

## Nuclear coalescence, collective behaviour and emission volume in small interacting systems

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The production of light nuclei and antinuclei in particle collisions can be described as the coalescence of final state nucleons that are close in phase space. In heavy ion collisions, it is usually assumed that the formation probability is controlled by the size of the interaction region, while nucleon momentum correlations are either neglected or treated as a collective effect. Interestingly, recent experimental data on nucleus and hadron production in  $pp$  collisions at LHC show evidence for such collective behaviour. Here, however, we argue that such data are naturally explained using QCD inspired event generators if both nucleon momentum correlations and the size of the emission volume of nucleons are considered. In order to consider both effects simultaneously, we employ a per-event coalescence model based on the Wigner function representation of the nucleus state. The model predicts the size and  $p_T$  dependence of the source volume measured at LHC, and it has therefore no free parameters. Finally, we comment on the validity of the underlying assumptions of the femtoscopy framework in small interacting systems and its relation to nuclear coalescence.

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## 1. Motivation

Light (anti-)nuclei, like (anti)deuteron, (anti)helion and (anti)triton, are sensitive probes for the QCD phase diagram due to their composite structure and small binding energies. At the same time, light antinuclei are of immense interest for the astroparticle community since they are ideal probes for new and exotic physics. In order to correctly interpret experimental results, a solid description of the formation process is needed.

## 2. Coalescence models

In the *coalescence model*, final state nucleons merge into a nucleus if they are sufficiently close in *phase space*. Traditionally, the yield was parametrised as

$$E_A \frac{d^3 N_A}{dP_A^3} = B_A \left( E_p \frac{d^3 N_p}{dP_p^3} \right)^Z \left( E_n \frac{d^3 N_n}{dP_n^3} \right)^N \Bigg|_{P_p=P_n=P_A/A}, \quad (1)$$

where  $B_A$  is known as the *coalescence factor*. In small interacting systems (e.g.  $e^+e^-$ ,  $pp$ , and dark matter), it is common to evaluate the coalescence condition in momentum space, in which case the coalescence factor scales with the *coalescence momentum* as  $B_A \propto p_0^{3(A-1)}$ . Meanwhile, in large interacting systems, only the emission volume is considered, implying that the coalescence momentum scales with the emission volume as  $B_A \propto V^{A-1}$ . These two limiting cases are, however, expected to be inaccurate, as we will demonstrate in the next section.

## 3. Time-scales

In the coalescence model, (anti)nuclei are produced by final state nucleons that have (nearly) completed their formation. This means that the largest time and distance scales of the problem in small systems are related to the hadronisation length,  $L_{\text{had}} \simeq \gamma L_0$ ,  $L_0 \sim R_p \simeq 1$  fm. Thus, one expects an emission length of  $\sigma_{(\text{point-like})} \sim 1$  fm even in point-like interactions. In extended processes, the emission length has an additional geometrical contribution from multiple parton-parton interactions:  $\sigma_{(\text{geom})} \sim R_N \sim$  fm. For example, in the particular case of  $pp$  and  $e^+e^-$  collisions, the transverse and longitudinal emission lengths are of the same order, and  $\sigma \equiv \sigma_{e^\pm} \simeq \sigma_{pp}/\sqrt{2} \simeq 1$  fm. Since the size of the deuteron, triton and helion wave functions are around the same size as the emission length,  $r_{\text{rms}}^d \sim 2$  fm, the size of the formation region and momentum correlations give both a sizeable contribution to the coalescence probability.

## 4. The WiFunC model

Momentum correlations and the emission volume are considered simultaneously in the WiFunC (Wigner Functions with Correlations) model [1, 2]. Here, the (anti)nucleus spectrum is found by projecting the nucleon density matrix onto the nucleus density matrix and assuming a Gaussian distribution for the nucleon emission. The (anti)deuteron yield can then be written as

$$\frac{d^3 N_d}{dP_d^3} = \frac{3\zeta}{(2\pi)^3} \int dq e^{-q^2 d^2} G(q, -q), \quad (2)$$

where

$$\zeta = \left( \frac{d^2}{d^2 + 4\sigma^2 m_T^2 / m^2} \right)^{1/2} \frac{d^2}{d^2 + 4\sigma^2}. \quad (3)$$

The model can be added to any Monte Carlo event generator by using a weight  $w = 3\Delta\zeta(d_1)e^{-d_1^2q^2} + 3(1-\Delta)\zeta(d_2)e^{-d_2^2q^2}$ , where  $\Delta = 0.581$ ,  $d_1 = 3.979$  fm,  $d_2 = 0.890$  fm are fixed by fitting a two-Gaussian wave function to the Hulthen wave function describing the deuteron. A similar expression has been derived for (anti)helion and (anti)triton. This model reproduces well various experimental data on antinucleus production in  $e^+e^-$ ,  $pp$  and  $pN$  collisions [1–4].

