Parker Bound and Monopole Production from Primordial Magnetic Fields

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We present new bounds on the cosmic abundance of magnetic monopoles based on the survival of primordial magnetic fields during the reheating and radiation-dominated epochs. The primordial magnetic fields accelerate the monopoles and the process of monopole acceleration extracts energy from the fields. The energy that the monopoles extract from the primordial magnetic field is consequently transferred to the primordial plasma through scattering processes with relativistic charged particles of the plasma. With a monopole number density large enough, this can cause the disappearance of the field. The new bounds can be stronger than the conventional Parker bound from galactic magnetic fields, as well as bounds from direct searches. We also apply our bounds to monopoles produced by the primordial magnetic fields themselves through the Schwinger effect, and derive additional conditions for the survival of the primordial fields. The discussion on this paper is based on [1].
1. Monopole dynamics in primordial magnetic fields

By noting that a population of monopoles would short out the magnetic fields inside galaxies, Parker obtained upper bounds on the flux of monopoles [2]. We show that, depending on the early cosmological history, the survival of primordial magnetic fields imposes bounds on the monopole abundance that are more stringent than the Parker limit from galactic magnetic fields and the bound presented in [3].

The equation of motion of a monopole with charge \( g \) and mass \( m \) in a Friedmann-Robertson-Walker metric from the time of the primordial magnetic field generation, \( t_i \), to the epoch of \( e^+e^- \) annihilation is:

\[
m \frac{d}{dt} (\gamma v) = gB - (f_p + mH \gamma) v. \tag{1}
\]

Here \(-f_p v\) is the frictional force due to the interactions of the monopoles with the particles of the primordial plasma and:

\[
f_p \sim \frac{e^2 g^2 N_c T^2}{16\pi^2}, \tag{2}
\]

with \( N_c \) the number the charged relativistic degrees of freedom.

The evolution of the magnetic field energy density can be derived by solving the equation:

\[
\frac{\dot{\rho}_B}{\rho_B} = -\Pi_{\text{red}} - \Pi_{\text{acc}}, \tag{3}
\]

where we define the dissipation rates due to redshifting and monopole acceleration as:

\[
\Pi_{\text{red}}(t) = 4H(t), \quad \Pi_{\text{acc}}(t) = \frac{4g}{B(t)}v(t)n(t). \tag{4}
\]

If \( \Pi_{\text{acc}}/\Pi_{\text{red}} \ll 1 \), the magnetic fields completely lose their energy and eventually disappear. If \( \Pi_{\text{acc}}/\Pi_{\text{red}} \gg 1 \), the back-reaction of the monopoles on the magnetic fields is negligible and the fields simply redshift as \( B \propto a^{-2} \), where \( a \) is the scale factor.

2. Bounds from the survival of primordial magnetic fields

From the condition \( \Pi_{\text{acc}}/\Pi_{\text{red}} \ll 1 \), we obtain upper bounds on the average monopole number density in the present universe. From the evolution during radiation domination, the result is the same of [3]. From the evolution during reheating we obtain new bounds that depend on the temperature at the end of reheating, \( T_{\text{dom}} \). We assume thermal equilibrium for the particles of the plasma during the reheating epoch, but stronger bounds can be obtained without this assumption.

For a process of magnetogenesis that ends sufficiently in the past, the bound for the monopole flux today is:

\[
F_0 \lesssim \begin{cases} 
10^{-10} \text{cm}^{-2}\text{s}^{-1}\text{s}^{-1} \left( \frac{v_0}{10^{-3}} \right) \left( \frac{B_0}{10^{-15} \text{G}} \right)^{3/5} \left( \frac{T_{\text{dom}}}{10^6 \text{GeV}} \right) \left( \frac{10}{g} \right)^{3/5}, & m \ll \bar{m}, \\
10^{-10} \text{cm}^{-2}\text{s}^{-1}\text{s}^{-1} \left( \frac{v_0}{10^{-3}} \right) \left( \frac{m}{10^{14} \text{GeV}} \right) \left( \frac{T_{\text{dom}}}{10^6 \text{GeV}} \right) \left( \frac{10}{g} \right)^2, & m \gg \bar{m},
\end{cases} \tag{5}
\]
Figure 1: Upper bounds on the magnetic monopole flux today. Here \( g = 2\pi/e \), \( B_0 = 10^{-15} \text{ G} \) and \( v_0 = 10^{-3} \). Blue: bounds from primordial magnetic fields during reheating in Eq. (5), for reheating temperatures \( T_{\text{dom}} = 100 \text{ MeV} \) (solid curve), \( 100 \text{ GeV} \) (dashed curve), \( 10^5 \text{ GeV} \) (dotted curve). Red: bound from primordial magnetic fields during radiation domination derived in [3]. Orange: original Parker bound [2]. Pink: extended Parker bound [4]. Black: direct search limit from the MACRO experiment [5]. Dashed grey: cosmological abundance bound.

where \( \bar{m} \) is given by:

\[
\bar{m} \approx 10^{14} \text{ GeV} \left( \frac{B_0}{10^{-15} \text{ G}} \right)^{3/5} \left( \frac{g}{10} \right)^{7/5} \left( \frac{N_{c, \text{dom}}}{100} \right)^{2/5}.
\]  

Here \( B_0 \) is the amplitude of the intergalactic magnetic field today.

In Fig. 1 we show the previous bounds on the monopole flux together with our new results. For a sufficiently low reheating temperature, our new bound is stronger than the original Parker bound and the limits from direct searches, even for GUT-scale monopoles.

3. Monopoles produced by primordial magnetic fields

Even in the absence of any initial monopole population, in the early universe the magnetic fields could have been strong enough to produce monopole pairs through the magnetic dual of the Schwinger effect [6, 7]. Hence we also apply our generic bounds to such pair-produced monopoles, in order to obtain the most conservative condition for the survival of primordial magnetic fields.

The rate of monopole-antimonopole pair production at arbitrary coupling in a static magnetic field has been derived through an instanton method:

\[
\Gamma = \frac{(gB)^2}{(2\pi)^3} \exp \left[ -\frac{\pi m^2}{gB} + \frac{g^2}{4} \right].
\]  

In the case of strong coupling, \( g \gg 1 \), this result is valid under the following weak field condition:

\[
B \leq \frac{4\pi m^2}{g^3}.
\]
Figure 2: Lower bounds for the monopole mass as a function of the Hubble scale at magnetic field generation, comparison between the weak field condition (dashed curves) and the bound from requiring negligible back-reaction on the primordial magnetic fields (solid curves). Here $g = 2\pi/e$ and $B_0 = 10^{-15}$ G. The results are shown for three different values of the reheating temperature $T_{dom}$: red: $10^{15}$ GeV, orange: $10^9$ GeV, pink: $10^3$ GeV. (a) During radiation domination; (b) during reheating.

We found that the condition $\Pi_{acc}/\Pi_{red} \ll 1$ applied to the pair-produced monopoles reduce to Eq. (8) on the initial strength of the primordial magnetic fields. This translates into lower bounds on the monopole mass as a function of the Hubble scale at magnetic field generation $H_i$, as shown in Fig. 2.

The bounds we derive are comparable to those obtained in [8] for the survival of the primordial magnetic fields to the Schwinger-production of monopole pairs. Thus, as long as the initial amplitude of the primordial magnetic field is sufficiently below the bound of the weak field condition in Eq. (8), any back-reaction from Schwinger-produced monopoles can be safely ignored.

References