

Hadron spectroscopy using holographic QCD and 't Hooft equation

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In supersymmetric formulation of light-front holographic QCD, brayons have two susy partners, mesons and tetraquarks and their degenerate mass in the chiral limit is generated by the transverse confinement scale. The mass degeneracy between these hadronic states is lifted by chiral symmetry breaking as well as the longitudinal confinement. In this talk, I present the results for hadron spectrum when $(1+1)$ -D 't Hooft Equation is used for the inclusion of longitudinal dynamics. I show that the predictions of this combined model are in good agreement with available spectroscopic data.

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1. Introduction

Light-front wavefunction (LFWF) provides for a Lorentz-invariant description of hadronic bound states. For a two-body system, meson quark-antiquark bound state for example, LFWF is written in terms of the fraction of the meson longitudinal momentum carried by the quark (x) and the light-front variable $\zeta = \zeta e^{i\phi}$ defined as:

$$\zeta = \sqrt{x(1-x)}\mathbf{b}_\perp, \quad (1)$$

where \mathbf{b}_\perp is the transverse separation between the quark and antiquark. The LFWF can then be factorized as

$$\Psi(x, \zeta, \phi) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\phi} X(x), \quad (2)$$

where $\Phi(\zeta)$ and $X(x)$ are transverse and longitudinal modes respectively, and L is the orbital angular momentum of the bound state.

In holographic light-front QCD (HLFQCD), in the semiclassical limit, i.e. zero quark mass and no quark loop, the transverse mode $\Phi(\zeta)$ is the solution of a Schrödinger-like wave equation [1, 2]

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U_\perp^{\text{LFH}}(\zeta) \right) \Phi(\zeta) = M_\perp^2 \Phi(\zeta). \quad (3)$$

The transverse confining potential U_\perp^{LFH} in (3) is uniquely determined from the conformal symmetry breaking mechanism and correspondence with weakly coupled string modes in AdS₅ space, which results in a light-front harmonic oscillator potential in physical spacetime with confinement scale κ :

$$U_\perp^{\text{LFH}}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1). \quad (4)$$

$J = L + S$ with S being the total quark-antiquark spin. The eigenvalue of Eq(3), M_\perp represents the mass of the bound state in the chiral limit. In fact, with the confining potential (4), one can solve (3) to obtain

$$M_\perp^2 = 4\kappa^2 \left(n_\perp + L + \frac{S}{2} \right), \quad (5)$$

which correctly predicts a massless pion when the quantum number n_\perp as well as L and S are all zero.

The longitudinal mode, on the other hand, has the form:

$$X(x) = \sqrt{x(1-x)}\chi(x), \quad (6)$$

where $\chi(x) = 1$ in HLFQCD by matching the pion EM or gravitational form factor in physical spacetime and AdS space [1]. Please note that this method of determination $X(x)$ is not based on any possible longitudinal dynamics.

The supersymmetric formulation of HLFQCD is based on the fact that a diquark (antidiquark) can be in the same representation of $SU_c(3)$ as an antiquark (quark) [3, 4]. Therefore, as shown in figure (1), assuming baryons and tetraquarks as two body systems of quark-diquark and diquark-antidiquark respectively, each baryon has two supersymmetric partners, a meson and a tetraquark.

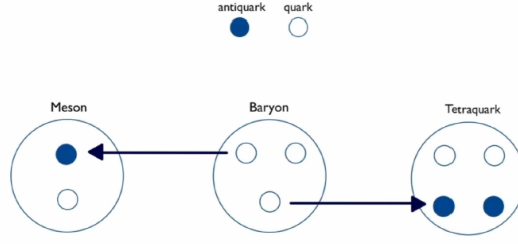


Figure 1: Mesons and tetraquarks as supersymmetric partners of baryons.

In this unified description of baryons, mesons and tetraquarks, the spectrum of these hadrons are obtained from:

$$M_{\perp,M}^2 = 4\kappa^2 \left(n_{\perp} + L_M + \frac{S_M}{2} \right), \quad (7)$$

$$M_{\perp,B}^2 = 4\kappa^2 \left(n_{\perp} + L_B + \frac{S_D}{2} + 1 \right), \quad (8)$$

and

$$M_{\perp,T}^2 = 4\kappa^2 \left(n_{\perp} + L_T + \frac{S_T}{2} + 1 \right), \quad (9)$$

where S_M , S_D and S_T are total quark-antiquark, diquark and diquark-antidiquark spin, respectively and. L_i , $i = M, B, T$ is the orbital angular momentum of the bound state.

In this work, supersymmetric HLFQCD is augmented by a longitudinal dynamics that encodes the inclusion of non-zero quark mass as well as longitudinal confinement [5]. The predicted hadronic masses are in good agreement with the available spectroscopic data.

