PROCEEDINGS OF SCIENCE

Chiral EFT for neutrinoless double beta decay

Wouter Dekens[a,*]

[a]*Institute for Nuclear Theory, University of Washington, Seattle WA 91195-1550, USA*

E-mail: wdekens@uw.edu

We discuss the effects of lepton-number-violating (LNV) interactions in neutrinoless double beta decay ($0\nu\beta\beta$), assuming that LNV arises either from light sterile neutrinos or from energy scales well above the electroweak scale. In this scenario, LNV can be described by an effective field theory (EFT), consisting of the standard model (SM), supplemented by higher-dimensional operators made up from SM fields and the sterile neutrino. We summarize the steps needed to obtain expressions for the $0\nu\beta\beta$ half-lives in terms of the LNV operators. We pay particular attention to the matching of the quark-level interactions onto chiral EFT, which involves nucleons and pions as the degrees of freedom. This theory allows one to derive LNV potentials between nucleons, which, together with many-body calculations, determine the decay rates of nuclei that are used in experiments. We will show that the matching onto chiral EFT requires the inclusion of a contact interaction at leading order and discuss a model estimate of the associated hadronic matrix element. We conclude by illustrating the constraints on the higher dimensional operators.

The 10th International Workshop on Chiral Dynamics - CD2021
15-19 November 2021
Online

*Speaker

PoS(CD2021)100

1. Introduction

The observation of neutrinoless double beta decay would demonstrate the Majorana nature of neutrinos [1], show that lepton number is violated, and provide a clear sign of physics beyond the SM (BSM). The current experimental constraints on the half-life of this process are already very stringent, e.g. $T^{0\nu}_{1/2} > 2.3\cdot 10^{26}$ yr in ^{136}Xe [2], while the next generation of experiments, for example [3, 4], are projected to probe half lives that are longer by one to two orders of magnitude. The interpretation of these constraints requires a theoretical framework that can describe the effects of BSM LNV interactions. If we assume that such BSM physics arises either from sterile neutrinos or from an energy scale, Λ, well above the electroweak scale, $v \simeq 246$ GeV, these LNV sources can be described by an EFT. This theory is the SM-EFT [5, 6] extended by sterile neutrinos, sometimes referred to as the νSMEFT [7], and involves the SM fields as well as sterile neutrinos as degrees of freedom, while effects due to heavy new particles are parametrized by higher-dimensional operators. The information about a particular UV complete scenario is contained in the couplings of the sterile neutrinos and the higher-dimensional operators. Thus, after deriving expressions for the $0\nu\beta\beta$ half-lives in terms of these couplings, assessing the impact of a UV completion on $0\nu\beta\beta$, reduces to matching that particular BSM scenario onto the EFT.

The required steps to obtain such expressions within this framework involve the evolution of the effective operators to the electroweak scale where the heavy SM fields are integrated out. The EFT is subsequently evolved to the scale where QCD becomes non-perturbative, Λ_χ ~GeV, and the quark-level theory is matched onto chiral EFT. The chiral Lagrangian allows one to derive a LNV potential between nucleons which can serve as the starting point for many-body calculations.

We start by discussing the set of operators at the scale Λ.

2. Lepton-number violation in the (ν)SM-EFT

LNV interactions arise at odd dimensions within the SM-EFT [8] so that the $SU(3)_c \times SU(2) \times U(1)_Y$ invariant Lagrangian describing heavy LNV sources can be written as

$$\mathcal{L}_{\nu_L} = \mathcal{L}_{SM} + \mathcal{L}^{(5)}_{\Delta L=2} + \mathcal{L}^{(7)}_{\Delta L=2} + \mathcal{L}^{(9)}_{\Delta L=2} + \dots , \tag{1}$$

where the dots stand for operators beyond dimension-nine. Although the dimension-seven (-nine) operators are suppressed by $1/\Lambda^2$ ($1/\Lambda^4$) compared to the dimension-five term, there are several BSM scenarios, such as the left-right model [9–11], in which operators up to dimension nine can play a role. Only one operator, which induces a Majorana mass for the left-handed neutrinos, appears at dimension five [12], while the complete set of dimension-seven operators includes 12 LNV interactions [7, 13–16]. Finally, the dimension-nine operators involving four quark and two lepton fields were classified in Refs. [17, 18], while a complete basis was derived more recently [19, 20].

