PROCEEDINGS OF SCIENCE

# Higher-order QCD corrections to the Higgs decay into bottom quarks from Padé approximants

**Cristiane Yumi London**[*]

*Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970 São Carlos, SP, Brazil*

*Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona, and Institut de Física d'Altes Energies (IFAE), The Barcelona Institute of Science and Technology (BIST), Campus UAB, E-08193 Bellaterra (Barcelona), Spain*

*E-mail:* cristiane.london@usp.br

We employ the method of Padé approximants to study the higher-order corrections of the massless scalar-current quark correlator. We begin by testing this method in the large-$\beta_0$ limit of QCD, where the perturbative series is known to all orders, using it as a testing ground to determine the best strategy to build the series at higher orders using only the first four coefficients. Applying the procedure in QCD, we estimate the yet unknown coefficient $c_5$ of order $\alpha_s^5$ (six loops) of the imaginary part of the correlator, directly related to $\Gamma(H \to b\bar{b})$, in a model-independent way as $c_5 = -6900 \pm 1400$. We conclude that with this correction the series is almost insensitive to renormalization scale variations. This corroborates that the QCD corrections to this decay are under excellent control and the uncertainty of $\Gamma(H \to b\bar{b})$ will continue to be dominated by the Standard Model parameters in the near future, mainly the strong coupling and the bottom-quark mass.

*41st International Conference on High Energy physics - ICHEP2022*
*6-13 July, 2022*
*Bologna, Italy*



[*]Speaker

# 1. Introduction

Without direct observation of physics beyond Standard Model (SM) in the Large Hadron Collider (LHC), it is necessary to increase the precision of both theoretical and experimental results in order to unravel whatever theory is underlying the SM. The decay width of the Higgs boson is dominated by the channel $H \to b\bar{b}$ and the higher-order corrections to this decay width are strongly dominated by Quantum Chromodynamics (QCD).

The decay rate of Higgs into $b\bar{b}$ is known up to fourth order ($\alpha_s^4$) in QCD for massless quarks [1] and is related to the imaginary part of the quark-antiquark scalar-current correlator $\Pi(s)$ as

$$\Gamma(H \to b\bar{b}) = \frac{1}{v^2\, m_H}\,\mathrm{Im}\,\Pi(m_H^2) = \frac{m_H}{v^2}\,\frac{N_c}{8\pi^2}\,m_b^2(m_H)\left[1 + \sum_{n=1}^{\infty} c_n\, a_s^n(m_H)\right] =$$
$$= \frac{m_H}{v^2}\,\frac{N_c}{8\pi^2}\,m_b^2(m_H)\left[1 + 5.667\,a_s + 29.15\,a_s^2 + 41.76\,a_s^3 - 825.7\,a_s^4 + \dots\right], \qquad (1)$$

where $N_c = 3$, $a_s = \alpha_s/\pi$ is the strong coupling, $v^2$ is the Higgs vacuum expectation value and the numerical values for the coefficients $c_n$ are for $N_f = 5$. Coefficients increase order by order and such growing gives rise to the question of whether yet-unknown-terms may spoil the sequence due to the renormalons of perturbation theory.

Since the perturbative series in QCD are at best asymptotic, it is useful to work with their Borel transform [2]. The Borel transform is the inverse of the Laplace transform and it suppresses the factorial divergence of the series coefficients. A fundamental feature of the Borel transform is its singularities along the real axis in the Borel plane. These singularities are the renormalons of perturbation theory and they govern the behavior of the perturbative series at intermediate and higher orders [2].

Due to the difficulty in determining the corrections for the perturbative series of $\Gamma(H \to b\bar{b})$ analytically, in this work we employ a method to estimate the missing higher orders with the available information in QCD. We apply Padé approximants to the Borel transform of the perturbative series of two physical observables — which are then renormalization group invariant — $\mathrm{Im}\,\Pi(s)$ and $\Pi''(s)$, in order to reconstruct the series of the decay rate to higher orders.

