PROCEEDINGS OF SCIENCE

Bubble wall dynamics at the electroweak scale

Stefania De Curtis,[a] **Luigi Delle Rose,**[b] **Andrea Guiggiani,**[a,*] **Ángel Gil Muyor**[c] **and Giuliano Panico**[a]

[a] *INFN sezione di Firenze and Dipartimento di Fisica e Astronomia, Università di Firenze, Via G. Sansone 1, I-50019, Sesto Fiorentino, Italy*

[b] *Dipartimento di Fisica, Università della Calabria, I-8703 Arcavacata di Rende, Cosenza, Italy*

[c] *IFAE and BIST, Universitat Autònoma de Barcelona, 08193-Bellaterra, Barcelona, Spain*

E-mail: stefania.decurtis@fi.infn.it, luigi.dellerose@unical.it, andrea.guiggiani@unifi.it, agil@ifae.es, giuliano.panico@unifi.it

Bubble nucleation is a key feature in a cosmological first-order phase transition. The non-equilibrium bubble dynamics and the properties of the transition are controlled by the density perturbations in the hot plasma. We present, for the first time, the full solution of the linearized Boltzmann equation. Our approach, differently from the traditional one based on the fluid approximation, does not rely on any ansatz. We focus on the contributions arising from the top quark species coupled to the Higgs field during a first-order electroweak phase transition. Our results significantly differ from the ones obtained in the fluid approximation with sizeable differences for the friction acting on the bubble wall.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

*Speaker

PoS(ICHEP2022)080

1. Introduction

The recent observation of gravitational waves has renewed a vivid interest in the study of first-order cosmological phase transition (PhT) at the electroweak (EW) scale. The dynamics of such PhT is source of a stochastic gravitational wave background whose frequency peak lies in the region that future space based interferometers [1–7] will probe. A key quantity that characterizes the gravitational wave spectrum together with many interesting relics left by the PhT such as the matter-antimatter asymmetry, is the terminal velocity of the PhT front, namely the bubble wall (BW) speed. In the steady state regime, the propagation velocity of the BW is a result of the balance between the internal pressure, due to the potential difference between the two phases and the friction that originates as a backreaction from perturbations around equilibrium in the plasma generated by the moving wall.

Perturbations around equilibrium are modelled by an effective Boltzmann equation as first shown in [8, 9] where a standard approach, dubbed "old formalism", to solve the Boltzmann equation was established. To deal with the collision operator, the authors of [8, 9] considered a perfect fluid distribution to model the perturbations and adopted a weighted strategy to recast the Boltzmann equation in a set of coupled differential equations whose solution provides the out-of-equilibrium perturbations.

The fluid approximation presents two main drawbacks, the presence of a singularity in the friction for wall propagating at the speed of sound and the arbitrariness on the choice of weights used to integrate the Boltzmann equation. Depending on the set of weights both friction and perturbations present quantitative differences. Despite recently new strategies such as the "new formalism" (NF) [10, 11] or the extended fluid approximation [12, 13] addressing the first issue, they still rely on an ansatz to solve the Boltzmann equation.

The previous discussion highlights the necessity to find a solution to the full Boltzmann equation without relying on any ansatz on the shape of the perturbation. The full solution (FS) obtained in such a way provides reliable quantitative predictions both on the out-of-equilibrium perturbation and the friction exerted on the bubble wall. Moreover, this will also clarify the issue regarding the presence of a singularity at the speed of sound. In [14] we presented, for the first time, a fully quantitative numerical solution to the Boltzmann equation. To present the methodology and to compare with the previous formalisms, we considered the EWPhT and we focused on the study of the top quark, the species that provide the leading contribution to the friction being the particle with the largest coupling to the Higgs field.