Extending this framework to include n possibly light sterile neutrinos, ν_R, leads to the following additional terms,

$$\mathcal{L}_{\nu_R} = \left[\frac{1}{2}\bar\nu_R i\partial\!\!\!/ \nu_R - \frac{1}{2}\bar\nu^c_R \bar M_R \nu_R - \bar L \tilde H Y_\nu \nu_R + \text{h.c.}\right] + \mathcal{L}^{(6)}_{\nu_R} + \mathcal{L}^{(7)}_{\nu_R} + \dots , \tag{2}$$

where the expression in brackets describes the kinetic term as well as the Dirac and Majorana masses of the sterile neutrinos, while $\mathcal{L}^{(6,7)}_{\nu_R}$ contain the complete set of dimension-six and -seven operators [7] that involve the sterile neutrino, which can be induced by heavy BSM physics coupling to both the right-handed sterile neutrino and SM fields. These operators can subsequently be evolved to the electroweak scale. After integrating out the heavy SM fields, and evolving to a scale of a few GeV, one ends up with a Lagrangian involving dimension-three, -six, -seven, and -nine operators.

After electroweak symmetry breaking, the dimension-three Lagrangian consists of the mass terms for the neutrinos

$$\mathcal{L}^{(3)} = -\frac{1}{2}\bar{N}^c M_\nu N + \text{h.c.}\,, \qquad M_\nu = \begin{pmatrix} M_L & M_D^* \\ M_D^\dagger & M_R^\dagger \end{pmatrix}, \tag{3}$$

where $N = (\nu_L,\, \nu_R^c)^T$, $M_{L,R}$ are the Majorana masses for the left- and right-handed neutrinos, induced by the dimension-five operator and $\bar{M}_R$, respectively, while $M_D = \frac{v}{\sqrt{2}}Y_\nu$ is the Dirac mass matrix. M_ν can be diagonalized by a $(3+n)\times(3+n)$ unitary matrix, U, such that the flavor and mass eigenstates are related by $N = UN_m$.

At dimension six, one encounters the following four-fermion interactions [21],

$$\begin{aligned}\mathcal{L}^{(6)} = \frac{2G_F}{\sqrt{2}}\Big\{&\bar{u}_L\gamma^\mu d_L\left[\bar{e}_R\gamma_\mu C^{(6)}_{\text{VLR}}\,\nu + \bar{e}_L\gamma_\mu C^{(6)}_{\text{VLL}}\,\nu\right] + \bar{u}_R\gamma^\mu d_R\left[\bar{e}_R\,\gamma_\mu C^{(6)}_{\text{VRR}}\,\nu + \bar{e}_L\,\gamma_\mu C^{(6)}_{\text{VRL}}\,\nu\right]\\ &+\bar{u}_L d_R\left[\bar{e}_L\,C^{(6)}_{\text{SRR}}\nu + \bar{e}_R\,C^{(6)}_{\text{SRL}}\nu\right] + \bar{u}_R d_L\left[\bar{e}_L\,C^{(6)}_{\text{SLR}}\nu + \bar{e}_R\,C^{(6)}_{\text{SLL}}\nu\right]\\ &+\bar{u}_L\sigma^{\mu\nu}d_R\,\bar{e}_L\sigma_{\mu\nu}C^{(6)}_{\text{TRR}}\,\nu + \bar{u}_R\sigma^{\mu\nu}d_L\,\bar{e}_R\sigma_{\mu\nu}C^{(6)}_{\text{TLL}}\,\nu\Big\} + \text{h.c.}\,,\end{aligned} \tag{4}$$

where $\nu = N_m + N_m^c$. These dimension-six terms receive contributions from the $\mathcal{L}^{(7)}_{\Delta L=2}$, as well as $\mathcal{L}^{(6,7)}_{\nu_R}$. Although the terms induced by $\mathcal{L}^{(6)}_{\nu_R}$ are not LNV by themselves, they can contribute to $0\nu\beta\beta$ after the inclusion of the Majorana mass term of the sterile neutrinos. The relevant dimension-seven operators have a similar form to the dimension-six interactions with the addition of a derivative.

