PROCEEDINGS OF SCIENCE

Current Status of Resummed Quantum Gravity

B.F.L. Ward[a,*]

[a] *Baylor University,*
Waco, TX, USA

E-mail: bfl_ward@baylor.edu

We present the current status of the theory of resummed quantum gravity. We focus on its prediction for the cosmological constant in the context of the Planck scale cosmology of Bonanno and Reuter and its relationship to Weinberg's asymptotic safety idea. We discuss its relationship to Weinberg's soft graviton resummation theorem. We also present constraints from and consistency checks of the theory as well as a possible new test of the theory.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

*Speaker

PoS(ICHEP2022)1215

1. Introduction

At this writing, we are at an important juncture in the development of the theory of elementary particles. It has been noted [1] the that success of the so-called Standard Model of Elementary Particles [2] is sufficient that it should now be called the Standard Theory of Elementary Particles. This we do this henceforward. Nonetheless, there does remain the search for a correct, testable theory of quantum gravity, where we mention the well-known possibilities represented by superstring theory [3], loop quantum gravity [4], exact field-space renormalization group methods [5–8], the running vacuum approach in Ref. [9], Weinberg's asymptotic safety idea [10], etc. – we apologize but we cannot list all of the still viable ideas due to the lack of space. Here, we give an update on the status of our resummed quantum gravity (RQG) approach [11] in this context.

The basic idea we exploit in our approach to quantum gravity is the demonstrated improvement of the accuracy of the quantum loop expansion with the systematic resummation of its large IR (infrared) contributions for a given order of exactness in the respective overall loop expansion as demonstrated in Refs. [12] following and extending the approach of Ref. [13] in the context of the ST EW theory. Our brief report is organized as follows. In the next Section, we give a concise review of the resummation theory which we use. Preliminary remarks in Section 3 are followed by a concise review of RQG theory in Section 4. Section 5 closes with some exemplary results and an outlook.

2. Exact Amplitude-Based Resummation Theory - Concise Review

The application of exact amplitude-based resummation theory to quantum gravity has as its starting point the master formula, for the process $q_1 + q_2 \to q'_1 + q'_2$, in an obvious notation for the kinematics,

$$d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}}$$
$$\prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}}$$
$$\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^0}\frac{d^3 q_2}{q_2^0}, \qquad (1)$$

where *new* (YFS-style) *non-Abelian* residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons. Definitions of the infrared functions $\rm SUM_{IR}(QCED)$ and $D_{\rm QCED}$ and of the residuals are given in Ref. [14]. The non-Abelian nature of QCD means that these methods can be directly applied to the Einstein-Hilbert Lagrangian as it was presented by Feynman [15, 16], as we discuss in what follows.

3. Preliminary Remarks

We present some preliminary observations to set the stage for our discussion. Specifically, the RQG theory which we present here will be seen to have to following properties::

- RQG is UV finite.
- RQG is consistent with Weinberg's asymptotic safety ansatz [10] and its realization by the exact field-space renormalization group program of Refs. [5–8].

- RQG, combined with the Planck scale cosmology of Bonanno and Reuter [7, 8], allows a prediction [17] for the cosmological constant with a relatively small error.
- RQG is consistent with Kreimer's [20] leg renormalizability results for quantum gravity.

We feel these remarks may be inviting for some.

4. Resummed Quantum Gravity - Concise Overview

Returning to the general development of the RQG theory, we write Feynman's formulation of the Einstein-Hilbert Lagrangian for the simplest case of matter, that of a Higgs-like scalar field of mass m:

$$\begin{aligned}\mathcal{L}(x) &= -\frac{1}{2\kappa^2}R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g}\\ &= \quad \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\}\\ &+ \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right]\\ &- \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots\,.\end{aligned} \tag{2}$$

R is the curvature scalar, g is the determinant of the metric of space-time $g_{\mu\nu} \equiv \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$, and $\kappa = \sqrt{8\pi G_N}$. We expand [15, 16] about Minkowski space with $\eta_{\mu\nu} = \text{diag}\{1,-1,-1,-1\}$. $\varphi(x)$ is our Higgs-like representative scalar field for matter and $\varphi(x)_{,\mu} \equiv \partial_\mu\varphi(x)$. We have introduced Feynman's notation $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu}y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$[1]. In (2) and in what follows, $m_0(m)$ is the bare (renormalized) scalar boson mass. We set presently the small observed [18, 19] value of the cosmological constant to zero so that our quantum graviton, $h_{\mu\nu}$, has zero rest mass in (2).

