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Grover's Quantum Search Algorithm of Causal Multiloop Feynman Integrals.

Andrés E. Rentería-Olivo[a,*]

[a] *Instituto de Física Corpuscular,*
Universitat de València,
Consejo Superior de Investigaciones Científicas,
Parc Científic, E-46980 Paterna, Valencia, Spain.

E-mail: andres.renteria@ific.uv.es

A proof-of-concept application of a quantum algorithm to multiloop Feynman integrals in the Loop-Tree Duality (LTD) framework is applied to a representative four-loop topology. Causality obtained through the LTD formalism, is a suitable problem to address with quantum computers given the straightforward possibility to encode the two on-shell states of a propagator on the two states of a qubit. A modification of Grover's quantum search algorithm is developed for querying multiple solutions over the unstructured set of solutions generated after the application of the LTD. The quantum algorithm is successfully implemented on IBM Quantum and QUTE simulators.

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1. Introduction

Computing high-precision theoretical predictions for current and future high-energy particle colliders requires novel techniques to deal with the higher orders in perturbative theory of scattering amplitudes. In this context, the loop-tree duality (LTD) [1–3] formalism exhibits interesting mathematical properties capable of overcoming current limitations. A remarkable property of LTD is the possibility of representing the causal nature of Feynman diagrams and scattering amplitudes, leading to an intuitive understanding of the singular structure of loop integrals. This manifestly causal representation allows to represent the original multiloop Feynman diagrams into a class of multiloop topologies defined by collapsing propagators into edges where the two on-shell states of these propagators can be naturally encoded by the two states of a qubit, leading to explore the application of quantum algorithms, for instance, a Grover's quantum search algorithm [4] and a Variational Quantum Eigensolver [5] approach. In this work we focus on the former approach.

2. Loop-Tree Duality and Causality

The causal representation of scattering amplitudes in the LTD formalism is obtained through the calculation of nested residues. In Refs. [3, 6] it is shown that scattering amplitudes can be written as

$$\mathcal{A}_D^{(L)} = \int_{\vec{\ell}_1 \ldots \vec{\ell}_L} \frac{1}{x_n} \sum_{\sigma \in \Sigma} \mathcal{N}_\sigma \prod_{i=1}^{n-L} \frac{1}{\lambda_{\sigma(i)}^{h_{\sigma(i)}}} + (\lambda^+ \leftrightarrow \lambda^-) \,, \tag{1}$$

with $x_n = \prod_i 2q_{i,0}^{(+)}$, $h_{\sigma(i)} = \pm 1$, $\mathcal{N}_\sigma$ a numerator determined by the interaction vertices of a specific theory and $\int_{\vec{\ell}_s} = -\mu^{4-d}(2\pi)^{1-d} \int \mathrm{d}^{d-1}\ell_s$, the integration measure in the loop three-momentum space. Eq. (1) only involves denominators with positive on-shell energies $q_{i,0}^{(+)} = (\vec{q}_i^{\,2} + m_i^2 - \imath 0)^{1/2}$, added together in same-sign combinations in the so-called causal propagators defined as

$$\lambda_{\sigma(i)}^{\pm} \equiv \lambda_p^{\pm} = \sum_{i \in p} q_{i,0}^{(+)} \pm k_{p,0} \,, \tag{2}$$

where $\sigma(i)$ stands for the partition p of the set of on-shell energies and the orientation of the energy components of the external momenta, $k_{p,0}$. The causal structure of $\lambda_p^{\pm}$ is defined by the sign of $k_{p,0}$ when the propagators in p are set on-shell. Each causal propagator is in a one-to-one correspondence with any possible threshold singularity of the amplitude, which contains overlapped thresholds that are known as causal entangled thresholds. The combinations of entangled causal propagators represent causal thresholds that can occur simultaneously, which are collected in the set Σ.

3. Causal query of multiloop Feynman integrals: the four-eloop case

We develop a modified version [4] of Grover's algorithm [7] for querying the so-called causal configurations from the LTD framework. We take as example the four-eloop topology, which consists in four loops made of edges (eloops) with a central four-interaction vertex and we implement it on IBM Quantum*. The implementation of this quantum algorithm requires four registers, namely,

*https://quantum-computing.ibm.com/

$|e\rangle$, $|c\rangle$, $|a\rangle$ and $|out\rangle$. In the first register, $|e\rangle$, we encode the state of the $N = 2^n$ edges of the topology and initialise it with a uniform superposition by applying Hadamard gates, $|e\rangle = H^{\otimes n}|0\rangle$. The second register, $|c\rangle$, encodes the comparison between two adjacent edges e_i and e_j in binary Boolean clauses defined by

$$c_{ij} \equiv (e_i = e_j) \qquad \text{and} \qquad \bar{c}_{ij} \equiv (e_i \neq e_j) \ . \tag{3}$$

