matrix elements of the Hamiltonian are determined by multiplying the single-baryon scaling of the operators and removing an overall factor of N_c to account for the scaling of the Hamiltonian. Finally, the large-N_c scalings can be mapped onto the LECs of the EFT corresponding to operators with the same spin-flavor structure Recently, these techniques have been applied to two-nucleon currents interacting with external fields in EFT$_{\not\pi}$ and ChEFT assuming that the Hartree form of the Hamiltonian and the factorization property still hold [58–60].

3. Lepton Number and Isospin Violation

In recent years, neutrinoless double beta decay has received significant attention in the context of effective field theory [61–69]. The light Majorana exchange mechanism has been a particular subject of interest [63, 65, 67]. A neutrino exchange potential has been derived in EFT$_{\not\pi}$ and ChEFT that is dressed to all orders by the leading order two-nucleon contact term in EFT$_{\not\pi}$ as well as one pion exchange in ChEFT. In both cases, the neutrino exchange dressed by contact interactions generates a divergence that requires the promotion of a contact term to leading order [65, 67], which was originally assumed to be suppressed. The term in the ChEFT Lagrangian for this contact term is [67]

$$\mathcal{L}^{NN}_{|\Delta L=2|} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta}\bar{e}_L C \bar{e}_L^T \frac{g_\nu^{NN}}{4}\left[\left(\bar{N}u\tilde{Q}^w_L u^\dagger N\right)^2 - \frac{1}{6}\operatorname{Tr}(\tilde{Q}^w_L\tilde{Q}^w_L)\left(\bar{N}\tau^a N\right)^2\right] + \text{H.c.}, \tag{6}$$

where N represents the doublet of nucleon fields, e_L is the left-handed electron, the charge conjugation matrix is $C = i\gamma^2\gamma^0$, G_F is the Fermi constant, V_{ud} is an element of the Cabibbo-Kobayashi-Maskawa matrix, and

$$\tilde{Q}^w_L = \tau^+ = \frac{1}{2}\left(\tau^1 + i\tau^2\right)\,. \tag{7}$$

PoS(CD2021)098

The matrix u is

$$u = \exp\left(\frac{i}{2F}\phi_a\tau^a\right), \tag{8}$$

where the ϕ_a $(a = 1, 2, 3)$ are the pion fields in Cartesian coordinates, the τ^a are Pauli matrices in isospin space, and F is the pion decay constant in the chiral limit. However, this introduces an LEC g_ν^{NN} at leading order that is *a priori* undetermined; therefore, it must be determined either from experiment or from a lattice QCD calculation.

Analagously, charge independence breaking (CIB) two-nucleon interactions mediated by pion exchange when including the electromagnetic pion mass splitting requires contact terms for renormalization. The pertinent interactions are [67]

$$\begin{aligned}\mathcal{L}_{CIB}^{NN} &= \frac{e^2}{4}\left\{C_1\left[\left(\bar{N}u^\dagger\tilde{Q}_R uN\right)^2 + \left(\bar{N}u\tilde{Q}_L u^\dagger N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_L^2 + \tilde{Q}_R^2\right)\left(\bar{N}\tau^a N\right)^2\right]\right.\\ &\left.\quad + C_2\left[2\left(\bar{N}u^\dagger\tilde{Q}_R uN\right)\left(\bar{N}u\tilde{Q}_L u^\dagger N\right) - \frac{1}{3}\mathrm{Tr}\left(U\tilde{Q}_L U^\dagger\tilde{Q}_R\right)\left(\bar{N}\tau^a N\right)^2\right]\right\},\end{aligned} \tag{9}$$

where $U = u^2$ and in this case

$$\tilde{Q}_L = \tilde{Q}_R = \frac{1}{2}\tau^3. \tag{10}$$

In fact, it has been shown that chiral symmetry requires $g_\nu^{NN} = C_1$. Now, the sum $C_1 + C_2$ can be determined from two-nucleon scattering data; however, the difference $C_1 - C_2$ is sensitive to two-nucleon-multi-pion interactions and is therefore currently inaccessible. While a determination of C_1 is equivalently a determination of g_ν^{NN}, C_1 can not be presently disentangled from C_2. In order to estimate the impact of this LEC on nuclear matrix elements, Ref. [67] assumed that $g_\nu^{NN} = \frac{1}{2}(C_1 + C_2)$.