Finally, dimension-nine operators [17, 18] with two electrons and four quarks can be induced by $\mathcal{L}^{(7)}_{\Delta L=2}$ and $\mathcal{L}^{(9)}_{\Delta L=2}$. In addition, if one of the sterile neutrinos has a mass $\Lambda_\chi < m_\nu < m_W$, terms in $\mathcal{L}^{(6)}_{\nu_R}$ can give rise to dimension-nine operators after integrating out the sterile neutrino. These interactions take the form,

$$\mathcal{L}^{(9)} = \frac{1}{v^5}\sum_i\left[\left(C^{(9)}_{i\,\text{R}}\,\bar{e}_R C\bar{e}_R^T + C^{(9)}_{i\,\text{L}}\,\bar{e}_L C\bar{e}_L^T\right)O_i + C^{(9)}_i\,\bar{e}\gamma_\mu\gamma_5 C\bar{e}^T\,O_i^\mu\right], \tag{5}$$

where O_i and O_i^μ are four-quark operators that are Lorentz scalars and vectors, respectively. Their definitions can be found in Ref. [22].

3. Chiral EFT

The chiral EFT should include all operators that have the same (chiral) symmetry properties as the original quark-level interactions of the previous section. Each of the interactions in the chiral Lagrangian comes with a hadronic matrix element, or low-energy constant (LEC), which

PoS(CD2021)100

is to be determined from experiment or lattice-QCD calculations. The matching is trivial for the dimension-three Lagrangian of Eq. (3), as both theories involve neutrinos as degrees of freedom. These neutrino mass terms can subsequently induce $0\nu\beta\beta$ in combination with two insertions of the weak current, which in chiral EFT take the form of $(\bar{n}p)(\bar{e}\nu)$ and $\pi(\bar{e}\nu)$ interactions, giving rise to an amplitude of the form $\mathcal{A}_3 \sim \sum_{i=1}^{3+n} G_F^2 m_{\nu_i} U_{ei}^2$. The dimension-six and -seven operators in Eqs. (4) mainly induce similar interactions to the Fermi operator, $\bar{p}n\bar{e}\nu$. The operators that are LNV can induce $0\nu\beta\beta$ after combination with a single insertion of G_F, $\mathcal{A}_6 \sim G_F C^{(6)}_{\Delta L=2}$, while the lepton-number conserving operators require insertions of m_ν, so that $\mathcal{A}_6 \sim m_\nu \left(C^{(6)}_{\Delta L=0}\right)^2$. The latter contribution is suppressed by additional powers of $1/\Lambda$, but allows for the possibility of sterile neutrino exchange, which can give significant contributions when $m_{\nu_R} \gg m_{\nu_{1,2,3}}$. Finally, the scalar dimension-nine operators mainly induce $\pi\pi\bar{e}e^c$ interactions, while the vector operators generate $\pi\bar{p}n\bar{e}e^c$ and $(\bar{p}n)(\bar{p}n)\bar{e}e^c$ terms [17], all of which do not need further insertions of G_F or m_ν to contribute to $0\nu\beta\beta$. The LECs for these operators have been calculated on the lattice for the scalar dimension-nine terms [23], most of those needed for $C_i^{(6,7)}$ are nucleon charges [21], while the LECs for the dimension-nine vector terms are currently unknown.

An additional issue arises for contributions involving sterile neutrinos. In the case of $m_{\nu_R} \gg 1$ GeV, the sterile neutrinos can be integrated out at the quark level, leading to dimension-nine operators discussed above. However, for $m_{\nu_R} \lesssim 1$ GeV, the ν_R have to be kept as degrees of freedom in chiral EFT. The case when the sterile neutrino is much lighter than the QCD scale can be described in chiral EFT, which, however, breaks down when m_{ν_R} is near Λ_χ. To obtain predictions for the contributions from ν_R in the whole m_{ν_R} range, we demand that the amplitudes correspond to the chiral EFT (perturbative QCD) prediction for $m_{\nu_R} \ll 1$ GeV ($m_{\nu_R} \gg 1$ GeV) and interpolate in between [24].

So far, we have been assuming the Weinberg power counting [25, 26] in order to determine the relative importance of the operators in the chiral Lagrangian. This power-counting scheme is known to break down in the strong sector [27–30], and more recently was shown to be unable to consistently renormalize the $nn \to ppee$ amplitude for $0\nu\beta\beta$. We review the arguments leading to this conclusion and how the power counting can be amended in the next section.

