Study of the heavy bottom baryons in a potential Model

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The singly bottom baryons are studied in the framework of a nonrelativistic quark model. We use the hypercentral approach to solve the six-dimensional Schrödinger equation of the baryons. Introducing a potential model, the ground state masses of the single-heavy $\Lambda$ and $\Sigma$ baryons are calculated. We investigate the $b \to c$ semileptonic decay width of the $\Lambda_b$ heavy baryon. Our results are in agreement with the available experimental data and those of other works.
1. Introduction

During the last few years many theoretical progresses have been achieved in spectroscopy [1–4] and semileptonic decays [5–8] of single and double-heavy baryons. The purpose of the present work is to give a description of the properties of heavy baryons containing one heavy quark within the framework of a non-relativistic quark model. Firstly, we calculate the mass spectra of the ground states of single-heavy $\Lambda$ and $\Sigma$ baryons states, in the hypercentral constituent quark model. Then, we focus on the $b \rightarrow c$ semileptonic transition of the $\Lambda_b$ baryon and evaluate the decay rate in the heavy quark limit. This paper is organized as follows. In Section 2 introducing our potential model, we present our predictions for the ground states (non-strange) single-heavy baryons. In Section 3 we introduce a universal function as the transition form factor to evaluate the semileptonic decay width of the $\Lambda_b$ baryon near to the zero recoil point. Section 4 includes conclusions.

2. Baryon masses

In order to calculate the baryon spectra we need to obtain the energy of the system. Using a phenomenological potential model we solve the three-body equation of the baryon system in a non-relativistic framework. In the hypercentral approach the hyperradial wave function $\psi_{\nu, \gamma}$ is obtained as a solution of the hyperradial equation [9–11]

$$\left[ \frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma (\gamma + 4)}{x^2} \right] \psi_{\nu, \gamma}(x) = -2\mu (E_{\nu, \gamma} - V(x)) \psi_{\nu, \gamma}(x),$$

where $x$ and $\mu$ are the hyperradius and reduced mass respectively. In the present model $\mu$ is taken as a free parameter. $\nu$ determines the number of the nodes of the wave function and $\gamma$ is the grand-angular quantum number. For the ground-state heavy baryons we have $\gamma = \nu = 0$. The non-perturbative potential used in our model is the Coulomb-plus-linear potential as $V(x) = ax - \frac{c}{x}$, where the coefficients of the confining and color-Coulombic parts $a$ and $c$ are constant. We use a Gaussian form of the spin and isospin hyperfine interactions [10, 11]. The quark masses used in our model are as $m_u = m_d = 330 \text{MeV}$, $m_c = 1650 \text{MeV}$ and $m_b = 4995 \text{MeV}$. The potential parameters $a = 1.61 \text{ fm}^{-2}$, $c = 4.59$ and $\mu = 0.844 \text{ fm}^{-1}$ are taken from our previous works [11, 12]. Here, we solve the Schrödinger equation numerically (for details see Refs. [10, 11]). Baryon mass is obtained by sum of the quark masses, energy eigenvalues and perturbative hyperfine interactions. The obtained masses for the single-heavy baryons are listed in table 1 and compared with the experimental data [13].

3. Semileptonic decay of $\Lambda_b$ heavy baryon

In the heavy-quark limit, all the form factors describing the semileptonic decays are proportional to the universal function $\zeta(\omega)$ known as the Isgur-Wise (IW) function. The form factors are parametrized in different approaches. In the heavy quark symmetry limit and close to zero recoil point, the expressions for the decay rates can be simplified and the weak transition form factors between single-heavy baryons can be expressed by a single universal Isgur-Wise (IW) function $\xi(\omega)$. To consider the $\Lambda_b \rightarrow \Lambda_c$ semileptonic transition, we use the following form of the IW function [5, 15]

$$\xi(\omega) = 0.99e^{-1.3(\omega-1)}$$

(2)
Table 1: Masses of single-heavy $\Lambda$ and $\Sigma$ baryons (in MeV).

