PROCEEDINGS OF SCIENCE

Finite system size correction in ϕ^4 theory NLO scattering using denominator regularization

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By first motivating the use of the newly developed denominator regularization we show how using it can be used to calculate the $2 \to 2$ scattering amplitude in a finite sized massive ϕ^4 theory. We consider a spacetime with periodic boundary conditions and asymmetric sizes of the periodic dimensions. By using denominator regularization we then derive the scattering amplitude which we show is consistent with the optical theorem and infinite volume limit. This showcases one of the strengths of denominator regularization over standard techniques such as dimensional regularization.

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1. Introduction

There are many questions surrounding the recent relativistic hydrodynamic calculations on the apparent formation of QGP in heavy-ion collisions at the LHC and RHIC [1–3]. This calculation uses lattice QCD methods to extrapolate the calculation of the equation of state to infinite volumes [4]. Crucially it is unknown whether the finite system size induced by the size of collided nuclei in heavy-ion collisions induces non-negligible contributions to the trace anomaly, and therefore equation of state, of QCD above its phase transition. In fact quenched lattice QCD calculations have suggested the possibility of the existence of significant such contributions [5]. The work shown here is a crucial first step in the direction of calculating these corrections by considering a finite sized scalar theory.

2. From dimensional to denominator regularization

When regularizing the divergent loop integral of the form $\int \frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2+\Delta^2)^2}$ we can employ dimensional regularization as used in [6] to analytically continue the number of dimensions to $d = 4 - 2\epsilon$ in order to isolate the divergence. We can however consider the following seemingly arbitrary manipulation:

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{\mu^{4-d}}{(\ell^2+\Delta^2)^2} = \int \frac{d^4\ell}{(2\pi)^4}\int \frac{d^{-2\epsilon}\ell_e}{(2\pi)^{-2\epsilon}}\frac{\mu^{2\epsilon}}{(\ell_e^2+\ell^2+\Delta^2)^2} \tag{1}$$

$$= \int \frac{d^4\ell}{(2\pi)^4}\int \frac{dr}{(2\pi)^{-2\epsilon}}\frac{2\pi^{-\epsilon}r^{-2\epsilon-1}}{\Gamma(-\epsilon)}\frac{\mu^{2\epsilon}}{(r^2+\ell^2+\Delta^2)^2} \tag{2}$$

$$= \int \frac{d^4\ell}{(2\pi)^4}\frac{(4\pi)^{\epsilon}\Gamma(2+\epsilon)\mu^{2\epsilon}}{(\ell^2+\Delta^2)^{2+\epsilon}} \tag{3}$$

suggesting that instead of analytically continuing the number of spatial dimensions, one can instead analytically continue the exponent of the denominator and introduce an appropriate function of ϵ that goes to 1 if one takes $\epsilon \to 0$ and carries the correct units. Indeed such a general new regularization scheme is not only possible, but has several desirable properties and advantages as discussed in [7]. The main advantage of denominator regularization we are after here is the easy treatment of non-symmetric compact spacetime dimensions.

3. Finite sized ϕ^4 theory

Let us consider the massive scalar ϕ^4 Lagrangian $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$ in an $n+1$ dimensional spacetime where we enforce periodic spatial boundary conditions, where the i^{th} dimension has size $[-\pi L_i, \pi L_i]$. Here different L_i don't have to be equal, and some may be infinite to characterize infinite spatial dimensions. We want to calculate the finite system size corrections to $2 \to 2$ NLO scattering. To this end we define $(-i\lambda)^2 iV(p^2) \equiv$  as usual [6], which gives us the scattering amplitude $\mathcal{M} = \lambda\left[1 + \lambda(\overline{V}(s) + \overline{V}(t) + \overline{V}(u))\right]$ in terms of the Mandelstam variables s, t, u and the renormalized loop "integral". Following [8] by setting $n = 3$ we find that we need to regularize the UV divergent sum

$$\sum_{\vec{k}\in\mathbb{Z}^3}\left[\sum_{i=1}^{3}(\frac{k_i}{L_i}+xp^i)^2+\Delta^2\right]^{-\frac{3}{2}} \tag{4}$$

