PROCEEDINGS OF SCIENCE

Possible discrepancies in the spectrum of GUT models*

Elizabeth Dobson,* **Axel Maas and Bernd Riederer**

Institute of Physics, NAWI Graz, University of Graz,
Universitätsplatz 5, Graz, Austria
E-mail: elizabeth.dobson@uni-graz.at, axel.maas@uni-graz.at, bernd.riederer@uni-graz.at

Analytical investigations and lattice simulations suggested that the spectrum of observable particles in BSM-like theories may differ from the elementary ones. We consider a GUT-like toy theory, which showed such qualitative discrepancies in exploratory lattice simulations, and perform a detailed quantitative investigation of its physical spectrum, covering 18 different channels in total. We find indeed a substantial different spectrum than the elementary one. In particular, stable states with quantum numbers different from the elementary ones are observed.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

*This project used the Vienna Scientific Cluster (VSC). ED and BR have been supported by the Austrian Science Fund FWF, grant P32760.

*Speaker

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PoS(ICHEP2022)425

1. Introduction

Many candidate models for new physics involve additional scalar fields coupled to an enlarged gauge group $\mathcal{G}$. Grand Unified Theories (GUTs), in particular, are appealing due to their minimal theoretical assumptions and their ability to explain anomaly cancellation and the quantization of electric charge in the Standard Model (SM) [1]. Typically, the 'unified' group $\mathcal{G}$ is broken down to the SM group by a Brout-Englert-Higgs (BEH) effect, with the scalars acquiring a symmetry-breaking vacuum expectation value (vev). A phenomenological necessity is to require that the low-energy effective spectrum of a GUT coincides with that of the SM. Importantly, how various degrees of freedom are 'frozen out' is not relevant, nor the nature of physics at intermediate scales. To find this low-energy spectrum, it is usually assumed both that a perturbative expansion in the couplings is valid, and also that, as a consequence, the freezing out of degrees of freedom can be described by the gauge group $\mathcal{G}$ 'spontaneously' breaking down to a 'low-energy' subgroup $\mathcal{H}$, coinciding with the SM one [1].

On a formal level, this picture is obstructed by Elitzur's theorem [2, 3], which shows that spontaneous breaking of local symmetries is not possible. In addition, the Gribov-Singer ambiguity invalidates perturbative Becchi-Rouet-Stora-Tyutin (BRST) construction as a means to identify the physical degrees of freedom in the non-Abelian case, even at arbitrarily weak coupling [4–6]. The consequence is that manifest, nonperturbative gauge-invariance is required for physical observables. By necessity, only composite states of the elementary fields can thus be observable. This is also true in the SM, but its special structure cancels consequences up to subleading contributions [3]. There, a one-to-one mapping between elementary and composite states arises, allowing them to be treated as (almost) equivalent: see [7] for a review.

Beyond the SM, this is generally no longer possible [7–9]. Especially in GUTs, the one-to-one mapping between elementary particles and physical, composite particles breaks down, which has also been confirmed in exploratory lattice calculations [7, 10, 11]. However, first studies [10] indicate it appears to be still possible to analytically describe the physical spectrum [8, 9] using perturbation theory augmented by the Fröhlich-Morchio-Strocchi (FMS) mechanism [3, 7].

Extending the exploratory investigations [10, 12] we find that, even for a simple toy model, qualitative differences arise between the elementary spectrum and the physical spectrum. Crucially, these discrepancies arise even at arbitrarily weak gauge coupling.

2. The physical spectrum of a toy GUT

Since simulating a realistic GUT is not computationally feasible, we instead consider a GUT-like toy model: SU(3) Yang–Mills theory coupled to a single fundamental scalar field φ:

$$\mathcal{L} = -\frac{1}{4} W_{\mu\nu}^{(a)} W^{\mu\nu}_{(a)} + (D_\mu \varphi)^\dagger (D^\mu \varphi) - V(\varphi^\dagger \varphi) \quad \text{with} \quad V = \lambda(\varphi^\dagger \varphi - f^2)^2 \,. \tag{1}$$