The large virtual IR effects in the respective loop integrals for the scalar propagator in quantum general relativity can be resummed [21–23] to the *exact* result, using an obvious notation from Ref. [21], $i\Delta'_F(k) = \frac{i}{k^2-m^2-\Sigma_s(k)+i\epsilon} = \frac{ie^{B''_g(k)}}{k^2-m^2-\Sigma'_s+i\epsilon}$ for $(\Delta = k^2 - m^2)$ where $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right)$. The form for $B''_g(k)$ holds for the UV(deep Euclidean) regime, so that $\Delta'_F(k)|_{\text{resummed}}$ falls faster than any power of $|k^2|$. It follows [21–23] that RQG is UV finite.

5. Some Exemplary Results

For a summary of results from RQG we refer the reader to Ref. [11]. Presently, we call attention to the running Newton and cosmological constants results [17, 21–23] $G_N(k) = G_N/(1 + \frac{c_{2,eff}k^2}{360\pi M_{Pl}^2})$, $g_* = \lim_{k^2\to\infty} k^2 G_N(k^2) = \frac{360\pi}{c_{2,eff}} \cong 0.0442$, and $\Lambda(k) \longrightarrow_{k^2\to\infty} k^2\lambda_*$, $\lambda_* = -\frac{c_{2,eff}}{2880}\sum_j(-1)^{F_j}n_j/\rho_j^2 \cong 0.0817$, where $c_{2,eff} \cong 2.56\times10^4$ is defined in Refs. [21–23], F_j is the fermion number of particle j, n_j is the effective number of degrees of freedom of j and $\rho_j = \rho(\lambda_c(m_j))$ for $\rho = \ln\frac{2}{\lambda_c}$ with $\lambda_c(j) = \frac{2m_j^2}{\pi M_{Pl}^2}$ for particle j with mass m_j . λ_* vanishes in an exactly supersymmetric theory. Finally, we note the pioneering RQG result for the cosmological constant itself in Ref. [17]:

$$\rho_\Lambda(t_0) \cong (2.4\times10^{-3}eV)^4. \tag{3}$$

$t_0 \cong 13.7\times10^9$ yrs. is the age of the universe, where the result is predicted to scale like t_0^{-2} and is seen to be in good agreement with experiment [19]: $\rho_\Lambda(t_0)|_{\text{expt}} \cong ((2.37\pm0.05)\times10^{-3}eV)^4$.

We conclude with the outlook that the analysis in Ref. [24] would seem to beckon more constraints on RQG [25].

[1]Our conventions for raising and lowering indices in the second line of (2) are the same as those in Ref. [16].

PoS(ICHEP2022)1215

References

[1] D.J. Gross, talk in *SM@50 Symposium*, Case West. Res. Univ., Cleveland, OH, June, 2018.

[2] S.L. Glashow, Nucl. Phys. **22** (1961) 579; S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264; A. Salam, in *Elementary Particle Theory*, ed. N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367; G. 't Hooft and M. Veltman, Nucl. Phys. B **44** (1972) 189, and references therein; D.J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343; H. David Politzer, *ibid*. **30** (1973) 1346.