These clauses compare the state of the adjacent edjes e_i and e_j; c_{ij} is required when the edjes are in the same state and $\bar{c}_{ij}$ otherwise. Each clause $\bar{c}_{ij}$ is implemented through two CNOT gates, each taking as control the corresponding qubits e_i and e_j respectively, and both taking as target the same qubit in the register $|c\rangle$. The clause c_{ij} requires an extra NOT gate on the target qubit on the register $|c\rangle$. The third register, $|a\rangle$, stores the eloop clauses that probe the adjacent edges that compose a cyclic circuit within the diagram. Specifically for the four-eloop topology, taking the conventional orientation of the edges in Fig. (1), the clauses are given by

$$a_0^{(4)} = \neg(c_{01} \wedge c_{12} \wedge c_{23}) \ , \qquad a_1^{(4)} = \neg(\bar{c}_{05} \wedge \bar{c}_{45}) \ , \qquad a_2^{(4)} = \neg(\bar{c}_{16} \wedge \bar{c}_{56}) \ , \tag{4}$$

$$a_3^{(4)} = \neg(\bar{c}_{27} \wedge \bar{c}_{67}) \ , \qquad a_4^{(4)} = \neg(\bar{c}_{34} \wedge \bar{c}_{47}) \ . \tag{5}$$

Each $a_i^{(4)}$ requires a multicontrolled Toffoli gate that takes as control its specific qubits $|c_{ij}\rangle$ and as target a qubit in the register $|a\rangle$ followed by a NOT gate. The last register, $|out\rangle$, requires a single qubit initialised in the $|-\rangle = X|0\rangle$ and is used as the Grover's marker. It stores the output of the quantum algorithm and is implemented through a multicontrolled Toffoli gate taking as control all the qubits from the register $|a\rangle$ and, if required, a qubit from the register $|e\rangle$. The oracle is defined as

$$U_w|e\rangle|c\rangle|a\rangle|out\rangle = (-1)^{f(a,e)}|e\rangle|c\rangle|a\rangle|out\rangle \ . \tag{6}$$

When the causal conditions are satisfied, $f(a,e) = 1$, and marks the corresponding states; otherwise, if $f(a,e) = 0$, they are left unchanged. For the four-eloop, the required Boolean marker is given by

$$f^{(4)}(a,e) = \left(\bigwedge_{i=0}^{4} a_i^{(4)} \right) \wedge e_0 \ . \tag{7}$$

After storing the marked states in $|out\rangle$, the registers $|a\rangle$ and $|c\rangle$ are rotated back to the state $|0\rangle$ by applying the oracle operations in inverse order. The last step in the algorithm is the amplification of the marked states by applying the diffuser opertator on the register $|e\rangle$. We use the diffuser defined on IBM Quantum[†].

4. Conclusions

The quantum algorithm developed in this work allowed us to successfully identify the causal singular configurations of the four-eloop topology in the LTD framework. The algorithm was successfully implemented in IBM Quantum and QUTE simulators[‡]. The result of the quantum algorithm is used to bootstrap the causal representation in LTD of representative multiloop topologies, allowing us a better understanding of this approach and enabling us to explore other techniques, such as a Variation Quantum approach, as well as finding that beyond particle physics, the identification of directed acyclic graphs is a challenging problem.

[†] https://qiskit.org

[‡] https://qute.ctic.es

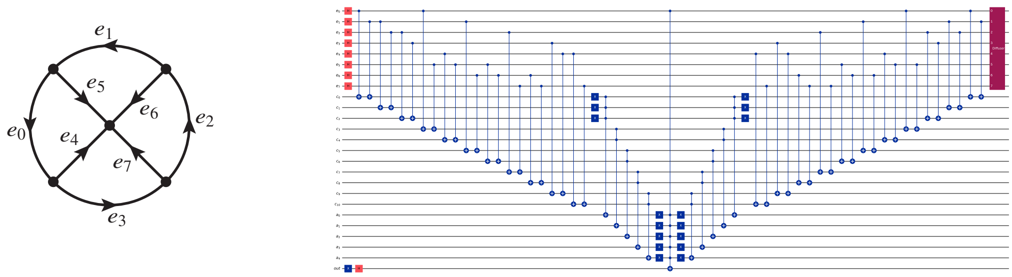

Figure 1: Left: Four-eloop topology with a fixed conventional orientation. Right: corresponding quantum circuit for the four-eloop topology.

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References

[1] S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J.-C. Winter, *From loops to trees by-passing Feynman's theorem*, *JHEP* **09** (2008) 065, [0804.3170].

[2] E. T. Tomboulis, *Causality and Unitarity via the Tree-Loop Duality Relation*, *JHEP* **05** (2017) 148, [1701.07052].

[3] J. de Jesús Aguilera-Verdugo et al., *A Stroll through the Loop-Tree Duality*, *Symmetry* **13** (2021) 1029, [2104.14621].

[4] S. Ramírez-Uribe, A. E. Rentería-Olivo, G. Rodrigo, G. F. R. Sborlini and L. Vale Silva, *Quantum algorithm for Feynman loop integrals*, *JHEP* **05** (2022) 100, [2105.08703].

[5] G. Clemente, A. Crippa, K. Jansen, S. Ramírez-Uribe, A. E. Rentería-Olivo, G. Rodrigo et al., *Variational quantum eigensolver for causal loop Feynman diagrams and acyclic directed graphs*, 2210.13240.

[6] J. J. Aguilera-Verdugo, F. Driencourt-Mangin, R. J. Hernández-Pinto, J. Plenter, S. Ramirez-Uribe, A. E. Renteria Olivo et al., *Open Loop Amplitudes and Causality to All Orders and Powers from the Loop-Tree Duality*, *Phys. Rev. Lett.* **124** (2020) 211602, [2001.03564].

[7] L. K. Grover, *Quantum mechanics helps in searching for a needle in a haystack*, *Phys. Rev. Lett.* **79** (1997) 325–328, [quant-ph/9706033].