2. Longitudinal dynamics

Longitudinal dynamics based on 't Hooft Equation [6] has already been used to predict the spectrum of light, heavy-light and heavy-heavy mesons [7]. In this approach, $\chi(x)$ in (6) is obtained from

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \chi(x) + U_L(x) \chi(x) = M_L^2 \chi(x), \quad (10)$$

with

$$U_L(x) \chi(x) = \frac{g^2}{\pi} \mathcal{P} \int dy \frac{\chi(x) - \chi(y)}{(x-y)^2}. \quad (11)$$

The hadron mass component due to longitudinal dynamics, M_L is sensitive to quark mass as well as the 't Hooft coupling g . The total mass can then be written as

$$M^2(n_L, n_T, J, L) = M_T^2(n_T, J, L) + M_L^2(n_L), \quad (12)$$

where n_L is longitudinal quantum number. Table 1 shows the input value of these parameters along with the universal transverse confinement scale $\kappa = 0.523 \text{ GeV}$ used for predicting hadronic spectrum.

Hadron	g	$m_{u/d}$	m_s	m_c	m_b
Light	0.128	0.046	0.357	-	-
Heavy-light	0.410	0.330	0.500	1.370	4.640
Heavy-heavy	0.523	-	-	1.370	4.640

Table 1: The quark masses and 't Hooft couplings in GeV. Note that we use $\kappa = 0.523$ GeV for all hadrons.

3. Predictions

To specify the predicted spectrum for the physical hadronic states the quantum numbers are assigned based on the following rules: Parity of mesons, baryons and tetraquarks is given as the following:

$$P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T} , \quad (13)$$

and the charge conjugation for mesons and tetraquarks is determined as:

$$C = (-1)^{n_L+L_M+S_M} = (-1)^{n_L+L_T+S_T-1} . \quad (14)$$

Tables (2), (3) and (4) show the mass predictions for light, heavy-light and heavy-heavy hadrons. Generally speaking, meson and baryon mass predictions are in good agreement with data. As for the tetraquark candidates however, the agreement with data is less impressive and indeed quite large for $f_0(500)$ and $f_0(980)(a_0(980))$. Please note that this latter discrepancy existed before introducing the longitudinal dynamics as reflected in the value of M_{\perp} .

4. Conclusion

Holographic light-front QCD augmented with a longitudinal dynamics based on the 't Hooft equation lead to predictions for masses of mesons, baryons and tetraquarks that are mostly in reasonable agreement with experimental data.

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Meson					Baryon					Tetraquark				
$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M	$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M	$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M
0^{-+}	$\pi(140)$	140	0	140	-	-	-	-	-	-	-	-	-	-
1^{+-}	$h_1(1170)$	335	1046	1098	$(1/2)^+$	$N(940)$	188	1046	1063	0^{++}	$f_0(500)$	335	1046	1098
2^{-+}	$\eta_2(1645)$	460	1479	1549	$(3/2)^-$	$N(1520)$	296	1479	1508	1^{-+}	-	408	1479	1534
1^{--}	$\rho(770), \omega(780)$	140	740	753	-	-	-	-	-	-	-	-	-	-
2^{++}	$a_2(1320), f_2(1270)$	335	1281	1324	$(3/2)^+$	$\Delta(1232)$	188	1281	1295	1^{++}	$a_1(1260)$	235	1281	1302
3^{--}	$\rho_3(1690), \omega_3(1670)$	460	1654	1717	$(3/2)^-$	$\Delta(1700)$	296	1654	1680	1^{+-}	$\pi_1(1600)$	335	1654	1688
4^{++}	$a_4(1970), f_4(2050)$	559	1957	2035	$(7/2)^+$	$\Delta(1950)$	372	1957	1992	-	-	-	-	-
0^-	$\bar{K}(495)$	456	0	456	-	-	-	-	-	-	-	-	-	-
1^+	$\bar{K}_1(1270)$	550	1046	1182	$(1/2)^+$	$\Lambda(1115)$	500	1046	1159	0^+	$K_0^*(1430)$	546	1046	1180
2^-	$K_2(1770)$	617	1479	1603	$(3/2)^-$	$\Lambda(1520)$	588	1479	1592	1^-	-	633	1479	1609
0^-	$\bar{K}(495)$	456	0	456	-	-	-	-	-	-	-	-	-	-
1^+	$\bar{K}_1(1270)$	550	1046	1182	$(1/2)^+$	$\Sigma(1190)$	500	1046	1159	0^{++}	$a_0(980)$ $f_0(980)$	920	1046	1393
1^-	$K^*(890)$	456	740	869	-	-	-	-	-	-	-	-	-	-
2^+	$K_2^*(1430)$	550	1281	1394	$(3/2)^+$	$\Sigma(1385)$	500	1281	1375	1^+	$K_1(1400)$	546	1281	1392
3^-	$K_3^*(1780)$	617	1654	1765	$(3/2)^-$	$\Sigma(1670)$	588	1654	1755	2^-	$K_2(1820)$	633	1654	1771
4^+	$K_4^*(2045)$	672	1957	2069	$(7/2)^+$	$\Sigma(2030)$	650	1957	2062	-	-	-	-	-
0^{-+}	$\eta'(958)$	759	0	759	-	-	-	-	-	-	-	-	-	-
1^{+-}	$h_1(1380)$	883	1046	1369	$(1/2)^+$	$\Xi(1320)$	805	1046	1320	0^{++}	$f_0(1370)$ $a_0(1450)$	920	1046	1393
2^{-+}	$\eta_2(1870)$	968	1479	1768	$(3/2)^-$	$\Xi(1620)$	876	1479	1719	1^{-+}	-	969	1479	1768
1^{--}	$\Phi(1020)$	759	740	1060	-	-	-	-	-	-	-	-	-	-
2^{++}	$f_2'(1525)$	883	1281	1556	$(3/2)^+$	$\Xi^*(1530)$	805	1281	1513	1^{++}	$f_1(1420)$ $a_1(1420)$	850	1281	1537
3^{--}	$\Phi_3(1850)$	968	1654	1916	$(3/2)^-$	$\Xi(1820)$	876	1654	1872	-	-	-	-	-
2^{++}	$f_2(1640)$	883	1281	1556	$(3/2)^+$	$\Omega(1672)$	1114	1281	1698	1^+	$K_1(1650)$	1159	1281	1728