# 2. Padé Approximants

In this section we give an outline of the Padé approximants concepts that are most relevant for this work. A Padé approximant (PA), denoted by $P_N^M(z)$, is used to approximate a function $f(z)$ whose Taylor series is known and it is given by a ratio of two polynomials, $Q_M(z)$ and $R_N(z)$ of orders $M$ and $N$ respectively [3]

$$P_N^M(z) = \frac{Q_M(z)}{R_N(z)} = \frac{a_0 + a_1\,z + a_2\,z^2 + \cdots + a_M\,z^M}{1 + b_1\,z + b_2\,z^2 + \cdots + b_N\,z^N}, \qquad (2)$$

where we used $R(0) = 1$. The PA parameters are determined by matching the Taylor coefficients of $f(z)$ with the expansion of the PA. Hence, the approximant $P_N^M(z)$ will reproduce the first $M+N+1$ coefficients of $f(z)$ and the coefficients of higher orders are all estimates. The PAs approximate more efficiently the original function when compared to the Taylor series. They are also systematic and model independent and they can reproduce the analytic properties of the function, such as the renormalons. In addition, they yield good predictions for the higher-order coefficients [4].

It is expected that the QCD perturbative series for $\Gamma(H \to b\bar{b})$ has superimposed cuts. Hence we employ another type of approximant called D-Log Padé approximants [3] because they are designed to approximate functions with branch cuts. The aim is to approximate a new function $F(z)$ that does not have cuts, only simple poles, which is meromorphic and obtained from $f(z)$ [3]. The D-Log Padé $\mathrm{Dlog}^M_N(z)$ to $f(z)$ is then [3, 4]

$$\mathrm{Dlog}^M_N(z) = f_{\text{norm}}(0)\ \exp\left\{\int \mathrm{d}z'\ \bar{P}^M_N(z')\right\}. \tag{3}$$

The Padé $\bar{P}^M_N(z')$ is build to $F(z)$ and the term $f_{\text{norm}}(0)$ is an adjustable constant that reproduces the function at $z = 0$. This value has to be added because, after calculating the derivative, this term disappears. The D-Log PAs can estimate the location of the branch point and also the multiplicity of the cut, since no assumptions about its position or its multiplicity are made.

## 3. Tests in the large-$\beta_0$ limit

Before we show the results of QCD we apply the method in the large-$\beta_0$ limit. This limit is a simplified model where higher-order corrections are known to all orders in the coupling $\alpha_s$, which can be used to evaluate quantitatively the method (more details can be found in Ref. [5]).

We performed a systematic study of different strategies in Ref. [5] for the use of the approximants to predict the higher-order coefficients of the perturbative series of the Borel transform of $\mathrm{Im}\,\Pi(s)$ and $\Pi''(s)$. We could observe that the PAs and the D-Log PA sequences appear to converge when the order of the approximant is raised and they were also able to reproduce the leading renormalons of the Borel transforms. It was also possible to note that the approximants that use only the first three coefficients as input are not sufficiently accurate in their predictions for the coefficients and the renormalons since they do not have enough information.

We could verify that the optimal method to determine the higher-order coefficients and the singularities using the same amount of information available in QCD (i.e. four coefficients) was the D-Log PAs applied to the Borel transform of the reduced second derivative. This relies on the fact that $B[\Pi''](u)$ displays a much simpler structure and the rernomalons — which are double poles in $\Pi''(s)$ — become simple poles due to the use of the function $F(z)$.

