PROCEEDINGS OF SCIENCE

Slow-roll inflation in Palatini $F(R)$ gravity

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We study inflation in presence of Palatini $F(R)$ gravity. Since it is not always possible to find analytic solutions for the equation of motion of the auxiliary field corresponding to an arbitrary $F(R)$, we propose a new method for the computation of the inflationary observables. We test this method on $F(R) = R + \alpha R^n$ and find that a large α suppresses the tensor-to-scalar ratio r. We also notice that the configuration with $n = 2$ is *unstable*, while the $n > 2$ case is ill-defined, with possible implications on the theoretically allowed UV behaviour of such Palatini models.

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1. Introduction

Past and recent observations of the cosmic microwave background radiation (CMB) support the flatness and homogeneity of the Universe at large scales. Such properties can be explained by assuming inflation i.e. an accelerated expansion during the very early Universe. Recently, a lot of attention has been gained by non-minimal models in the Palatini formulation of gravity, where the metric and the connection are treated as independent variables. In this proceeding we study a scalar inflaton in presence of Palatini $F(R)$ gravity. More details of the analysis are given in [1].

2. Slow-roll computations

We start by considering the following action for a real scalar inflaton ϕ minimally coupled to a Palatini $F(R)$ gravity (in Planck units: $M_{\rm P} = 1$):

$$S_J = \int \mathrm{d}^4x\sqrt{-g_J}\left[\frac{1}{2}F(R(\Gamma)) - \frac{1}{2}k(\phi)g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \tag{1}$$

where $k(\phi) > 0$ is the non-minimal kinetic function for the inflaton and $V(\phi)$ its positive scalar potential. As customary, we introduce the auxiliary field ζ and move to the Einstein frame [1]

$$S_E = \int \mathrm{d}^4x\sqrt{-g_E}\left[\frac{M_{\rm P}^2}{2}R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi,\zeta)\right], \tag{2}$$

where the canonically normalized scalar χ is defined by

$$\frac{\partial\chi}{\partial\phi} = \sqrt{\frac{k(\phi)}{F'(\zeta)}}, \tag{3}$$

and the full scalar potential is

$$U(\chi,\zeta) = \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}. \tag{4}$$

As shown in [1], we cannot solve exactly the equation of motion (EoM) of ζ for an arbitrary $F(R)$. Nevertheless, it is possible to perform inflationary computations by using the auxiliary field as the computational variable. First of all we need the EoM for ζ under slow-roll (SR), which is [1]

$$G(\zeta) \equiv \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right] = V(\phi). \tag{5}$$

Then, we use (5) as a constraint and we replace $V(\phi)$ with $G(\zeta)$ in (4), obtaining a scalar potential[1] that depends only on the auxiliary field ζ (see [1] for more details):

$$U(\zeta) = \frac{1}{4}\frac{\zeta}{F'(\zeta)}. \tag{6}$$

[1]Note that $U(\zeta)$ is an actual scalar potential only when the kinetic term of ϕ is negligible like in SR.