Table 2: Computed masses (in MeV), using the 't Hooft potential, for the light hadrons compared to the PDG data [8].

Meson					Baryon					Tetraquark				
$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M	$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M	$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M
0 ⁻	$D(1870)$	1861	0	1861	-	-	-	-	-	-	-	-	-	-
1 ⁺	$D_1(2420)$	2135	1046	2377	(1/2) ⁺	$\Lambda_c(2290)$	2191	1046	2428	0 ⁺	$\bar{D}_0^*(2400)$	2510	1046	2719
2 ⁻	$D_J(2600)$	2326	1479	2756	(3/2) ⁻	$\Lambda_c(2625)$	2460	1479	2870	1 ⁻	-	2751	1479	3123
0 ⁻	$\bar{D}(1870)$	1861	0	1861	-	-	-	-	-	-	-	-	-	-
1 ⁺	$\bar{D}_1(2420)$	2135	1046	2377	(1/2) ⁺	$\Sigma_c(2455)$	2191	1046	2428	0 ⁺	$D_0^*(2400)$	2510	1046	2719
1 ⁻	$D^*(2010)$	1861	740	2003	-	-	-	-	-	-	-	-	-	-
2 ⁺	$D_2^*(2460)$	2135	1281	2490	(3/2) ⁺	$\Sigma_c^*(2520)$	2191	1281	2538	1 ⁺	$D(2550)$	2510	1281	2818
3 ⁻	$D_3^*(2750)$	2326	1654	2854	(3/2) ⁻	$\Sigma_c(2800)$	2460	1654	2964	-	-	-	-	-
0 ⁻	$D_s(1968)$	2025	0	2025	-	-	-	-	-	-	-	-	-	-
1 ⁺	$D_{s1}(2460)$	2283	1046	2511	(1/2) ⁺	$\Xi_c(2470)$	2348	1046	2570	0 ⁺	$\bar{D}_{s0}^*(2317)$	2676	1046	2873
2 ⁻	$D_{s2}^?$	2464	1479	2874	(3/2) ⁻	$\Xi_c(2815)$	2586	1479	2979	1 ⁻	-	2908	1479	3262
1 ⁻	$D_s^*(2110)$	2025	740	2156	-	-	-	-	-	-	-	-	-	-
2 ⁺	$D_{s2}^*(2573)$	2283	1281	2618	(3/2) ⁺	$\Xi_c^*(2645)$	2348	1281	2675	1 ⁺	$D_{s1}(2536)$	2676	1281	2967
3 ⁻	$D_{s3}^*(2860)$	2464	1654	2968	-	-	-	-	-	-	-	-	-	-
1 ⁺	$\bar{D}_{s1}^?$	2283	1046	2511	(1/2) ⁺	$\Omega_c(2695)$	2524	1046	2732	0 ⁺	-	2845	1046	3031
2 ⁺	D_{s2}^*	2283	1281	2618	(3/2) ⁺	$\Omega_c(2770)$	2524	1281	2830	1 ⁺	-	3012	1281	3273
0 ⁻	$\bar{B}(5280)$	5130	0	5130	-	-	-	-	-	-	-	-	-	-
1 ⁺	$\bar{B}_1(5720)$	5385	1046	5486	(1/2) ⁺	$\Lambda_b(5620)$	5460	1046	5559	0 ⁺	$B_J(5732)$	5775	1046	5869
2 ⁻	$\bar{B}_J(5970)$	5560	1479	5753	(3/2) ⁻	$\Lambda_b(5920)$	5714	1479	5902	1 ⁻	-	5999	1479	6179
0 ⁻	$B(5280)$	5130	0	5130	-	-	-	-	-	-	-	-	-	-
1 ⁺	$B_1(5720)$	5385	1046	5486	(1/2) ⁺	$\Sigma_b(5815)$	5460	1046	5559	0 ⁺	$\bar{B}_J(5732)$	5775	1046	5869
1 ⁻	$B^*(5325)$	5130	740	5183	-	-	-	-	-	-	-	-	-	-
2 ⁺	$B_2^*(5747)$	5385	1281	5535	(3/2) ⁺	$\Sigma_b^*(5835)$	5460	1281	5608	1 ⁺	$B_J(5840)$	5775	1281	5915
0 ⁻	$B_s(5365)$	5292	0	5292	-	-	-	-	-	-	-	-	-	-
1 ⁺	$B_{s1}(5830)$	5528	1046	5626	(1/2) ⁺	$\Xi_b(5790)$	5610	1046	5707	0 ⁺	$\bar{B}_{s0}^?$	5936	1046	6027
1 ⁻	$B_s^*(5415)$	5292	740	5343	-	-	-	-	-	-	-	-	-	-
2 ⁺	$B_{s2}^*(5840)$	5528	1281	5674	(3/2) ⁺	$\Xi_b^*(5950)$	5610	1281	5754	1 ⁺	$B_{s1}^?$	5936	1281	6073
1 ⁺	$B_{s1}^?$	5528	1046	5626	(1/2) ⁺	$\Omega_b(6045)$	5791	1046	5885	0 ⁺	-	6110	1046	6199