3.1 A contact interaction at leading order

The inconsistency related to the Weinberg power counting can be seen by considering the $nn \to ppee$ amplitude in chiral EFT. The calculation requires diagrams involving LNV vertices, dressed with the strong interactions in all possible ways. The leading-order strong interactions involve pion exchange and a contact interaction between nucleons. The needed diagrams are depicted in the first three lines of the left panel of Fig. 1, for the case in which $0\nu\beta\beta$ is mediated by light Majorana neutrinos. The classes of diagrams in the first two lines do not lead to inconsistencies, while those in the third line give rise to a divergence at leading order. The $0\nu\beta\beta$ amplitude therefore obtains a dependence on the regulator, as depicted for the $\overline{\text{MS}}$ scheme and a coordinate-space cut-off in the right panel of Fig. 1, implying it is not properly renormalized.

This unphysical behavior can be removed by including a contact interaction of the form $\mathcal{L} \sim g_\nu^{NN}(\bar{p}n)(\bar{p}n)\bar{e}e^c$ in the Lagrangian already at leading order. This additional interaction, depicted in the last line of Fig. 1, can absorb the regulator dependence, so that the total amplitude

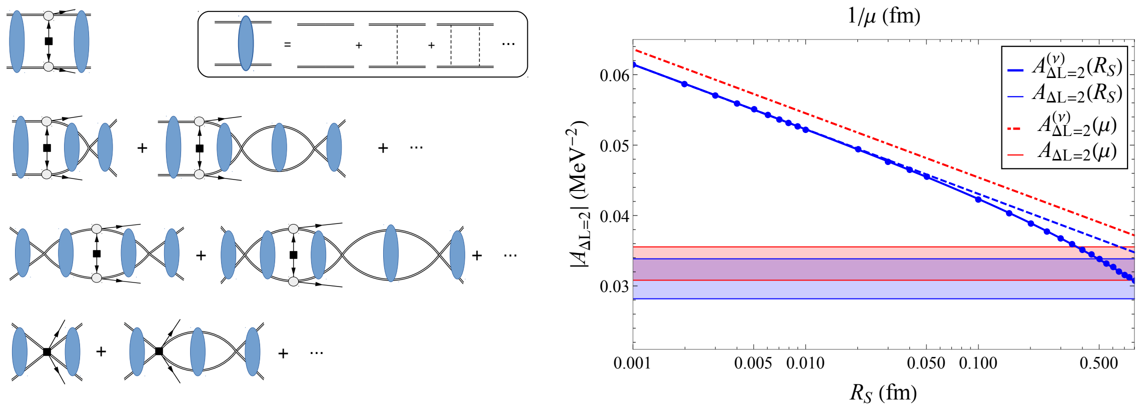

Figure 1: Left panel: Contributions to $nn \to ppee$. Double, dashed, and plain lines denote nucleons, pions, and leptons, respectively. Gray circles denote the weak current, and the black square an insertion of $m_{\beta\beta}$ (or g_ν^{NN} in the case of the fourth line). Right panel: The $nn \to ppee$ amplitude as a function of the regulator. The $\overline{\text{MS}}$ scheme is shown in red, while a cut-off scheme is shown in blue.

becomes independent. This is illustrated by the horizontal bands in the right panel of Fig. 1. Although this leads to a renormalizable amplitude for $nn \to ppee$, it also introduces an unknown LEC, g_ν^{NN}. Given the lack of experimental data, one would preferably obtain this LEC from first principles through a lattice calculation. There are ongoing efforts in this direction [31–37], however, currently, the available estimates involve model calculations [38, 39] or large-N_c determinations [40].