2. The Boltzmann equation

For large enough bubbles we can consider a planar wall in a steady state that moves with a velocity v_w. Orienting the reference frame in such a way that the wall moves along the negative side of the $z-$axis and boosting in the wall reference frame the Boltzmann equation can be written in the following form

$$\mathcal{L}[f] = \left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right) f = -\mathbf{C}[f] \quad (1)$$

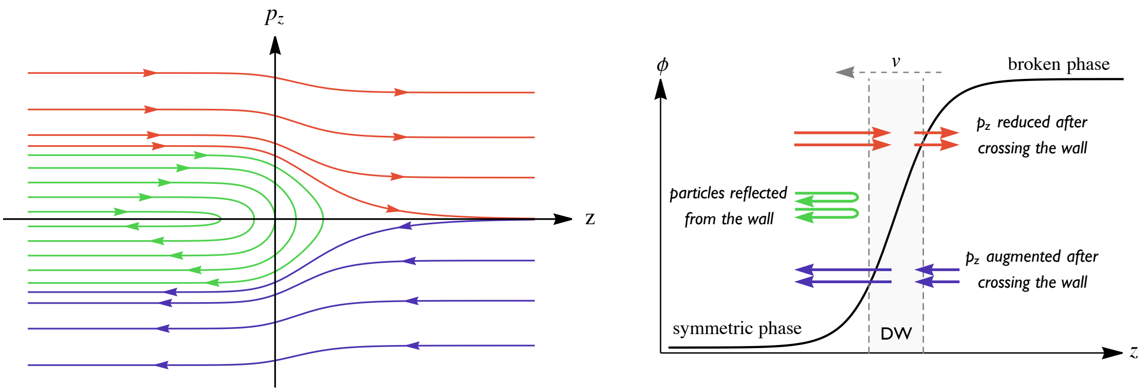

Figure 1: Left panel: Paths with fixed energy and transverse momentum in the $z - p_z$ phase space for the choice $m(z) \propto 1 + \tanh(z/L)$. The red, green and purple colors denote sets of contours with different behavior. The arrows show the flow of a particle within the phase space. Right panel: Schematic representation of the behaviour of the particles across the DW.

where $m(z)$ is the mass of the particle, $(m^2)'$ stands for $d(m^2)/dz$. **C**[f] is the collision operator that describes the microscopic interactions taking place in the plasma, while $\mathcal{L}$ is the Liouville operator.

The collision operator ensures that the plasma recovers equilibrium far from the DW. Thus for $z \to \pm\infty$ the distribution function is the standard Fermi-Dirac or Bose-Einstein distribution, namely

$$f_v = \frac{1}{1 \pm e^{\beta\gamma_w (E - v_w p_z)}} \tag{2}$$

where $\beta = 1/T$ and $\gamma = 1/\sqrt{1 - v_w^2}$. Deviations from equilibrium are expected to be small, thus writing $f = f_v + \delta f$, we can linearize the Boltzmann equation in terms of δf obtaining

$$\left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right)\delta f + \bar{\mathbf{C}}[\delta f] = \frac{(m(z)^2)'}{2E}\partial_{p_z} f_v = \beta\gamma v \frac{(m(z)^2)'}{2E} f_v', \tag{3}$$

where we defined

$$f_v' = -\frac{e^{\beta\gamma(E - v p_z)}}{(e^{\beta\gamma(E - v p_z)} \pm 1)^2}, \tag{4}$$

and $\bar{\mathbf{C}}[\delta f]$ denotes the collision integral linearized in δf. The source term rapidly decays far from the DW where the Higgs profile is constant. Close to the DW, instead, the source term is present, in agreement with the expectation that perturbations are localized on the DW and rapidly vanish away from it.

2.1 Flow paths and collision operator

Along the paths on which both the transverse momentum $p_\perp$ and the quantity $p_z^2 + m^2(z)$ are constant, the Liouville operator reduces to a total derivative with respect to z :

$$\mathcal{L} = \left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right) \to \frac{p_z}{E}\frac{d}{dz} \tag{5}$$

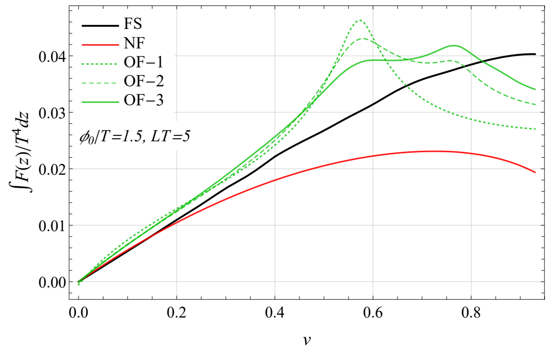

Figure 2: Friction acting on the bubble wall as a function of the velocity. The black solid line corresponds to the solution of the full Boltzmann equation (FS, our result), the dotted, dashed and solid green lines are obtained with the old formalism (OF) at order 1, 2 and 3 respectively [9, 12], while the red line corresponds to the new formalism (NF) [11].