Starting from this toy model, we measure the physical spectrum via lattice simulations and compare it to the elementary spectrum. This allows to assess the impact of enforcing gauge invariance. Despite its simplicity, this model still has several important features: the potential allows for a BEH effect and the nonabelian gauge group implies the failure of the BRST construction. In addition, there is a nontrivial U(1) global group under which the scalars carry a charge of $\frac{1}{3}$. Thus, in contrast to the SM, the global group and the gauge group are not of the same type, and the global group is smaller than the gauge group. It is this feature, which is common to most GUTs, which appears to be decisive in the emerging mismatch between the elementary spectrum and the physical spectrum [7, 9]. Investigating this model therefore allows to study generic features that arise with larger gauge groups than that of the SM. In addition, predictions from FMS-augmented perturbation theory are available for some channels of this theory [8, 12].

Gauge-invariant states can only be characterized by their spin J, parity P, and either a global U(1) charge or a charge parity C. We denote uncharged channels by J^{PC} and unit-charged channels by J_1^P. Note that to every particle with non-vanishing U(1) charge there is a corresponding anti-particle. We will not consider higher charged states, and in the following, will consider all states with $J \leq 2$, summing up to 18 different channels.

The physical composite operators are created in analogy to QCD. These are gaugeballs made entirely out of gauge bosons, uncharged states containing an equal amount of scalars and antiscalars, and U(1)-charged states which contain a multiple of three (anti-)scalars as well as an equal amount of further scalars and anti-scalars. Of course, operators in the same channel mix, especially gaugeballs with scalar-antiscalar states. Gaugeball operators are constructed in complete analogy to glueballs in QCD. In the other cases, we consider operators with the minimum number of scalar fields. These can be constructed as tensor products with a Clebsch-Gordan construction from the basis operators [8, 13, 14]

$$O^M_{\mu_1,\ldots\mu_N} = \varphi^\dagger D_{\mu_N}\ldots D_{\mu_1}\varphi \tag{2}$$

$$O^B_{\{\mu_i\},\{\nu_i\},\{\rho_i\}} = \epsilon_{abc}(D_{\mu_{N_\mu}}\ldots D_{\mu_1}\varphi)_a(D_{\nu_{N_\nu}}\ldots D_{\nu_1}\varphi)_b(D_{\rho_{N_\rho}}\ldots D_{\rho_1}\varphi)_c, \tag{3}$$

where O^M creates uncharged states and O^B creates charged states, and Latin indices are gauge indices. The similarity to mesons and baryons in QCD is striking.

On the other hand, in presence of a BEH effect which breaks the gauge group SU(3) down to SU(2), the elementary spectrum would be [8] 3 massless, 4 degenerate and 1 heavy vector bosons as well as a single, uncharged, massive scalar. All other channels would only contain scattering states.

3. The spectrum from lattice simulations

Since O^M and O^B are bound state operators, it is necessary to perform a nonperturbative evaluation of the induced spectrum, at the very least to identify suitable analytical approximation methods. For that purpose, we used lattice simulations. We obtained lattice versions of the operators O^M and O^B using the subduction method of [15], including the different representations of the discrete lattice rotation group. We combined this with semaring as well as multiple distinct operators in most channels, including some scattering operators in some channels. We used the enhanced variational analysis of [16] to extract the energy levels in the different channels. Simulations were done on multiple symmetric volumes up to 32^4. This turned out be sufficient for reliable infinite-volume extrapolation. More technical details can be found in [10, 13, 14]. We used the results of [10] to identify useful simulation points. We will present here results for two cases, one in the region where the BEH effect occurs in a gauge-fixed setup and the (gauge) coupling is weak, and one where this is not the case. This theory, as scalar-gauge theories in general, have a low signal-to-noise ratio. Thus, our results are statistics-limited.