A recent estimate followed a similar approach to the one used to obtain the electromagnetic contributions to the mass differences of hadrons [41, 42]. Here, one uses the fact that the hadronic part of the amplitude is proportional to

$$\begin{aligned}\mathcal{A} &\sim \int \frac{d^4k}{(2\pi)^4} \int d^4x \frac{e^{ix\cdot k}}{k^2+i\epsilon} \langle h_f(p_f)|T\{J_W^\mu(0), J_{W\,\mu}(x)\}|h_i(p_i)\rangle \\ &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+i\epsilon} \mathcal{T}(k, p_i, p_f)\,,\end{aligned} \tag{6}$$

where the two insertions of J_W arise from the needed weak interactions, $h_{i,f}$ are the hadronic initial and final states, k is the momentum flowing through J_W, and the propagator, $\sim 1/k^2$, arises from the exchanged neutrino. A determination of the correlator $\mathcal{T}(k, p_i, p_f)$ as a function of k, then allows for an estimate of the total amplitude. This correlator can be thought of as the amplitude for $h_i(p_i)W^+(k) \to h_f(p_f)W^-(k)$, with $h_i = nn$ and $h_f = pp$, which, for small virtualities $k^2 \ll \Lambda_\chi^2$, can be computed in chiral EFT. For momenta much larger than the QCD scale, $k^2 \gg \text{GeV}^2$, the correlator can instead be evaluated using the operator product expansion. In the intermediate region we supplement the chiral EFT result with phenomenological form factors and off-shell effects from NN intermediate states, while neglecting inelastic intermediate states. This gives an LEC of $O(1)$ with an estimated 30% uncertainty, in $\overline{\text{MS}}$, $\tilde{g}_\nu^{NN}(\mu = m_\pi) = 1.3(6)$. This result is consistent with a large-N_c estimate [40] and implies a leading-order effect due to the contact interaction.

Although this discussion focused on the light Majorana neutrino exchange mechanism, the same power-counting issues appear for several of the higher-dimensional operators. Analogous arguments

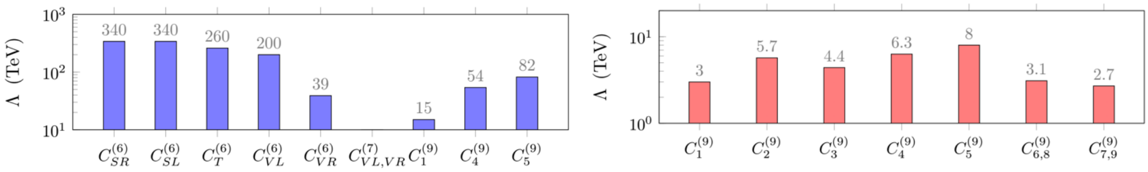

Figure 2: Limits on the Wilson coefficients in Eqs. (4) and (5), where the $C_i^{(6)}$ coefficients with labels, SR, SL, T, VL, VR, correspond to, SRL, SLL, TLL, VLL, VRL, in Eq. (4). See Refs. [21, 22] for explicit definitions of the dimension-seven and -nine terms. The left panel depicts the limits on the couplings generated at dimension seven and assumes $C_i = v^3/\Lambda^3$, while the right panel shows the constraints on couplings induced by dimension-nine operators, assuming $C_i = v^5/\Lambda^5$.

then imply that several contact interactions, which would be higher-order terms in Weinberg's power counting, have to be promoted to leading order in order to be able to renormalize the $nn \to ppee$ amplitude. These modifications to the power counting arise for the scalar dimension-nine operators, and some of the $C_i^{(6,7)}$.

4. Nuclear matrix elements

Having constructed the chiral Lagrangian and ensured that the leading-order $nn \to ppee$ amplitude can be renormalized consistently, the resulting LNV potential can be used to move from the nucleon level to decay rates in nuclei. This requires the calculation of phase-space integrals over the lepton momenta as well as the matrix elements of the LNV potential between initial and final nuclear states. While the phase-space integrals are well known [43], the needed nuclear matrix elements (NMEs) involve complicated many-body calculations. Only a limited set of NMEs is needed in order to describe any source of LNV that can arise from purely heavy BSM physics [21], all of which have been determined in the literature. The relevant NMEs due to light sterile neutrino exchange take very similar forms, but acquire a dependence on the mass of the neutrino. These contributions can again be determined in the limits $m_{\nu_R} \ll \Lambda_\chi$ and $m_{\nu_R} \gg \Lambda_\chi$, but require dedicated determinations in the region $m_{\nu_R} \sim m_\pi$. The m_{ν_R} dependence is not known for all NMEs, so that we again use the known low and high m_{ν_R} behavior as constraints and interpolate in the intermediate region.