In order to determine the scaling of the LECs with respect to N_c, it is necessary to derive an overcomplete set of operators and eliminate all redundancies through Fierz transformations while keeping track of the leading order in N_c operators. This procedure was performed in Ref. [60]. For the electromagnetic spurion operators, the minimal Lagrangian to LO and NLO in the large-N_c expansion is

$$\begin{aligned}\mathcal{L}_{\text{LO-in-}N_c} =& e^2\left\{\frac{1}{2}\left[2\bar{C}_{1,1} + \bar{C}_{12,1} - \bar{C}_3\right]\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger N\right)^2\right.\\ &\left.+ \bar{C}_3\left[\left(N^\dagger\sigma^i\tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger\sigma^i\tau^a N\right)^2\right]\right\},\end{aligned} \tag{11}$$

$$\begin{aligned}\mathcal{L}_{\text{NLO-in-}N_c} =& e^2\left\{\frac{1}{2}\left[2\bar{C}_{1,1} - \bar{C}_{12,1} - \bar{C}_6\right]\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger N\right)^2 + \bar{C}_2\left(N^\dagger N\right)\left(N^\dagger\tilde{Q}_+ N\right)\right.\\ &\left.+ \bar{C}_6\left[\left(N^\dagger\sigma^i\tilde{Q}_- N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger\sigma^i\tau^a N\right)^2\right]\right\}.\end{aligned} \tag{12}$$

The terms proportional to $\bar{C}_3$ and $\bar{C}_6$ are related to those in Eq. (9) according to

$$C_1 = -3\bar{C}_3 - 3\bar{C}_6 = -3\bar{C}_3\left[1 + O(1/N_c)\right], \tag{13}$$

$$C_2 = -3\bar{C}_3 + 3\bar{C}_6 = -3\bar{C}_3\left[1 + O(1/N_c)\right]. \tag{14}$$

Thus, $C_1 = C_2$ up to $O(1/N_c)$ corrections, which justifies the assumptions $C_1 = \frac{1}{2}(C_1 + C_2)$ as a reasonable approximation.

References [68, 69] recently used an approach analogous to the Cottingham formula in order to estimate $\tilde{C}_1$ and $\tilde{C}_1 + \tilde{C}_2$, where $\tilde{C}_i = \left(\frac{m_N}{4\pi}C\right)^2 C_i$, $i = 1, 2$ and C is $C = C_0$ in EFT$_{\not\pi}$ and $C = C_0 + \frac{g_A^2}{4\pi F^2}$ in ChEFT. If we only consider the central values of the estimates, then the ratio of the LECs is

$$\frac{\tilde{C}_1(\mu = m_\pi)}{\tilde{C}_2(\mu = m_\pi)} \approx 0.81\,, \tag{15}$$

where μ is the renormalization scale in the minimal subtraction scheme. This ratio is well within 30% of 1, the large-N_c value of this ratio. The combination $\tilde{C}_1 + \tilde{C}_2(\mu = m_\pi) = 5.1$ was also determined from two-nucleon CIB data in Ref. [69]. If we approximate $\tilde{C}_1 = \tilde{C}_2$, then

$$\tilde{C}_1\big|_{\text{large-}N_c} = \tilde{C}_2\big|_{\text{large-}N_c} = 2.55\,, \tag{16}$$

and if we allow for roughly 30% corrections then

$$1.7 \lesssim \tilde{C}_1,\ \tilde{C}_2 \lesssim 3.4\,. \tag{17}$$

Therefore, the large-N_c constraints in combination with experiment favor a slightly larger value $\tilde{C}_1$ compared to the Cottingham approach. However, the large-N_c constraints are in rough agreement with the extraction of the couplings from the Cottingham approach.