[3] M.B. Green and J. H. Schwarz, Phys. Lett. B**149** (1984) 117; *ibid.* **151** (1985) 21 (1985); D.J. Gross *et al.*, Phys. Rev. Lett. **54** (1985) 502; Nucl. Phys. B**256** (1985) 253; *ibid.* 267 (1986) 75; see also M. Green, J. Schwarz and E. Witten, *Superstring Theory, v. 1 and v. 2*, (Cambridge U. Press, Cambridge, 1987); J. Polchinski, *String Theory, v. 1 and v. 2*, (Cambridge U. Press, Cambridge, 1998).

[4] See for example V.N. Melnikov, Gravit. Cosmol. **9** (2003) 118; L. Smolin, hep-th/0303185; A. Ashtekar and J. Lewandowski, Class. Quantum Grav. **21** (2004) R53; A. Perez, gr-qc/0409061; M. Bojowald *et al.*, arXiv.org:0806.3929, and references therein.

[5] M. Reuter, Phys. Rev. D**57** (1998) 971, and references therein.

[6] O. Lauscher and M. Reuter, Phys. Rev. D**66** (2002) 025026.

[7] A. Bonanno and M. Reuter, Phys. Rev. D**65** (2002) 043508.

[8] A. Bonanno and M. Reuter, Jour. Phys. Conf. Ser. **140** (2008) 012008, and references therein.

[9] J. Sola Percaula, Phil. Trans. Roy. Soc. Lond. A **380** (2022) 20210182, and references therein.

[10] S. Weinberg, in *General Relativity, an Einstein Centenary Survey*, eds. S. W. Hawking and W. Israel, (Cambridge Univ. Press, Cambridge, 1979).

[11] B.F.L. Ward, Int. J. Mod. Phys. A **33** (2018) 1830028.

[12] S. Jadach and B.F.L. Ward, Phys. Rev. **D38** (1988) 2897;*ibid.* **D39** (1989) 1471; *ibid.* **D40** (1989) 3582; Comp. Phys. Commun. **56** (1990) 351; S.Jadach, B.F.L. Ward and Z. Was, Comput. Phys. Commun. **66** (1991) 276; *ibid.* **79** (1994) 503; *ibid.* **124** (2000) 233; *ibid.***130** (2000) 260; Phys. Rev.D**63** (2001) 113009; S.Jadach, W. Placzek and B.F.L. Ward, Phys. Lett. **B390** (1997) 298; S. Jadach *et al.*, Phys. Lett. **B417** (1998) 326; Comput. Phys. Commun. **119** (1999) 272; *ibid.***140** (2001) 432, 475; Phys. Rev. **D61** (2000) 113010; *ibid.* **D65** (2002) 093010.

[13] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. **13** (1961) 379; see also K. T. Mahanthappa, Phys. Rev. **126** (1962) 329, for a related analysis.

[14] S.K. Majhi *et al.*, Phys. Lett. B**719** (2013) 367, arXiv:1208.4750; A. Mukhopadhyay and B.F.L. Ward, Mod. Phys. Lett. A **31** (2016) 1650063, arXiv:1412.8717.

[15] R.P. Feynman, Acta Phys. Pol.**24** (1963) 697-722.

[16] R.P. Feynman, *Lectures on Gravitation*, eds. F. Moringo and W. Wagner, (CIT, Pasadena, 1971).

[17] B.F.L. Ward, Phys. Dark Univ. **2** (2013) 97.

[18] A.G. Riess *et al.*, Astron. J. **116** (1998) 1009; S. Perlmutter *et al.*, Astrophys. J. **517** (1999) 565.

[19] C. Amsler *et al.*, Phys. Lett. B**667** (2008) 1.

[20] D. Kreimer, Ann. Phys. **323** (2008) 49; *ibid.* **321** (2006) 2757.

[21] B.F.L. Ward, Open Nucl.Part.Phys.Jour. **2**(2009) 1.

[22] B.F.L. Ward, Mod. Phys. Lett. A**17** (2002) 237.

[23] B.F.L. Ward, Mod. Phys. Lett. A**19** (2004) 143.

[24] G. Calcagni *et al.*, JCAP **10** (2019) 012.

[25] B.F.L. Ward, to appear.

PoS(ICHEP2022)1215