Apart from the fact that the m_{ν_R} dependence is not known for several NMEs, the determinations of NMEs varies by a factor of two to three between different methods [43–46], see Ref. [47] for a recent review. This is the case both for the NMEs related to the exchange of light Majorana neutrinos and for those needed for the contributions due to higher-dimensional operators. Recently, preliminary *ab initio* calculations, using potentials inspired by chiral EFT, have started to appear for nuclei that are used in experiments [48–51]. These calculations are especially interesting as they in principle allow for controlled uncertainty estimates. The same methods have been used to gauge the impact of the contribution due to g_ν^{NN} using the model estimate described above, leading to an $O(40\%)$ effect in ^{48}Ca [51], similar in size to the effect found previously in light nuclei [52].

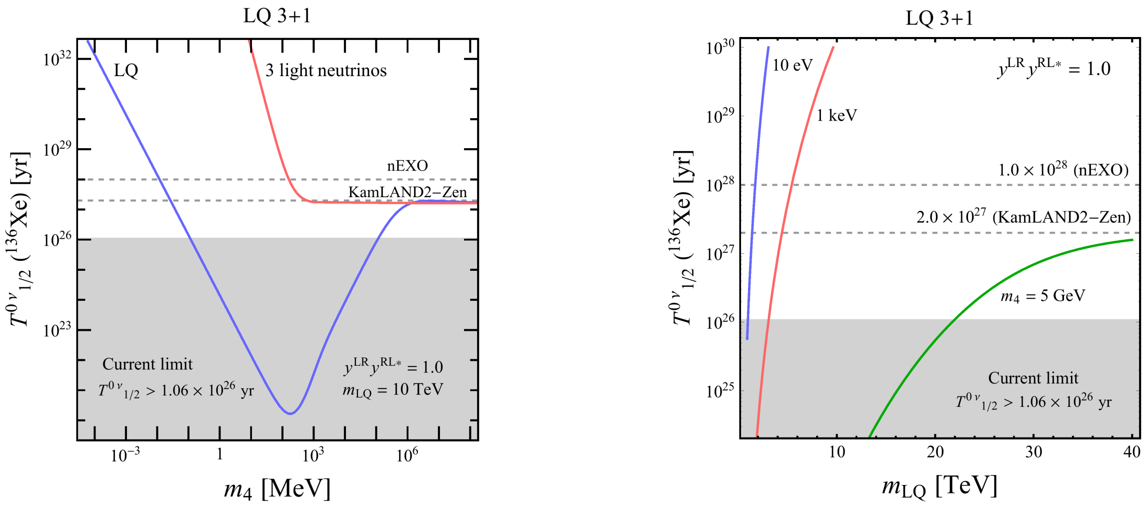

Figure 3: Left panel: $T^{0\nu}_{1/2}(^{136}\text{Xe})$ as a function of $m_{\nu_R} = m_4$ in the 3 + 1 leptoquark model described in the text. The blue line is the total half life whereas the red line depicts the contribution from the exchange of neutrinos interacting via left-handed currents. Right panel: Similar but now we fixed $m_4 = 10$ eV (blue), $m_4 = 1$ keV (red), and $m_4 = 5$ GeV (green) and vary m_{LQ}. In both panels the NMEs of Ref. [44] were used, see [24] for details.

5. Constraints

Although there are significant theoretical uncertainties related to both the hadronic and nuclear matrix elements (LECs and NMEs), here we briefly discuss the constraints that can be set on the LNV sources. Using the NMEs of Ref. [45], together with estimates for the LECs [22] and the experimental limit [53], gives rise to the constraints on the dimension-seven and -nine operators depicted in Fig. 2. One sees that $0\nu\beta\beta$ can probe very high scales, up to hundreds of TeVs, in the case of dimension-seven operators, while the limits on the dimension-nine interactions lie in the $1 - 10$ TeV range.

Finally, we illustrate the impact of sterile neutrinos on the $0\nu\beta\beta$ half life in Fig. 3. The red line in the left panel shows the half-life as a function of m_{ν_R} in a '3+1' scenario in which one sterile neutrino is added to the SM. The blue line depicts the half life that results from the same scenario with the addition of interactions that are induced by a leptoquark field, which generates $C^{(6)}_{\text{SRR}}$ and $C^{(6)}_{\text{TRR}}$, with a mass of $m_{\text{LQ}} = 10$ TeV. The right panel shows the half life as a function of m_{LQ}. Clearly, these non-standard sterile neutrino interactions can have a significant impact on the half life, especially in the region $m_4 \sim$GeV.