,where $\Delta^2 = -x(1-x)p^2 + m^2 - i\varepsilon$. By analytically continuing the exponent of the denominator $\frac{3}{2} \to \frac{3}{2} + \epsilon$, we can find that our sum is in the form of a generalized Epstein zeta function [9]

$$\zeta(\{a_i\},\{b_i\},c;s) \equiv \sum_{\vec{k}\in\mathbb{Z}^p}[a_i^2k_i^2+b_ik_i+c]^{-s} \tag{5}$$

where repeat indices are summed over. Using the analytic continuation of the generalized Epstein zeta function as derived in [8] allows us to isolate the divergence and find the renormalized

$$\overline{V}(p^2) = -\frac{1}{2}\frac{1}{(4\pi)^2}\int_0^1 dx\left[\ln\frac{\mu^2}{\Delta^2}+2\sum_{\vec{k}\in\mathbb{Z}^3}{}' \cos\left(2\pi x\sum k_ip^iL_i\right)K_0\left(2\pi\sqrt{\Delta^2\sum(k_iL_i)^2}\right)\right] \tag{6}$$

where $\sum'$ denotes the exclusion of the zero vector from the sum and K_ν is the usual modified Bessel function of the second kind. We can identify the first term as the infinite volume result as derived in [6], and the second term we can see smoothly goes to 0 as $L_i \to \infty$, as one would expect.

4. Unitarity

The optical theorem requires that the imaginary part of our scattering amplitude obeys $2\,\mathrm{Im}[\mathcal{M}] = \sigma_{\text{tot}}$, which would be equivalent to the fact that unitarity is respected. If we consider now the case where m of the 3 spatial dimensions has size L and the other $3-m$ are infinite, it is easy to show that the total cross-section up to NLO is given by

$$\sigma_{\text{tot}} = \frac{\lambda^2}{16\pi}\frac{\pi^{\frac{1-m}{2}}}{\Gamma\left(\frac{3-m}{2}\right)}\frac{1}{L\sqrt{s}}\sum_{0\le l\le R^2}^{*}\frac{r_m(l)}{\sqrt{R^2-l}^{\,m-1}} \tag{7}$$

where $R = L\sqrt{\frac{s}{4}-m^2}$, r_m is the sum of squares function and $\sum^{*}_{0\le l\le R^2}$ means that if R^2 is an integer its term has half weight. It can also be shown that the imaginary part of the scattering amplitude at NLO takes the form

$$2\,\mathrm{Im}[\mathcal{M}] = \frac{\lambda^2}{16\pi}\frac{2R}{L\sqrt{s}}\sum_{\vec{k}\in\mathbb{Z}^m}\mathrm{sinc}\left(2\pi R\|\vec{k}\|\right). \tag{8}$$

By using the formula

$$\sum_{\vec{k}\in\mathbb{Z}^m}\mathrm{sinc}\left(2\pi R\|\vec{k}\|\right) = \frac{\pi^{\frac{1-m}{2}}}{2R\Gamma\left(\frac{3-m}{2}\right)}\sum_{0\le l\le R^2}^{*}\frac{r_m(l)}{\sqrt{R^2-l}^{\,m-1}} \tag{9}$$

as derived in [8] we can see that the optical theorem, and equivalently unitarity, holds.

5. Conclusions

We have shown how one can see denominator regularization arise from a well-behaved case of dimensional regularization. We further showed that this new regularization scheme works when we apply it to a finite sized massive scalar field theory, where other techniques such as the aforementioned dimensional regularization poses non-trivial technical and conceptual issues. We have shown that the optical theorem and equivalently unitarity holds for $2 \to 2$ NLO scattering for the considered finite sized ϕ^4 theory.

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