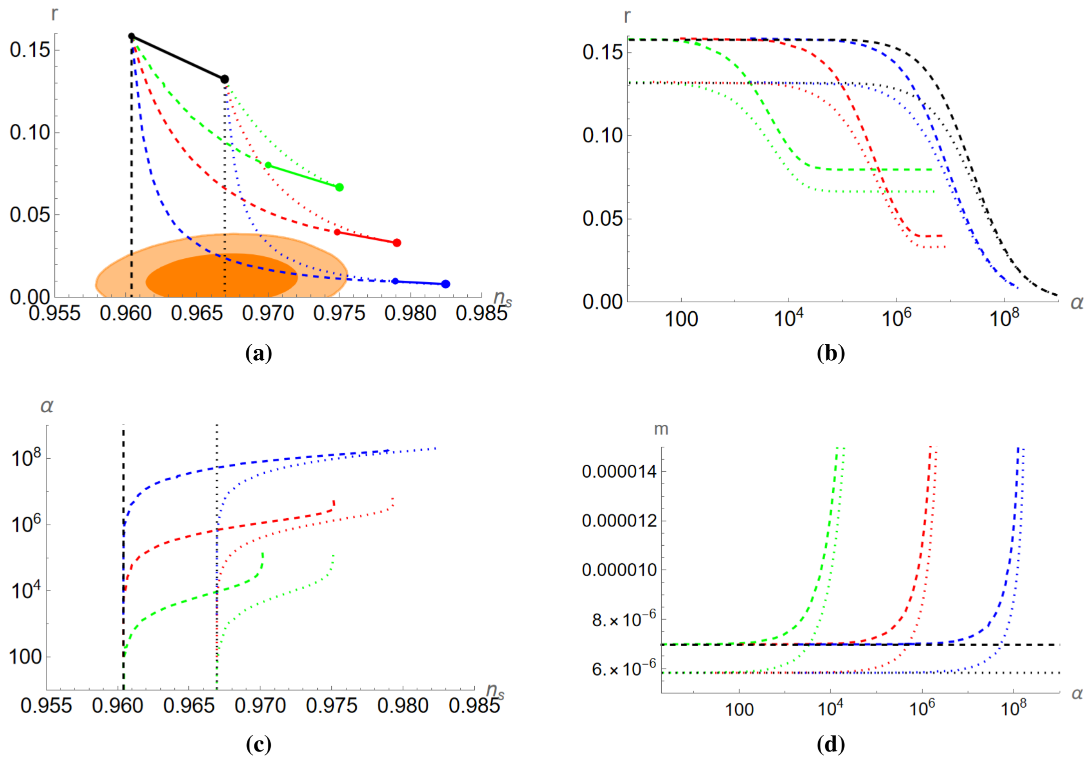

Figure 1: r vs. n_s (a), r vs. α (b), α vs. n_s (c) and m vs. α (d) for the model in (8) with $n = 3/2$ (green), $n = 7/4$ (red), $n = 31/16$ (blue) and $n = 2$ (black) for $N_e = 50$ (dashed) and $N_e = 60$ (dotted). For reference, the predictions for $N_e \in [50, 60]$ of the corresponding asymptotic solutions (see [1] for more details) (same color code, continuous line) and of quadratic inflation (black). In orange the 1,2σ allowed regions coming from the latest combination of Planck, BICEP/Keck and BAO data [3].

Next, we need the derivatives of $U(\zeta)$ with respect to χ. By applying the chain rule for the derivative of composite functions, we have

$$\frac{\partial}{\partial\chi} = \frac{\partial\zeta}{\partial\chi}\frac{\partial}{\partial\zeta} = \frac{\partial\phi}{\partial\chi}\frac{\partial\zeta}{\partial\phi}\frac{\partial}{\partial\zeta} = \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}}\left(\frac{\partial G}{\partial\zeta}\frac{\partial V^{-1}}{\partial G}\right)^{-1}\frac{\partial}{\partial\zeta}\,, \tag{7}$$

where eq. (3), and $V^{-1}(G)$ is the inverse function of $V(\phi)$ defined via (5). Now (the SR parameters and) the inflationary observables can be computed straightforwardly. See [1] for the exact expressions of the tensor-to-scalar ratio r, the scalar spectral index n_s, the amplitude of the scalar power spectrum A_s and the number of e-folds N_e.

3. Test scenarios

In this section we test our method with

$$F(R) = R + \alpha R^n\,, \quad k(\phi) = 1\,, \quad V(\phi) = \frac{m^2}{2}\phi^2\,. \tag{8}$$

