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Black holes and nilmanifolds: quasinormal modes as the fingerprints of extra dimensions?

A. Chrysostomou,[a,b,*] **A. S. Cornell,**[a] **A. Deandrea,**[b] **E. Ligout**[c] **and D. Tsimpis**[b]

[a] *Department of Physics, University of Johannesburg, PO Box 524, Auckland Park 2006, South Africa*
[b] *Institut de Physique des Deux Infinis de Lyon, Université de Lyon, UCBL, UMR 5822, CNRS/IN2P3, 4 rue Enrico Fermi, 69622 Villeurbanne Cedex, France*
[c] *École Normale Supérieure de Lyon, Site Monod, 46 Allée d'Italie, 69364 Lyon Cedex 07, France*
E-mail: chrysostomou@ip2i.in2p3.fr, acornell@uj.ac.za, deandrea@ipnl.in2p3.fr, etienne.ligout-de-oliveira1@ens-lyon.fr, tsimpis@ipnl.in2p3.fr

Quasinormal modes (QNMs), the damped oscillations in spacetime that emanate from a perturbed body as it returns to an equilibrium state, have served for several decades as a theoretical means of studying n-dimensional black hole spacetimes. These black hole QNMs can in turn be exploited to explore beyond the Standard Model (BSM) scenarios and quantum gravity conjectures. With the establishment of the LIGO-Virgo-KAGRA network of gravitational-wave (GW) detectors, there now exists the possibility of comparing computed QNMs against GW data from compact binary coalescences. Encouraged by this development, we investigate whether QNMs can be used in the search for signatures of extra dimensions. To address a gap in the BSM literature, we focus here on higher dimensions characterised by negative Ricci curvature. As a first step, we consider a product space comprised of a 4D Schwarzschild black hole spacetime and a 3D nilmanifold (twisted torus); we model the black hole perturbations as a scalar test field. We find that the extra-dimensional geometry can be stylised in the QNM effective potential as a squared mass-like term. We then compute the corresponding QNM spectrum using three different numerical methods and determine possible constraints for extra dimensions.

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*Speaker

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The parameter space of models with higher-dimensional negative compact spaces appears relatively under-explored when compared with their flat and positively-curved counterparts. Phenomenologically, these models could be used to address the hierarchy problem and cosmological observations [1]. Motivated by these, we investigate the manifold $\mathcal{M}_4 \times \mathcal{N}_3$, where $\mathcal{M}_4$ is a flat $(3+1)$ spacetime and $\mathcal{N}_3$ is a 3D negative compact space,

$$\mathrm{d}s^2_{\mathcal{N}_3} = \delta_{ab} e^a e^b = (r^1 \mathrm{d}y^1)^2 + (r^2 \mathrm{d}y^2)^2 + (r^3 \mathrm{d}y^3 + N r^1 r^3 \mathrm{d}y^2)^2 \ . \tag{1}$$

This is the most general minimal left-invariant metric for a “nilmanifold", a twisted torus constructed from the nilpotent Heisenberg algebra in Ref. [1]. Here, r^i are constant “radii‘ and $N = r^1 r^2 \mathtt{f}/r^3$ with the structure constant $\mathtt{f} = -f^3{}_{12} \neq 0$ serving as the “twist parameter".

In light of the regular detection of gravitational-wave (GW) events from compact binary coalescences by the LIGO-Virgo-KAGRA (LVK) collaboration, interest in using GW data to constrain extra-dimensional models is building. However, it is known that GW observations are not developed enough to constrain extra-dimensional hypotheses; unlike collider searches, we have yet to obtain precise final state signatures for which we can search [2]. For these reasons, we suggest a new approach by which to probe extra dimensions within GW data that compares the quasinormal frequency (QNF) spectrum of a black hole (BH) embedded in the $\mathcal{M}_4 \times \mathcal{N}_3$ manifold against searches for parametric deviations from general relativity (GR) in post-merger GW emissions.