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$\Lambda_c$</th>
<th>$\Sigma_c$</th>
<th>$\Sigma^*_c$</th>
<th>$\Lambda_b$</th>
<th>$\Sigma_b$</th>
<th>$\Sigma^*_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ours</td>
<td>2286</td>
<td>2457</td>
<td>2503</td>
<td>5631</td>
<td>5803</td>
<td>5847</td>
</tr>
<tr>
<td>Exp</td>
<td>2286</td>
<td>2453</td>
<td>2518</td>
<td>5619</td>
<td>5815</td>
<td>5832</td>
</tr>
</tbody>
</table>

The differential decay width has the form [5, 6]

$$\frac{d\Gamma}{d\omega} = \frac{2}{3}m^4_{\Lambda_c} m_{\Lambda_b} A \xi^2(\omega) \sqrt{\omega^2 - 1} \left[3\omega(\eta + \eta^{-1}) - 2 - 4\omega^2\right],$$

(3)

where $\eta = \frac{m_{\Lambda_b}}{m_{\Lambda_c}}$ and

$$A = \frac{G_F}{(2\pi)^3} |V_{cb}|^2 Br(\Lambda_c \rightarrow ab).$$

(4)

The parameter $G_F$ is the Fermi Coupling constant. $V_{bc}$ is the CKM matrix element and its value is $V_{bc} = 0.04$. $m_{\Lambda_b}$ and $m_{\Lambda_c}$ are the masses of the initial $\Lambda_b$ and final $\Lambda_c$ baryons, respectively. $Br(\Lambda_c \rightarrow ab)$ is the branching ratio through which $\Lambda_c$ is detected.

For the considered universal function $\xi(\omega)$ we get the slope as

$$\rho^2 = -\left.\frac{d\xi}{d\omega}\right|_{\omega=1} = 1.28,$$

(5)

Using the obtained masses listed in table 1 and equations. 2-4, the semileptonic decay rate of the heavy $\Lambda_b$ baryon is calculated as

$$\Gamma = \int_{\omega=1}^{\omega=\omega_{max}} \frac{2}{3}m^4_{\Lambda_c} m_{\Lambda_b} A \xi^2(\omega) \sqrt{\omega^2 - 1} \left[3\omega(\eta + \eta^{-1}) - 2 - 4\omega^2\right] d\omega,$$

(6)

where

$$\omega_{max} = \frac{m^2_{\Lambda_b} + m^2_{\Lambda_c}}{2m_{\Lambda_b} m_{\Lambda_c}}.$$  

(7)

In Fig. 1 we plot the variation of the differential decay width for the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ semileptonic transition, where $\ell$ stands for a light lepton, $\ell = e, \mu$. For the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ transition, we obtain $\omega_{max}[\Lambda_b \rightarrow \Lambda_c] = 1.43$. Using the obtained baryon masses we get $\Gamma = 8.92 \times Br(\Lambda_c \rightarrow ab) \times 10^{10} s^{-1}$ which is consistent with the value $\Gamma = 8.32 \times Br(\Lambda_c \rightarrow ab) \times 10^{10} s^{-1}$ reported by Ref. [14]. Guo and Meta have also reported $\Gamma = 5.74 \times Br(\Lambda_c \rightarrow ab) \times 10^{10} s^{-1}$ [15].

4. Conclusions

We have developed a non-relativistic quark model to study the single-bottom baryons. Using a phenomenological potential model, we have obtained the masses of the non-strange single-heavy baryons and then, using the evaluated masses the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ semileptonic transition was studied. Working in the heavy quark limit and close to zero recoil point, the form factors of the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ transition were simplified to a unique universal IW function and finally, the semileptonic decay rate is calculated.
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Figure 1: The variation of differential decay rate for the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ semileptonic decay.

References