Table 3: Computed masses (in MeV), using the 't Hooft potential, for hadrons with one heavy quark, compared to the PDG data [8].

Meson					Baryon					Tetraquark				
$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M	$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M	$J^{P(C)}$	Name	M_{\parallel}	M_{\perp}	M
0 ⁺⁺	$\eta_c(2984)$	2927	0	2927	-	-	-	-	-	-	-	-	-	-
1 ⁺⁻	$h_c(3525)$	3440	1046	3596	(1/2) ⁺	$\Xi_{cc}^{\text{SELEX}}(3520)$	3254	1046	3418	0 ⁺⁺	$\chi_{c0}(3415)$	3864	1046	4003
1 ⁻⁻	$J/\psi(3096)$	2927	740	3019	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{c2}(3556)$	3440	1281	3671	(3/2) ⁺	$\Xi_{cc}^{\text{LHCb}}(3620)$	3254	1281	3497	1 ⁺⁺	$\chi_{c1}(3510)$	3580	1281	3802
1 ⁻⁻	$\psi'(3686)$	3440	1281	3671	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{c2}(3927)$	3794	1654	4139	(3/2) ⁺	Ξ_{cc}^*	3751	1654	4099	1 ⁺⁺	$X(3872)$	4062	1654	4386
1 ⁻⁻	$\Upsilon(2S)(10020)$	9776	1281	9860	-	-	-	-	-	1 ⁺⁻	$Z_c(3900)$	4240	1654	4551
0 ⁺⁺	$\eta_b(9400)$	9424	0	9424	-	-	-	-	-	-	-	-	-	-
1 ⁺⁻	$h_b(9900)$	9776	1046	9832	(1/2) ⁺	$\Xi_{bb}^?$	9750	1046	9806	0 ⁺⁺	$\chi_{b0}(9860)$	10285	1046	10338
1 ⁻⁻	$\Upsilon(9460)$	9424	740	9453	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{b2}(9910)$	9776	1281	9860	(3/2) ⁺	$\Xi_{bb}^?$	9750	1281	9834	1 ⁺⁺	$\chi_{b1}(9893)$	10081	1281	10162
1 ⁻⁻	$\Upsilon(2S)(10020)$	9776	1281	9860	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{b2}(10270)$	10024	1654	10160	(3/2) ⁺	$\Xi_{bb}^?$	10100	1654	10234	1 ⁺⁺	$X_b^?$	10425	1654	10555

Table 4: Computed masses (in MeV), using the 't Hooft potential, for hadrons with two heavy quarks, compared to the PDG data [8].