4. Dark Matter Direct Detection

The direct detection of dark matter has been studied in the context of ChEFT [70–80], a nonrelativistic EFT for the nucleus as a whole [81], and a nonrelativistic EFT for single nucleons [82–86]. Recently, we have applied the combined large-N_c and EFT$_{\not\pi}$ expansion to one- and two-nucleon currents relevant for dark matter direct detection [59]. There, we analyzed the impact of large-N_c constraints for nucleon, deuteron, triton, and helium-3 targets. Here, we summarize the results relevant for nucleon and deuteron targets. Specifically, the large-N_c scalings of the LECs of one- and two-nucleon currents coupled to a spin-1/2 weakly interacting massive particle (WIMP) were considered. The one-nucleon operators that conserve both parity (P) and time-reversal invariance (T) with zero derivatives are

$$\mathcal{L}_{\chi N}^{(PT)} = C_{1,\chi N}^{(PT)}\left(N^\dagger N\right)\left(\chi^\dagger\chi\right) + C_{2,\chi N}^{(PT)}\left(N^\dagger\sigma^i\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) \tag{18}$$

$$+ C_{3,\chi N}^{(PT)}\left(N^\dagger\sigma^i N\right)\left(\chi^\dagger\sigma^i\chi\right) + C_{4,\chi N}^{(PT)}\left(N^\dagger\tau^3 N\right)\left(\chi^\dagger\chi\right)\,. \tag{19}$$

The one-derivative operators that violate parity but conserve time-reversal invariance are

$$\begin{aligned}\mathcal{L}_{\chi N}^{(\not P T)} &= C_{5,\chi N}^{(\not P T)}\epsilon^{ijk}\nabla^j\left(N^\dagger\sigma^k\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) + C_{6,\chi N}^{(\not P T)}\epsilon^{ijk}\nabla^j\left(N^\dagger\sigma^k N\right)\left(\chi^\dagger\sigma^i\chi\right)\\
&+ iC_{7,\chi N}^{(\not P T)}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^i\sigma^i\tau^3 N\right)\left(\chi^\dagger\chi\right) - \frac{1}{2m_\chi}\left(N^\dagger\sigma^i\tau^3 N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^i\chi\right)\right]\\
&+ iC_{8,\chi N}^{(\not P T)}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^i N\right)\left(\chi^\dagger\sigma^i\chi\right) - \frac{1}{2m_\chi}\left(N^\dagger N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^i\sigma^i\chi\right)\right]\\
&+ iC_{9,\chi N}^{(\not P T)}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^i\sigma^i N\right)\left(\chi^\dagger\chi\right) - \frac{1}{2m_\chi}\left(N^\dagger\sigma^i N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^i\chi\right)\right]\\
&+ iC_{10,\chi N}^{(\not P T)}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^i\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) - \frac{1}{2m_\chi}\left(N^\dagger\tau^3 N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^i\sigma^i\chi\right)\right],\end{aligned} \tag{20}$$

and the parity and time-reversal invariance violating operators are

$$\begin{aligned}\mathcal{L}_{\chi N}^{(\not P \not T)} &= C_{11,\chi N}^{(\not P \not T)}\nabla^i\left(N^\dagger\sigma^i\tau^3 N\right)\left(\chi^\dagger\chi\right) + C_{12,\chi N}^{(\not P \not T)}\nabla^i\left(N^\dagger N\right)\left(\chi^\dagger\sigma^i\chi\right)\\
&+ C_{13,\chi N}^{(\not P \not T)}\nabla^i\left(N^\dagger\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) + C_{14,\chi N}^{(\not P \not T)}\nabla^i\left(N^\dagger\sigma^i N\right)\left(\chi^\dagger\chi\right)\\
&+ i\epsilon^{ijk}C_{15,\chi N}^{(\not P \not T)}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) - \frac{1}{2m_\chi}\left(N^\dagger\sigma^k\tau^3 N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^j\sigma^i\chi\right)\right]\\
&+ i\epsilon^{ijk}C_{16,\chi N}^{(\not P \not T)}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k N\right)\left(\chi^\dagger\sigma^i\chi\right) - \frac{1}{2m_\chi}\left(N^\dagger\sigma^k N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^j\sigma^i\chi\right)\right].\end{aligned} \tag{21}$$