The BH response to a perturbation is dominated by quasinormal modes (QNMs) [3]; the corresponding QNFs are referred to as the “fingerprints" of BHs since they can be computed directly from the characteristic parameters of their BH source and vice-versa [4]. Here, we consider the non-rotating, spherically-symmetric Schwarzschild BH (under units $G = c = 1$),

$$\mathrm{d}s^2_{\mathrm{BH}} = g^{\mathrm{BH}}_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2(\sin^2 \mathrm{d}\theta^2 + \mathrm{d}\phi^2) \ , \tag{2}$$

where $f(r) = 1 - 2M/r$. Per the “no-hair" conjecture, such a black hole is fully characterised by its mass M [5]. Classically, energy cannot escape from within the event horizon $r_{\mathrm{H}} = 2M$; at $r = \infty$, radiation may “leak out" but it cannot (re)enter the system. Thus, the intrinsic boundary conditions of the system dictate that gravitational radiation is purely ingoing at the horizon and purely outgoing at spatial infinity, rendering the QNM problem inherently dissipative.

From Eqs. (1) and (2), we describe the BH embedded in this higher-dimensional manifold with a “Schwarzschild-nilmanifold metric", $ds^2_{7\mathrm{D}} = ds^2_{\mathrm{BH}} + ds^2_{\mathrm{nil}}$. To model the behaviour of the BH perturbations at the lowest linear approximation (where the perturbations are much smaller than $g^{\mathrm{BH}}_{\mu\nu}$), we may study the evolution of a scalar test field propagating on a fixed background (see Ref. [6] for details). We express this behaviour through the QNM and its discrete set of QNFs,

$$\Psi^s_{n\ell m}(\mathbf{z}) = \sum_{n=0}^{\infty} \sum_{\ell,m}^{\infty} \frac{\psi_{sn\ell}(r)}{r} \, Y^s_{m\ell}(\theta,\phi) \, Z(y^1,y^2,y^3) \, e^{-i\omega t} \ , \quad \omega_{sn\ell} = \omega_R - i\omega_I \ . \tag{3}$$

Here, $\mathbb{R}e\{\omega\}$ is the physical oscillation frequency and $\mathbb{I}m\{\omega\}$ is the inverse damping rate. The spin of the perturbing field is given by s and m, ℓ are the usual azimuthal and angular numbers associated with the spherical harmonic decomposition in θ, ϕ. The overtone number n labels QNMs by monotonically increasing multiples of $|\mathbb{I}m\{\omega\}|$. The QNF spectrum is in turn dominated by the least-damped, the longest-lived "fundamental mode": $n = 0$, $\ell = m = 2$.

The evolution of a scalar field is described through the Klein-Gordon equation. For this spacetime, recall that the Laplacian of a product space is the sum of its parts:

$$\nabla^2 \Psi(\mathbf{z}) = \left(\nabla^2_{\mathrm{BH}} + \nabla^2_{\mathrm{nil}}\right) \Phi^s_{n\ell m}(\mathbf{x}) Z(\mathbf{y}) \ . \tag{4}$$

However, if we choose to encode the higher-dimensional behaviour through an effective mass term representing a Kaluza-Klein tower of states, then we may describe the 7D scalar field evolution through a 4D "massive" Klein-Gordon equation,

$$\nabla^2_{\rm nil} Z(\mathbf{y}) = -\mu^2 Z(y^1, y^2, y^3) \qquad \Rightarrow \qquad \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi\right) - \mu^2\Psi = 0 \,. \tag{5}$$

While spherical harmonics can be used to describe the angular part, it is the radial behaviour that presents a characteristic wavelike equation containing the QNF and the effective scalar potential,

$$\frac{d^2\psi}{dr_*^2} + \left(\omega^2 - V(r)\right)\psi = 0\,, \qquad V(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2\right)\,. \tag{6}$$

We employ three numerical methods to calculate the QNF spectrum. Dolan and Ottewill's expansion method [7] expresses the QNF as a series in inverse parameters of ℓ. When we apply their method to Eq. (6), we obtain a series expansion in terms of μ and L, and QNFs in excellent agreement with those computed using the modified WKB [8] and Pöschl-Teller [9] methods. See Table 1 for pertinent results. Note that when $\mu^2 > \mathbb{R}e\{\omega^2\}$, the QNMs are no longer "propagative"; they become "evanescent" [10] and cannot be relied upon for this analysis.