We conclude that EFTs allow one to determine the contributions of LNV sources to $0\nu\beta\beta$ in a systematic way. This approach relies on a tower of effective theories, where the most involved steps involve the matching of the quark-level theory onto chiral EFT. Regardless of significant nuclear and hadronic uncertainties, this leads to stringent constraints on LNV sources from heavy BSM physics, as well as the contributions due to sterile neutrinos.

Acknowledgments

I would like to thank the organizers for the opportunity to present this work at Chiral Dynamics, and for an interesting and enjoyable meeting.

References

[1] J. Schechter and J. W. F. Valle, Phys. Rev. **D25**, 2951 (1982).

[2] S. Abe *et al.* [KamLAND-Zen Collaboration], [arXiv:2203.02139 [hep-ex]].

[3] J. B. Albert *et al.* [nEXO Collaboration], Phys. Rev. C **97**, 065503 (2018), [arXiv:1710.05075 [nucl-ex]].

[4] N. Abgrall *et al.* [LEGEND Collaboration], AIP Conf. Proc. **1894**, 020027 (2017), [arXiv:1709.01980 [physics.ins-det]].

[5] W. Buchmüller and D. Wyler, Nucl. Phys. B **268**, 621 (1986).

[6] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, JHEP **1010**, 085 (2010), [arXiv:1008.4884 [hep-ph]].

[7] Y. Liao and X.-D. Ma, Phys. Rev. **D96**, 015012 (2017), [arXiv:1612.04527 [hep-ph]].

[8] A. Kobach, Phys. Lett. **B758**, 455 (2016), [arXiv:1604.05726 [hep-ph]].

[9] J. C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974), [Erratum: Phys. Rev.D11,703(1975)].

[10] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975).

[11] G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975).

[12] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[13] L. Lehman, Phys. Rev. **D90**, 125023 (2014), [arXiv:1410.4193 [hep-ph]].

[14] K. S. Babu and C. N. Leung, Nucl. Phys. **B619**, 667 (2001), [arXiv:hep-ph/0106054].

[15] N. F. Bell, M. Gorchtein, M. J. Ramsey-Musolf, P. Vogel, and P. Wang, Phys. Lett. **B642**, 377 (2006), [arXiv:hep-ph/0606248].

[16] Y. Liao and X.-D. Ma, JHEP **11**, 043 (2016), [arXiv:1607.07309 [hep-ph]].

[17] G. Prezeau, M. Ramsey-Musolf, and P. Vogel, Phys. Rev. **D68**, 034016 (2003), [arXiv:hep-ph/0303205].

[18] M. L. Graesser, JHEP **08**, 099 (2017), [arXiv:1606.04549 [hep-ph]].

[19] Y. Liao and X.-D. Ma, JHEP **11**, 152 (2020), [arXiv:2007.08125 [hep-ph]].

[20] H.-L. Li, Z. Ren, M.-L. Xiao, J.-H. Yu, and Y.-H. Zheng, JHEP **06**, 138 (2021), [arXiv:2012.09188 [hep-ph]].

[21] V. Cirigliano, W. Dekens, J. de Vries, M. L. Graesser, and E. Mereghetti, JHEP **12**, 082 (2017), [arXiv:1708.09390 [hep-ph]].

[22] V. Cirigliano, W. Dekens, J. de Vries, M. L. Graesser, and E. Mereghetti, JHEP **12**, 097 (2018), [arXiv:1806.02780 [hep-ph]].

[23] A. Nicholson *et al.*, Phys. Rev. Lett. **121**, 172501 (2018), [arXiv:1805.02634 [nucl-th]].

[24] W. Dekens, J. de Vries, K. Fuyuto, E. Mereghetti, and G. Zhou, JHEP **06**, 097 (2020), [arXiv:2002.07182 [hep-ph]].

[25] S. Weinberg, Phys. Lett. **B251**, 288 (1990).

[26] S. Weinberg, Nucl. Phys. **B363**, 3 (1991).

[27] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl. Phys. **B478**, 629 (1996), [arXiv:nucl-th/9605002].