These paths are the trajectories of the particles in the $(p_\perp, p_z, z)$ phase space along which energy and $p_\perp$ are conserved. They are determined by the condition $p_z^2 + m^2(z) = c$. We show in Fig. 1 the curves for $m(z) \propto 1 + \tanh(z/L)$ where L denotes the wall thickness. We recognize three different families, red paths describe particles with enough momentum p_z to eventually cross the wall, green paths, on the contrary, describe particles which are reflected after the interaction with the DW and finally, purple paths describe particles that travel in the negative z direction and eventually exit from the bubble.

The collision operator for the $2 \to 2$ processes is given by

$$\mathbf{C}[f] = \sum_i \frac{1}{4N_p E_p} \int \frac{d^3\mathbf{k} d^3\mathbf{p}' d^3\mathbf{k}'}{(2\pi)^5 2E_k 2E_{p'} 2E_{k'}} |\mathcal{M}_i|^2 \delta^4(p + k - p' - k')\mathcal{P}[f] \tag{6}$$

with the population factor

$$\mathcal{P}[f] = f(p)f(k)(1 \pm f(p'))(1 \pm f(k')) - f(p')f(k')(1 \pm f(p))(1 \pm f(k)), \tag{7}$$

where the sum runs over the relevant processes with amplitude $\mathcal{M}_i$. N_p is the number of degrees of freedom of the incoming particle with momentum p, k is the momentum of the second incoming particle while p' and k' are the momenta of the outgoing particles. The signs $\pm$ are $+$ for bosons and $-$ for fermions.

3. Numerical analysis and results

As explained in [14] it is possible to recast the Boltzmann equation in the following form

$$\frac{d}{dz}\delta f - \frac{Q(p)}{2p_z}\delta f = \mathcal{S}. \tag{8}$$

Such equation can be exactly solved for δf imposing the boundary conditions $\delta f \to 0$ as $z \to \pm\infty$, as we recover thermal equilibrium away from the DW. The term $\mathcal{S}$ contains contributions where the perturbation is integrated. To solve Eq. (8) we thus developed a numerical iterative procedure as described in [14].

As a testing setup we considered a single species out-of-equilibrium in the plasma: the top quark. Being the species with the largest coupling with the Higgs it is expected that the most relevant effects on the PhT dynamics arise from it. We modelled the bubble wall assuming that the Higgs profile can be approximated by

$$\phi(z) = \frac{\phi_0}{2}[1 + \tanh(z/L)], \tag{9}$$

where $L = 5/T$ is the wall thickness, $\phi_0 = 150$ GeV is the Higgs VEV in the broken phase and T is the PhT temperature that we fixed $T = 100$ GeV.

A relevant quantity for the computation of the terminal velocity v_w is the friction acting on the bubble wall, namely

$$F(z) = \frac{dm^2}{dz} N_p \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E} \delta f(p) \tag{10}$$

In Fig. 2 we show the integral of the friction over z as a function of the velocity. We compare our results (full solution, FS) with the ones already present in the literature. The red line corresponds to the result obtained in the NF of ref. [11], while the green lines represent the total friction in the OF of ref. [9] when we include additional perturbations in the fluid approximation (1, 2 and 3).

Our solution predicts a smooth behaviour across the whole range of velocities with a plateau for $v_w \gtrsim 0.8$ and a linear behaviour for small velocities instead. We found a fair numerical agreement between the OF and the FS for $v_w \lesssim 0.2$. However, at higher velocities, the OF develops peaks corresponding to the zero eigenvalues of the Liouville operator pointing out an instability in the weighted strategy. Since in the FS such peaks are absent we conclude that they are an artifact of the OF meaning that no strong effect is present at the barrier of sound when only top contributions are taken into account.