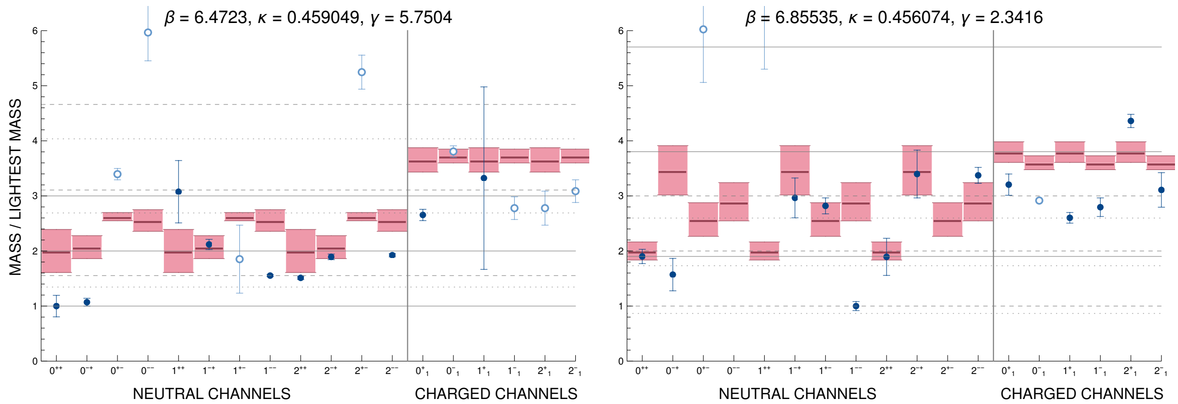

Figure 1: Spectra, rescaled to the lightest state, for simulation at parameters without BEH effect (left, $\beta = 6.4723$, $\kappa = 0.459049$, $\gamma = 5.7504$ [10]) and with BEH effect (right, $\beta = 6.85535$, $\kappa = 0.456074$, $\gamma = 2.3416$ [10]). The results are extrapolated to infinite volume and the errors are statistical only. Empty circles indicate that only an upper bound could be obtained. The grey horizontal lines give (multiples) of the basic scales of the FMS mechanism, extracted from the uncharged scalar and vector masses [10]. These correspond to the Higgs mass (line), the heaviest gauge boson mass (dashed line) and the lighter gauge boson mass (dotted line) (being $\sqrt{3/4}$ of the heavier mass at tree-level). The relevant elastic thresholds in each channel are shown as red blocks.

To understand the low-lying spectrum, we identified the lightest stable charged and neutral states and from these the trivial scattering states in all channels. This allows us to identify genuine bound states, which lie below the elastic threshold. Excited states could in principle be obtained as in [16], but at the moment statistics are insufficient.

4. The fundamental spectrum

The spectra for the different parameter sets are plotted in fig. 1, and are consistent with previous results [10, 12] in available channels. A striking feature is that we always observe a mass gap, in stark contrast to the ungapped elementary spectrum. So far, previous investigations [10] suggest that this is not caused by nonperturbative, strong interactions in the unbroken SU(2) group. Especially, ungapped physical spectra have been observed in other theories with a BEH effect [11]. Furthermore, we confirm previous results [7, 11] that in the 'Higgs-like' case the lightest state is a vector, while otherwise it is a scalar particle. In the elementary spectrum, this is not necessary.

Besides this, we find another uncharged stable state in both cases in the 0^{-+} channel. Within errors, it is found to be degenerate with the expected [8] 0^{++} channel. This requires further investigation, as this state had not been studied before, and the presence of a bound state here is unexpected. In particular, although in the 'QCD-like' case the tensor state 2^{++} is stable, we note that it is much heavier than would be expected from SU(3) Yang–Mills theory in comparison to the (stable) 0^{-+} and 1^{--} states. Thus, the couplings are such that the scalar dynamics is not suppressed. In contrast, in the 'Higgs-like' case we do not observe any further uncharged stable states, which are statistically unambiguously below the elastic threshold, in agreement with [8, 9].