## 5. Space-time correlations and going beyond the equal-time approximation

Some event generators have implemented a (semi) classical description of the space-time evolution of the cascade. One can thus in principle take into account space-time correlations by evaluating

$$\frac{d^3N_d}{dP_d^3} = \frac{3}{8(2\pi)^3} \int d^3q d^3r \mathcal{D}^{(3)}(\vec{q}, \vec{r}) W_{np}(\vec{q}, \vec{r}), \quad (4)$$

where  $\mathcal{D}^{(3)}(\vec{q}, \vec{r})$  is the deuteron Wigner function. Even if it is trivial to compute  $\mathcal{D}$  numerically for any given wave function, one should use either a Gaussian or a two-Gaussian wave function [1], because a different choice will necessarily lead to negative coalescence probabilities [5].

The framework underlying Eq. (4) relies on the equal-time approximation. That is, it is assumed that the nucleons are produced at the same time. More concretely, it is assumed that  $q \ll m\sigma/t \sim$  GeV [6]. Since the bulk of (anti)nuclei are produced by nucleons with  $q \sim \mathcal{O}(0.1)$  GeV, this approximation may lead to an uncertainty of around 10–20 % (see also the comment in Ref. [7]). By using the methodology described in Ref. [6], one can show that the effect of non-equal emission times on Eq. (4) is simply the substitution [8]

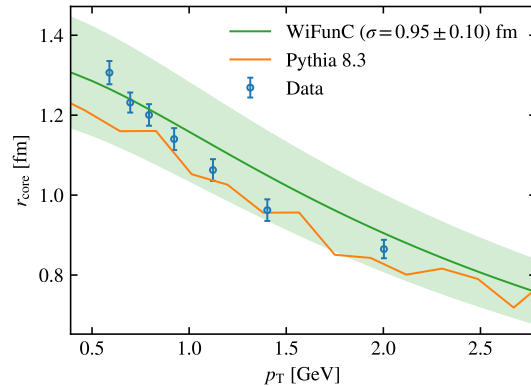
$$\mathcal{D}^{(3)}(\vec{q}, \vec{r}) \rightarrow \mathcal{D}^{(3)}(\vec{q}, \vec{r} + \vec{q}t/m_p), \quad (5)$$

i.e. the nucleons move classically in the pair rest frame and the interaction occurs when both of them have been produced. When Eq. (5) is used, only the Gaussian wave function should be used for the deuteron, since the antisymmetric part that arises when using the two-Gaussian wave function no longer drops out.

It is important to emphasize that QCD inspired event generators are considering the cascade in momentum space, and that one cannot know the position and momentum of a particle simultaneously. Hence, the space-time picture should not be interpreted as anything but a representation of the probability distribution of the position. The method has, however, some clear advantages compared to the semi-classical ansatz underlying Eq. (2): (1) the dependence on collision parameters such as impact parameter, multiplicity, energy and collision type are trivial to consider, (2) one can go beyond the equal-time approximation, (3) the Lorentz boost in Eq. (3) does not have to be defined relative to the beam axis.

## 6. Femtoscopy experiments and emission volume

The emission size  $\sigma$  can be directly measured in femtoscopy experiments. In Fig. 1, the emission size measured by the ALICE collaboration in  $pp$  collisions at 13 TeV is compared to the prediction by the WiFunC model [4]. The agreement between the experimental data and the prediction is a strong validation of the basic assumptions of the model. Moreover, it implies that the WiFunC model contains no free parameters.



**Figure 1:** The predicted source size  $r_{\text{core}}$  by the WiFunC model is compared to the experimental data by the ALICE collaboration. The blue line is obtained using the ansatz in Eq. (2) and red line using Eq. (4) and Pythia.

In addition, we plot the emission volume predicted by Pythia 8 [9]: Due to the equal-time approximation, femtoscopy experiments measure the distance between the nucleons in the *lab* frame, *length contracted* in the pair rest frame. One should note that there exists currently no “official tune” for the space-time picture in Pythia. Thus, femtoscopy experiments and (anti)deuteron measurements can in principle be used to tune the parameters.

The steepening at large  $p_T$  which increases with multiplicity is in the WiFunC model and Pythia explained by a combination of the non-trivial source function and two-particle correlations.

## 7. Summary

Both momentum correlations and the emission volume have to be taken into account when describing the production of (anti)nuclei in small interacting system. This can be achieved using the WiFunC model, which has proven to describe well a variety experimental data. Using the same approach, one can alternatively use an event generator to describe the emission size on an event-by-event basis, in which case one can go beyond the equal-time approximation.

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