[28] S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, Nucl. Phys. **A700**, 377 (2002), [arXiv:nucl-th/0104030].

[29] A. Nogga, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. **C72**, 054006 (2005), [arXiv:nucl-th/0506005].

[30] B. Long and C.-J. Yang, Phys. Rev. **C86**, 024001 (2012), [arXiv:1202.4053 [nucl-th]].

[31] X. Feng, L.-C. Jin, X.-Y. Tuo, and S.-C. Xia, Phys. Rev. Lett. **122**, 022001 (2019), [arXiv:1809.10511 [hep-lat]].

[32] X.-Y. Tuo, X. Feng, and L.-C. Jin, Phys. Rev. D **100**, 094511 (2019), [arXiv:1909.13525 [hep-lat]].

[33] V. Cirigliano, W. Detmold, A. Nicholson, and P. Shanahan, [arXiv:2003.08493 [nucl-th]].

[34] W. Detmold and D. Murphy [NPLQCD Collaboration], [arXiv:2004.07404 [hep-lat]].

[35] X. Feng, L.-C. Jin, Z.-Y. Wang, and Z. Zhang, Phys. Rev. D **103**, 034508 (2021), [arXiv:2005.01956 [hep-lat]].

[36] Z. Davoudi, W. Detmold, K. Orginos, A. Parreño, M. J. Savage, P. Shanahan, and M. L. Wagman, Phys. Rept. **900**, 1 (2021), [arXiv:2008.11160 [hep-lat]].

[37] Z. Davoudi and S. V. Kadam, Phys. Rev. Lett. **126**, 152003 (2021), [arXiv:2012.02083 [hep-lat]].

[38] V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, and E. Mereghetti, Phys. Rev. Lett. **126**, 172002 (2021), [arXiv:2012.11602 [nucl-th]].

[39] V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, and E. Mereghetti, JHEP **05**, 289 (2021), [arXiv:2102.03371 [nucl-th]].

[40] T. R. Richardson, M. R. Schindler, S. Pastore, and R. P. Springer, [arXiv:2102.02184 [nucl-th]].

[41] W. Cottingham, Annals Phys. **25**, 424 (1963).

[42] H. Harari, Phys. Rev. Lett. **17**, 1303 (1966).

[43] M. Horoi and A. Neacsu, [arXiv:1706.05391 [hep-ph]].

[44] J. Hyvärinen and J. Suhonen, Phys. Rev. **C91**, 024613 (2015).

[45] J. Menéndez, J. Phys. **G45**, 014003 (2018).

[46] J. Barea, J. Kotila, and F. Iachello, Phys. Rev. **C91**, 034304 (2015), [arXiv:1506.08530 [nucl-th]].

[47] M. Agostini, G. Benato, J. A. Detwiler, J. Menéndez, and F. Vissani, [arXiv:2202.01787 [hep-ex]].

[48] J. M. Yao, B. Bally, J. Engel, R. Wirth, T. R. Rodríguez, and H. Hergert, Phys. Rev. Lett. **124**, 232501 (2020), [arXiv:1908.05424 [nucl-th]].

[49] A. Belley, C. G. Payne, S. R. Stroberg, T. Miyagi, and J. D. Holt, Phys. Rev. Lett. **126**, 042502 (2021), [arXiv:2008.06588 [nucl-th]].

[50] S. Novario, P. Gysbers, J. Engel, G. Hagen, G. R. Jansen, T. D. Morris, P. Navrátil, T. Papenbrock, and S. Quaglioni, Phys. Rev. Lett. **126**, 182502 (2021), [arXiv:2008.09696 [nucl-th]].

[51] R. Wirth, J. M. Yao, and H. Hergert, Phys. Rev. Lett. **127**, 242502 (2021), [arXiv:2105.05415 [nucl-th]].

[52] S. Pastore, J. Carlson, V. Cirigliano, W. Dekens, E. Mereghetti, and R. B. Wiringa, Phys. Rev. **C97**, 014606 (2018), [arXiv:1710.05026 [nucl-th]].

[53] A. Gando *et al.* [KamLAND-Zen Collaboration], Phys. Rev. Lett. **117**, 082503 (2016), [arXiv:1605.02889 [hep-ex]], [Addendum: Phys. Rev. Lett.117,no.10,109903(2016)].