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Next-to-leading power corrections for soft photon bremsstrahlung

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A long-standing discrepancy in the soft photon bremsstrahlung has attracted a renewed attention in view of the proposed measurements with a future upgrade of the ALICE detector in the upcoming runs of the LHC. We discuss the possibility to implement techniques that have been recently developed for soft gluon resummation at Next-to-Leading-Power (NLP) to the soft photon spectrum. Specifically, we discuss two different corrections to the formula that has been used so far to compare with data.

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1. An experimental conundrum

Theoretical descriptions of soft photon emissions typically rely on the Leading Power (LP) eikonal approximation, where the photon momentum $k \to 0$. In this limit, the bremsstrahlung spectrum in a $2 \to n$ process is given by [1–3]

$$\frac{d\sigma_{\rm LP}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3 \cdots \int d^3p_n \left(\sum_{i,j=1}^{n} -\eta_i\eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}\right) d\sigma_H(p_1,\ldots,p_n)\,, \quad (1)$$

where ω_k is the photon energy, $\eta_i = \pm 1$ is equal (opposite) to the sign of the charge of the i-th particle in the final (initial) state, while $d\sigma_H$ denotes the non-radiative differential cross-section depending on the hard momenta p_i. The spectrum is governed by the eikonal factor in bracket, which is universal, insensitive to spin and recoil of the hard emitter and in agreement with classical power spectrum $\frac{d\sigma}{d\omega_k} \sim \frac{1}{\omega_k}$.

However, the LP formula in eq. (1) disagrees with data when final state hadrons are present [4–6]. In the light of planned future measurements with the ALICE detector [7, 8], it is therefore of primary importance to have reliable theoretical predictions and investigate corrections to eq. (1). Building on recent progress in threshold resummation (where the soft boson is an *undetected* gluon) [9–11], we study the impact of next-to-leading power (NLP) corrections to the strict $k \to 0$ limit in the case of a *detected* photon. Specifically, we consider two sets of NLP corrections to eq. (1).

2. NLP photons via LBK theorem

The first kind of corrections, first studied by Low, Burnett and Kroll (LBK) [1, 2], is valid only at the tree-level and consists in simply Taylor expanding the soft photon momentum at NLP. In the modern language of next-to-soft theorems [12], the result of this expansion yields the radiative amplitude $\mathcal{A}_{n+1}$ in terms of the non-radiative amplitude $\mathcal{A}_n$ and reads

$$\begin{aligned} &\mathcal{A}_{n+1}(k, p_1, \ldots, p_n) = (\mathcal{S}_{LP} + \mathcal{S}_{NLP})\, \mathcal{A}_n(p_1, \ldots, p_n)\,, \\ &\mathcal{S}_{LP} = \sum_{i=1}^{n} \eta_i \frac{\epsilon^*(k)\cdot p_i}{p_i \cdot k}\,, \qquad \mathcal{S}_{NLP} = \sum_{i=1}^{n} \eta_i \frac{\epsilon^*_\mu(k) k_\nu (S^{\mu\nu} + L^{\mu\nu})}{p_i \cdot k}\,. \end{aligned} \quad (2)$$

Note the coupling with the spin generator $S^{\mu\nu}$ and the orbital angular momentum generator $L^{\mu\nu}$. Hence, NLP soft emissions are sensitive to the spin and the recoil of the hard emitter.

Although the theorems in eq. (2) can be applied to a generic scattering process, the non-radiative amplitude $\mathcal{A}_n$ depends on momenta that violate momentum conservation at NLP. This feature becomes problematic for numerical implementations, where the momentum k is always finite. A possibility to bypass this problem has been proposed in [13] where, building on [14], the LBK theorem has been rewritten in terms of a shifted kinematics. Squaring and summing over polarizations, it reads in this formulation

$$|\mathcal{A}_{n+1}(p_1, \ldots, p_n, k)|^2 = \left(\sum_{i,j=1}^{n} -\eta_i\eta_j \frac{p_i \cdot p_j}{p_i \cdot k\; p_j \cdot k}\right) |\mathcal{A}_n(p_1 + \delta p_1, \ldots, p_n + \delta p_n)|^2\,, \quad (3)$$

where the shifts δp_i are defined as

$$\delta p_\ell^\mu = \left(\sum_{i,j=1}^{n} \eta_i \eta_j \frac{p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k} \right)^{-1} \sum_{m=1}^{n} \left(\eta_m \eta_\ell \frac{(p_m)_\nu G_\ell^{\mu\nu}}{p_m \cdot k} \right) , \tag{4}$$

and

$$G_i^{\mu\nu} = g^{\mu\nu} - \frac{(2p_i - k)^\mu k^\nu}{2p_i \cdot k} = g^{\mu\nu} - \frac{p_i^\mu k^\nu}{p_i \cdot k} + O(k) . \tag{5}$$