The new formalism, instead, correctly predicts a smooth behaviour for the integrated friction. However, a good numerical agreement is found only for small velocities.

4. Conclusions and outlook

In [14] we presented for the first time the fully quantitative solution to the Boltzmann equation that describes particle diffusion in the presence of a moving wall. Differently from previous approaches, our solution does not rely on any particular ansatz nor momentum dependence. This allows one to solve exactly the Boltzmann equation. We compared our results with the ones already known in the literature , namely the OF [9], its extended version [12, 13] and the NF [11] and we showed that reliable quantitative results are obtained using our method in the whole range of propagating velocity.

This clearly represents a necessary step towards a reliable understanding of the bubble wall dynamics. The friction computed using the numerical method that we developed in [14] allows to compute the relevant quantities that determine the PhT dynamics, such as the terminal velocity and the wall thickness, by solving the Higgs equation of motion. These parameters crucially have an impact on the prospects of any BSM scenario to predict cosmological signals.

The inclusion of all the SM particles is clearly important to obtain quantitatively reliable predictions. In a future work [15] we plan to include electroweak gauge bosons and to treat the light degrees of freedom as a background plasma in local equilibrium as explained in [16].

PoS(ICHEP2022)080

References

[1] C. Caprini et al., *Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions*, *JCAP* **04** (2016) 001 [1512.06239].

[2] C. Caprini et al., *Detecting gravitational waves from cosmological phase transitions with LISA: an update*, *JCAP* **03** (2020) 024 [1910.13125].

[3] S. Kawamura et al., *The Japanese space gravitational wave antenna DECIGO*, *Class. Quant. Grav.* **23** (2006) S125.

[4] S. Kawamura et al., *The Japanese space gravitational wave antenna: DECIGO*, *Class. Quant. Grav.* **28** (2011) 094011.

[5] W.-R. Hu and Y.-L. Wu, *The Taiji Program in Space for gravitational wave physics and the nature of gravity*, *Natl. Sci. Rev.* **4** (2017) 685.

[6] W.-H. Ruan, Z.-K. Guo, R.-G. Cai and Y.-Z. Zhang, *Taiji program: Gravitational-wave sources*, *Int. J. Mod. Phys. A* **35** (2020) 2050075 [1807.09495].

[7] TianQin collaboration, J. Luo et al., *TianQin: a space-borne gravitational wave detector*, *Class. Quant. Grav.* **33** (2016) 035010 [1512.02076].

[8] G. D. Moore and T. Prokopec, *Bubble wall velocity in a first order electroweak phase transition*, *Phys. Rev. Lett.* **75** (1995) 777 [hep-ph/9503296].

[9] G. D. Moore and T. Prokopec, *How fast can the wall move? A Study of the electroweak phase transition dynamics*, *Phys. Rev. D* **52** (1995) 7182 [hep-ph/9506475].

[10] J. M. Cline and K. Kainulainen, *Electroweak baryogenesis at high bubble wall velocities*, *Phys. Rev. D* **101** (2020) 063525 [2001.00568].

[11] B. Laurent and J. M. Cline, *Fluid equations for fast-moving electroweak bubble walls*, *Phys. Rev. D* **102** (2020) 063516 [2007.10935].

[12] G. C. Dorsch, S. J. Huber and T. Konstandin, *On the wall velocity dependence of electroweak baryogenesis*, *JCAP* **08** (2021) 020 [2106.06547].

[13] G. C. Dorsch, S. J. Huber and T. Konstandin, *A sonic boom in bubble wall friction*, *JCAP* **04** (2022) no.04, 010 [2112.12548].

[14] S. De Curtis, L. Delle Rose, A. Guiggiani, Á. Gil Muyor and G. Panico, *Bubble wall dynamics at the electroweak phase transition*, *JHEP* **03** (2022), 163 [2201.08220].

[15] S. De Curtis, L. Delle Rose, A. Guiggiani, Á. Gil Muyor and G. Panico, *In preparation*.

[16] B. Laurent and J. M. Cline, *First principles determination of bubble wall velocity*, *Phys. Rev. D* **106** (2022) no.2, 023501 [2204.13120].