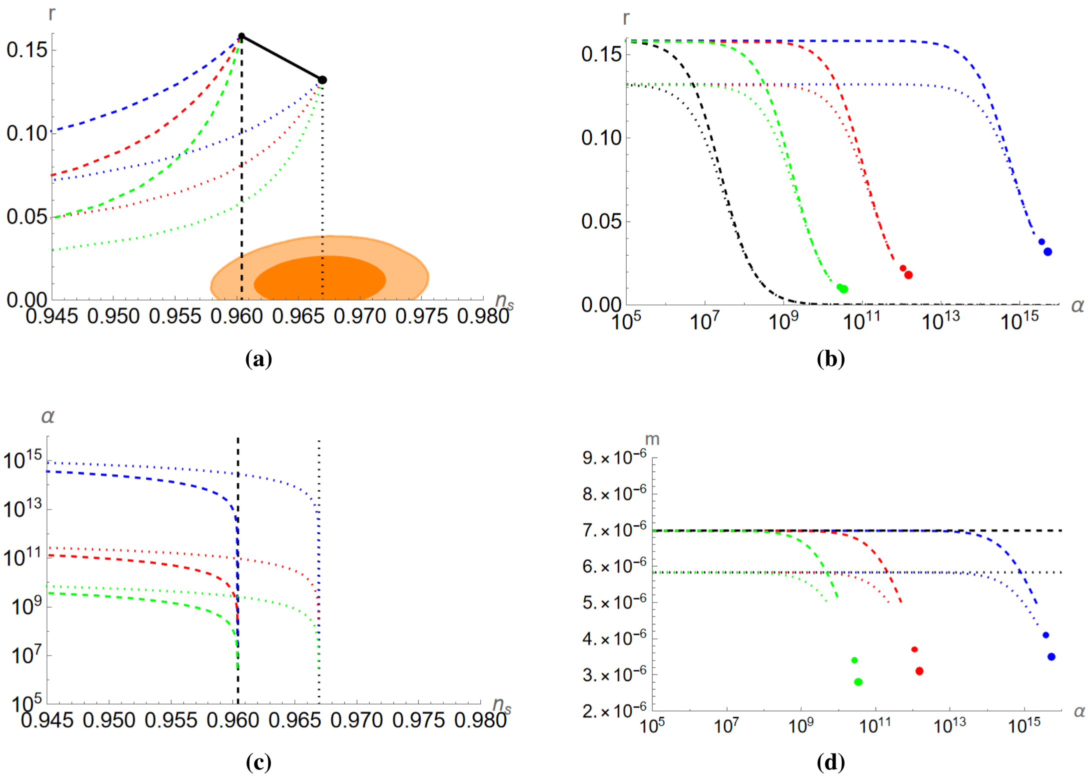

Figure 2: r vs. n_s (a), r vs. α (b), α vs. n_s (c) and m vs. α (d) for the model in (8) with $n = 3$ (blue), $n = 5/2$ (red), $n = 9/4$ (green) and $n = 2$ (black) for $N_e = 50$ (dashed) and $N_e = 60$ (dotted). In the same color code, the corresponding limits when $\alpha = \bar{\alpha}$ for $N_e = 50$ (small dot) and $N_e = 60$ (large dot). In orange the 1,2σ allowed regions coming from the latest combination of Planck, BICEP/Keck and BAO data [3].

We consider two different scenarios, $n < 2$ and $n > 2$. We stress that, during SR, the positivity of both $U(\zeta)$ and $F'(\zeta)$ implies $\zeta > 0$ and $\alpha > 0$. We also notice that our procedure reproduces the results of [2] with $n = 2$ for any kind of $V(\phi)$. Check [1] for more details.

3.1 $n < 2$

In this subsection we apply our method on the model in (8) for $1 < n < 2$. In this case the solution of (5) always exists. We show in Fig. 1 our numerical results. For a full analysis of such results see [1]. In here we briefly comment on the two most important features. First of all we notice that the net effect of the αR^n term is to lower r. Second, we stress that the limits for $\alpha \to +\infty$ with $n = 2$ and $n \neq 2$ are two completely different configurations. At a given $\alpha \gg 1$, a slight variation from $n = 2$ might completely jeopardize the inflationary predictions of the $n = 2$ case. This happens when $\alpha \gg \frac{1}{|n-2|}$.

3.2 $n > 2$

Now we study the setup of eq. (8) for $n > 2$. We start by checking the constraint (5):

$$G(\zeta) = \frac{1}{4}\left[\zeta - \alpha(n-2)\zeta^n\right] = V(\phi)\,. \tag{9}$$

For $n > 2$, $G(\zeta)$ is unbounded from below and exhibits a local maximum at $\zeta_{\max} = [\alpha\, n\, (n-2)]^{\frac{1}{1-n}}$. Therefore we can only treat the model as an effective theory if SR is realized between 0 and $\zeta_{\max}$ [1]. Our numerical results are shown in Fig. 2. For a deep analysis of such results see [1]. In here we briefly comment on the most important features. Once again the net effect of the αR^n term is to lower r. However, the effect on n_s and m is the opposite with the respect to the $n < 2$ case. In fact, now, by increasing α both n_s and m are decreasing. We can also see that, for a given n, there is an upper limit on α. Such an upper limit can be roughly estimated as $\alpha = \bar{\alpha}$ when $\zeta_N = \zeta_{\max}$. The limit is only rough because n_s has a pole at $\zeta = \zeta_{\max}$ meaning the loss of validity of the SR approximation [1]. Nevertheless, we can still provide useful estimates for the limit values of r, m and α by using $\zeta_N = \zeta_{\max}$. In fact, the numerical values for r approach closely the limit ones. Analogously, $m_{\bar{\alpha}}$ and the actual limit of m have the same order of magnitude.

4. Conclusions

We studied single field inflaton in the presence of Palatini $F(R)$ gravity and developed a new method that allows SR computations if the Jordan frame potential $V(\phi)$ is an invertible function of ϕ. We applied our method to quadratic inflation embedded in $R + \alpha R^n$ gravity. We computed the inflationary predictions for $n < 2$ and $n > 2$. In both cases for α increasing, r decreases. On the other hand, for α increasing when $n < 2$ ($n > 2$), n_s and m are increasing (decreasing). Moreover, we have also discovered that the $n = 2$ case is *unstable*: in the strong coupling limit $\alpha \gg 1$, a slight variation from $n = 2$ can induce a large change in n_s. Finally, for $n > 2$ and given n and N_e, α shows an upper limit, which is a consequence of the $n > 2$ case being ill-defined and only valid as an effective description. Therefore, it appears that a UV consistent theory of Palatini gravity should prevent the generation of terms with $n > 2$ (unless part of a converging series that makes (5) consistent). Such a result might be relevant in understanding loop corrections in Palatini gravity.

Acknowledgments

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