Note that the eikonal factor in eq. (3) is the same as in eq. (1) and therefore eq. (3) can be analogously implemented in the bremsstrahlung cross section, thus providing a correction of order $(\omega_k)^0$ to eq. (1).

3. NLP photons with QCD loop corrections

The second type of corrections is due to the fact that NLP soft theorems receive loop corrections. Virtual collinear effects are captured by radiative jet functions J^μ [15]. In particular, for a process with a single quark-antiquark pair in the massless limit (such as $e^+e^- \to q\bar{q}\gamma$), the leading QCD correction to the LBK contribution comes from the quark radiative jet, which in dimensional regularization (with $d = 4 - 2\epsilon$, $\bar{\mu}$ the $\overline{\text{MS}}$ scale and n^μ a light-like reference vector) reads [16]

$$\begin{aligned} J^{\mu(1)}(p,n,k) = \left(\frac{\bar{\mu}^2}{2p \cdot k}\right)^{\epsilon} \Bigg[& \left(\frac{2}{\epsilon} + 4 + 8\epsilon\right) \left(\frac{n \cdot k}{p \cdot k} \frac{p^\mu}{p \cdot n} - \frac{n^\mu}{p \cdot n} \right) - (1 + 2\epsilon) \frac{i k_\alpha S^{\alpha\mu}}{p \cdot k} \\ & + \left(\frac{1}{\epsilon} - \frac{1}{2} - 3\epsilon \right) \frac{k^\mu}{p \cdot k} + (1 + 3\epsilon) \left(\frac{\gamma^\mu \not{n}}{p \cdot n} - \frac{p^\mu}{p \cdot k} \frac{\not{k} \not{n}}{p \cdot n} \right) \Bigg] + O(\epsilon^2, k) . \end{aligned} \tag{6}$$

Correspondingly, the next-to-soft theorem of eq. (3) receives a logarithmic correction, given by

$$\left(\sum_i \epsilon_\mu^*(k) \, q_i \, J_i^{\mu(1)} \right) \mathcal{A}_n = \frac{2}{p_1 \cdot p_2} \left[\sum_{ij} \left(\frac{1}{\epsilon} + \log\left(\frac{\bar{\mu}^2}{2p_i \cdot k} \right) \right) q_j \, p_i \cdot k \, \frac{p_j \cdot \epsilon}{p_j \cdot k} \right] \mathcal{A}_n , \tag{7}$$

with q_i the electric charge of the quark. The soft photon bremsstrahlung at $O(\alpha_s)$ then becomes

$$\frac{d\sigma}{d^3k} = \frac{d\sigma_{\text{NLP-tree}}}{d^3k} + \frac{\alpha_s}{4\pi} \frac{d\sigma_{\text{NLP-}J}}{d^3k} , \tag{8}$$

where $\frac{d\sigma_{\text{NLP-tree}}}{d^3k}$ is obtained from $|\mathcal{A}_{n+1}|^2$ in eq. (3) and

$$\frac{d\sigma_{\text{NLP-}J}}{d^3k} = \frac{\alpha}{(2\pi)^2} \frac{1}{\omega_k} \int d^3p_3 \cdots \int d^3p_n \left(\sum_{i=1}^{n} \eta_i \frac{8 \log\left(\frac{\bar{\mu}^2}{2p_i \cdot k} \right)}{p_i \cdot k} \right) d\sigma_H(p_1, \ldots, p_n) . \tag{9}$$

This result provides a correction of order $\alpha_s \log\left(\frac{\bar{\mu}^2}{2p_i \cdot k}\right)$ to eq. (1), and as such it is particularly enhanced for small ω_k and small k_t.

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