The couplings scale as

$$C_{1,\chi N}^{(PT)},\ C_{2,\chi N}^{(PT)},\ C_{5,\chi N}^{(\not P T)},\ C_{7,\chi N}^{(\not P T)},\ C_{8,\chi N}^{(\not P T)},\ C_{11,\chi N}^{(\not P \not T)},\ C_{12,\chi N}^{(\not P \not T)},\ C_{15,\chi N}^{(\not P \not T)} \sim O(N_c), \tag{22}$$

$$C_{3,\chi N}^{(PT)},\ C_{4,\chi N}^{(PT)},\ C_{6,\chi N}^{(\not P T)},\ C_{9,\chi N}^{(\not P T)},\ C_{10,\chi N}^{(\not P T)},\ C_{13,\chi N}^{(\not P \not T)},\ C_{14,\chi N}^{(\not P \not T)},\ C_{16,\chi N}^{(\not P \not T)} \sim O(1). \tag{23}$$

The terms proportional to $C_{7,\chi N}^{(\not P T)}$, $C_{8,\chi N}^{(\not P T)}$, $C_{9,\chi N}^{(\not P T)}$, $C_{10,\chi N}^{(\not P T)}$, $C_{15,\chi N}^{(\not P \not T)}$, $C_{16,\chi N}^{(\not P \not T)}$ contain explicit suppressions from $m_N \sim O(N_c)$ or m_χ which is treated as being large with respect to other parameters in the theory. Therefore, we consider $C_{1,\chi N}^{(PT)}$, $C_{2,\chi N}^{(PT)}$, $C_{5,\chi N}^{(\not P T)}$, $C_{11,\chi N}^{(\not P \not T)}$, and $C_{12,\chi N}^{(\not P \not T)}$ to be subleading in a combined large-N_c and heavy WIMP expansion.

The Lagrangian component that couples WIMPs to two-nucleon currents is

$$\begin{aligned}\mathcal{L}_{\chi,NN} &= C_{1,\chi NN}^{(\mathrm{SI},\,s)}\left(\chi^\dagger\chi\right)\left(N^\dagger N\right)\left(N^\dagger N\right) + C_{2,\chi NN}^{(\mathrm{SI},\,s)}\left(\chi^\dagger\chi\right)\left(N^\dagger\sigma^i N\right)\left(N^\dagger\sigma^i N\right)\\
&+ C_{1,\chi NN}^{(\mathrm{SD},\,s)}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^i N\right)\left(N^\dagger N\right) + C_{1,\chi NN}^{(\mathrm{SI},\,v)}\left(\chi^\dagger\chi\right)\left(N^\dagger\tau^3 N\right)\left(N^\dagger N\right)\\
&+ \epsilon^{ijk}\epsilon^{3ab}C_{1,\chi NN}^{(\mathrm{SD},\,v)}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^k\tau^b N\right) + \epsilon^{ijk}C_{2,\chi NN}^{(\mathrm{SD},\,v)}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^j\tau^3 N\right)\left(N^\dagger\sigma^k N\right)\\
&+ C_{1,\chi NN}^{(\mathrm{SI},\,t)}\left(\chi^\dagger\chi\right)\left[\left(N^\dagger\sigma^i\tau^3 N\right)\left(N^\dagger\sigma^i\tau^3 N\right) - \frac{1}{2}\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i\tau^a N\right)\right],\end{aligned} \tag{24}$$

The LEC superscripts s, v, and t indicate isoscalar, isovector, and isotensor operators, respectively. The superscripts SI and SD indicate whether the operator is dark matter spin independent or spin dependent, respectively. where

$$C^{(\mathrm{SI},\,s)}_{1,\chi NN}, C^{(\mathrm{SD},\,v)}_{1,\chi NN}, C^{(\mathrm{SI},\,t)}_{1,\chi NN} \sim O(N_c)\,, \tag{25}$$

$$C^{(\mathrm{SD},\,s)}_{1,\chi NN}, C^{(\mathrm{SI},\,v)}_{1,\chi NN}, C^{(\mathrm{SD},\,v)}_{2,\chi NN} \sim O(1)\,, \tag{26}$$