## 4. Results in QCD

In QCD, the renormalons are branch cuts [2, 6] and are located at the same position as the renormalons in the large-$\beta_0$ limit. Hence, the use of D-Log PAs, which was the most efficient strategy found in our studies in large-$\beta_0$, remains appealing since they tend to be a superior choice for functions with branch cuts. Our final results were determined as follows: the central value is the average between the largest and the smallest estimated coefficients and the uncertainty is given by half of the maximum spread found between two PAs. Applying this prescription to the higher-order coefficients of $\mathrm{Im}\,\Pi(s)$ gives us up to eighth order [5]

$$c_5 = -6900 \pm 1400 \qquad c_6 = (0.3 \pm 3.5) \times 10^4 \qquad c_7 = (3.7 \pm 2.5) \times 10^5 \qquad c_8 = (0.2 \pm 2.4) \times 10^6\ . \tag{4}$$

One can notice that the errors of the coefficients of order higher than six are larger than 100%, however they do not lead to significant uncertainties in the perturbative series due to their strong $\alpha_s$ suppression. Our result for $c_5$, the first unknown coefficient in QCD, is in agreement with other estimates in the literature obtained through different methods [7].

Our results for higher-order coefficients can be used to estimate the uncertainty in the Standard Model calculation of the decay rate of Higgs into bottom quarks. Employing our final value for the six loop coefficient, $c_5$, we have at N5LO

$$\Gamma(H \to b\bar{b}) = 2.3806(^{+0.041}_{-0.027})_{m_b} \pm (0.0042)_{\alpha_s} \pm (0.0032)_{m_H} \pm (0.0002)_{\mu} \pm (0.0003)_{\text{PAs}}\ \text{MeV}, \tag{5}$$

where we used $m_b(m_b) = 4.18^{+0.03}_{-0.02}$ GeV, $\alpha_s(m_Z) = 0.1179 \pm 0.0010$, $m_H = 125.25 \pm 0.17$ GeV [8]. The error identified with $\mu$ is related to the renormalization scale variation and was determined as the maximum spread of the decay rate in the interval $[m_H/2, 2m_H]$ divided by two. We can notice that the renormalization scale dependence is small at fifth order and the dominant uncertainties arise from the QCD parameters — the bottom quark mass, the strong coupling as well as the Higgs mass. The largest uncertainty remains the $b$ mass even employing a value with an error 27% lower [9].

## Acknowledgements

I thank Diogo Boito and Pere Masjuan for the collaboration during this project. This work was financed in part by São Paulo Research Foundation (FAPESP) grants No. 2018/21050-0 and No. 2020/15532-1 and by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

## References

[1] P. A. Baikov, K. G. Chetyrkin and J. H. Kuhn, Phys. Rev. Lett. **96**, 012003 (2006) [arXiv:hep-ph/0511063 [hep-ph]]; F. Herzog, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, JHEP **08**, 113 (2017) [arXiv:1707.01044 [hep-ph]].

[2] M. Beneke, Phys. Rept. **317**, 1-142 (1999) [arXiv:hep-ph/9807443 [hep-ph]].

[3] G. A. Baker Jr *et al*, Elsevier, 1975; G. A. Baker Jr and P. Graves-Morris, Cambridge University Press, Vol. 59, 1996.

[4] D. Boito, P. Masjuan and F. Oliani, JHEP **08**, 075 (2018) [arXiv:1807.01567 [hep-ph]].

[5] D. Boito, C. Y. London and P. Masjuan, JHEP **01**, 054 (2022) [arXiv:2110.09909 [hep-ph]].

[6] M. Jamin, Eur. Phys. J. ST **230**, no.12-13, 2609-2624 (2021) [arXiv:2106.01614 [hep-ph]].

[7] A. L. Kataev and V. V. Starshenko, Mod. Phys. Lett. A **10**, 235-250 (1995) [arXiv:hep-ph/9502348 [hep-ph]]; A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, JHEP **06**, 085 (2010) [arXiv:1004.4125 [hep-ph]].

[8] P. A. Zyla *et al.* [Particle Data Group], PTEP **2020**, no.8, 083C01 (2020).

[9] J. Erler, P. Masjuan and H. Spiesberger, [arXiv:2203.02348 [hep-ph]].