To perform parameter estimation, tests of GR, and other analyses in the QNM regime, PyRing [11–13] was developed. It is integrated within the LVK software infrastructure, combining observed GW data with simulation and numerically-generated waveform templates. As a first step, we use PyRing to run an agnostic test of GR-deviation in GW data from the GW150914 event using a Kerr waveform template (see Figure 1). However, for improved accuracy, we report the results from the hierarchical combination of LVK's strongest bounds on GR deviations to date [14],

$$\delta\omega = 0.02^{+0.07}_{-0.07}\,, \qquad \delta\tau = 0.13^{+0.21}_{-0.22}\,. \tag{7}$$

Suppose we set $\omega^{\rm GR} = \omega^{\mu=0}$ and use the computed QNF series expansion for the dominant QNM $\omega(\ell = 2, \mu)$. Then we can use the real part of the QNF to constrain μ, which yields the result

$$0.1747 < \mu < 0.3681\,. \tag{8}$$

Thus, by using searches for parametric deviations from GR, we can place naive constraints on an extra-dimensional scenario through a QNM analysis. Our next immediate step is to subject the mass spectrum of the nilmanifold model studied in Ref. [1] to this constraint in order to extract tangible bounds on the radius of the nilmanifold extra dimensions herein constructed.

μ	$\omega(\ell,\mu)$	$\delta\omega$	$\delta\tau$
0.0	$0.4836 - 0.0968i$	0.0000	0.0000
0.1	$0.4868 - 0.0968i$	0.0065	0.0113
0.2	$0.4963 - 0.0924i$	0.0262	0.0473
0.3	$0.5124 - 0.0868i$	0.0594	0.1149
0.4	$0.5352 - 0.0787i$	0.1066	0.2302
0.5	$0.5653 - 0.0676i$	0.1687	0.4306
0.6	$0.6032 - 0.0532i$	0.2472	0.8206
0.7	$0.6500 - 0.0343i$	0.3440	1.8181

Table 1: For increasing μ, we compute the QNF spectrum using the Dolan-Ottewill method to order $O(L^{-6})$. To correspond to the search for parametric deviations in GR, we structure our results for $\omega = \mathbb{R}e\{\omega\}$ and the damping time $\tau = -1/\mathbb{I}m\{\omega\}$ as $\omega = \omega^{\mu=0}(1+\delta\omega)$ and $\tau = \tau^{\mu=0}(1+\delta\tau)$, respectively.

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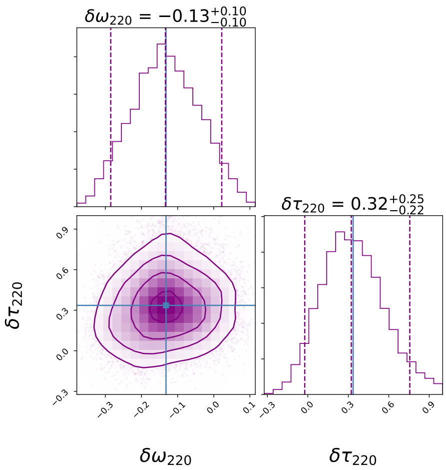

Figure 1: As a proof-of-concept, we perform a rudimentary parameter estimation of the GR deviations using PyRing for event GW150914 (GW data sampled at 4096 Hz). We narrow priors to reduce computation cost. With Corner, we plot the 2D posteriors and 1D histograms on $(\delta\omega, \delta\tau)$, where $(0, 0)$ is the GR-predicted value. Dashed lines and contours demarcate the 90% credible region; the blue line indicates the mean.

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