$$C^{(\mathrm{SI},\,s)}_{2,\chi NN} \sim O(1/N_c)\,. \tag{27}$$

The constraints for the couplings may be translated into constraints for the cross sections for WIMP-nucleon scattering, which is relevant for cosmological searches for dark matter. Generally, the unpolarized amplitude can be decomposed as

$$|\mathcal{M}|^2 = 16\pi \left(m_T + m_\chi\right)^2 \left[\sigma_0^{\mathrm{SI}} F^2_{\mathrm{SI}}\left(E_R\right) + \sigma_0^{\mathrm{SD}} F^2_{\mathrm{SD}}\left(E_R\right)\right]\,. \tag{28}$$

where m_T is the mass of the target. Thus, at LO-in-N_c the ratio of the spin-independent and spin-dependent cross sections for WIMP-nucleon elastic scattering in the limit of zero momentum transfer is

$$\frac{\sigma^{\mathrm{SI}}_{0,N}}{\sigma^{\mathrm{SD}}_{0,N}} = \frac{C^{(PT)2}_{1,\chi N}\left(1 \pm \frac{C^{(PT)}_{4,\chi N}}{C^{(PT)}_{1,\chi N}}\right)^2}{3C^{(PT)2}_{2,\chi N}\left(1 \pm \frac{C^{(PT)}_{3,\chi N}}{C^{(PT)}_{2,\chi N}}\right)^2} \approx \frac{C^{(PT)2}_{1,\chi N}}{3C^{(PT)2}_{2,\chi N}} \sim \frac{1}{3}\,, \tag{29}$$

for all proton and neutron combinations if both $C^{(PT)}_{1,\chi N}$ and $C^{(PT)}_{2,\chi N}$ are considered to be of natural size apart from their N_c scalings. This ratio can receive 30% corrections from the large-N_c expansion, but this indicates that we expect the SI and SD cross sections to be of the same size.

Similar constraints may be obtained for WIMP-deuteron elastic scattering. In principle, the two-body currents can contribute at next-to-leading-order if they connect two S-wave states. For elastic WIMP-deuteron scattering, these operators must connect two $^3S_1$ states, so the necessary operators are proportional to $C^{(\mathrm{SI},s)}_{1,\chi NN}$, $C^{(\mathrm{SI},s)}_{2,\chi NN}$, and $C^{(\mathrm{SD},s)}_{1,\chi NN}$. Therefore, the cross sections in the limit $q^2 \to 0$ are

$$\sigma^{\mathrm{SI}}_{0,d} = \frac{m^2_{\chi d}}{\pi}\left[2C^{(PT)}_{1,\chi N} - \frac{\gamma}{\pi}\left(\mu-\gamma\right)^2\left(C^{(\mathrm{SI},\,s)}_{1,\chi NN} + C^{(\mathrm{SI},\,s)}_{2,\chi NN}\right)\right]^2\,, \tag{30}$$

$$\sigma^{\mathrm{SD}}_{0,d} = \frac{2m^2_{\chi d}}{\pi}\left[2C^{(PT)}_{3,\chi N} - \frac{\gamma}{\pi}\left(\mu-\gamma\right)^2 C^{(\mathrm{SD},\,s)}_{1,\chi NN}\right]^2\,. \tag{31}$$

As a result, we expect the SD cross section to be $1/N_c^2$ or about $1/10$ suppressed relative to the SI cross section. Additionally, the WIMP-deuteron and WIMP-nucleon SI cross sections should be about the same size apart from other numerical enhancements while the SD WIMP-deuteron cross section should be $1/N_c^2$ suppressed with respect to the SD WIMP-nucleon cross section.

5. Summary

Large-N_c expansions can be used to provide novel constraints for few-nucleon currents relevant for BSM physics. In particular, the LECs in nuclear EFTs that contribute to neutrinoless double beta decay, WIMP-nucleon, and WIMP-deuteron scattering have been organized into large-N_c

hierarchies. These constraints can be implemented in *ab initio* calculations that make use of EFT interactions in order to simplify the input required at a given order of the EFT. Additionally, these constraints can complement ongoing lattice calculations by indicating which couplings should be dominant according to QCD.

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