The charged sector is very interesting. In both cases we observe that charged states are generically heavier than uncharged ones, as would be naively expected in a constituent model based on (2-3) [8]. In the 'QCD-like' case, most of the charged states are heavy, and it is not yet statistically significant, whether the lightest state is a scalar or a vector. This is different in the 'Higgs-like' case, where the vector states are lighter. Furthermore, there are two vector states of opposite parity, which are, within statistical uncertainty, degenerate. In addition, both in the scalar and the tensor channel states appear which seem to be bound. All of these states carry three times the custodial charge of the elementary scalar, and would not have been expected in a tree-level analysis. Indeed, even beyond the elementary spectrum only the vector state has been anticipated and previously observed [8, 10, 12]. For the other states not yet an understanding exists, and requires further investigations. Such a rich charged spectrum would have not been anticipated based on usually analyses of GUTs [1].

5. Analytically understanding the spectrum

FMS augmented perturbation theory [3, 7] has been used to analyse the spectrum of this theory [8]. Essentially, this approach expands gauge-invariant composite operators in a fixed gauge in the Higgs vev in a suitable gauge, e. g. 't Hooft gauge, as $\varphi(x) = v\hat{n} + \eta(x)$, and then proceeds perturbatively. Consider the 0^{++} channel. A suitable, gauge-invariant, composite operator is $(\varphi^\dagger\varphi)(x)$. Substituting the vev splitting into its two-point function yields a gauge-invariant sum

$$\langle(\varphi^\dagger\varphi)(x)(\varphi^\dagger\varphi)(y)\rangle_{\text{con.}} = v^2\langle h(x)h(y)\rangle_{\text{con.}} + 2v\langle h(x)(\eta^\dagger\eta)(y)\rangle_{\text{con.}} + \langle(\eta^\dagger\eta)(x)(\eta^\dagger\eta)(y)\rangle_{\text{con.}}\,, \quad (4)$$

where $h(x) = \text{Re}[\hat{n}^\dagger\eta(x)]$ denotes the usual elementary (unphysical) Higgs field. This is still an exact statement: while the individual terms are gauge-dependent, their sum is not. The next step is to expand the matrix element in the usual perturbative series. At leading order, this leaves only the elementary Higgs itself and a scattering state of two Higgs particles. Thus, one obtains a pole at the Higgs mass for the right-hand side in this approximation, predicting the bound state to have the same mass. This can be extended beyond leading-order [17, 18], and appears to remain true to all orders [17]. It has also been confirmed in lattice simulations [10].

This establishes a one-to-one mapping in the SM [3, 7, 17, 18]. In the present theory this does not remain true for other channels [8]. E. g. in the uncharged vector channel to leading order

$$\langle(i\varphi^\dagger D_\mu\varphi)(x)(i\varphi^\dagger D_\nu\varphi)(y)\rangle_{\text{con.}} = v^2\langle W_\mu^8(x)W_\nu^8(y)\rangle_{\text{con.}} + O\left(\eta/v\right) + \ldots\,, \quad (5)$$

and thus is mapped to the heaviest gauge boson. For other investigated channels [8, 9] no two-point function on the right-hand side has been found — in fact, one does not expect that degenerate states exist in the physical spectrum [9]. This led to the prediction of the theory being gapped [8, 12].

This has two consequences. On the one hand, it is to be expected that only the elementary mass scales, the mass of the Higgs and the two masses in the gauge boson spectrum, determine the masses of the physical spectrum. These have been included in fig. 1. In the 'Higgs-like' case it indeed appears that states cluster around multiples of these scales, confirming the prediction. The other is that the charged states cannot be mapped to an elementary state. Thus, the only explanation for the lightest state is that it is a non-trivial bound state. It was attempted to understand it within a leading-order constituent model [8, 10]. Lattice data seemed to agree with it at first, though with larger errors [10]. The data shown in fig. 1, which are more precise, seem to be less in favor. This also requires further understanding. Especially, if augmented perturbation theory beyond the leading order can determine their masses, or whether these states are genuinely nonperturbative.

6. Conclusions and outlook

We have shown that the physical spectrum of a 'toy GUT' does not coincide with the elementary spectrum, in agreement with theoretical expectations [8] and previous exploratory lattice simulations [10, 12]. At least the studied [8, 9] uncharged states seem to be in reasonable agreement with FMS-augmented perturbative predictions, but other channels seem to hold further challenges. Our results therefore strongly support the general conclusion [8, 9] that the elementary spectrum is not an adequate proxy for the low-energy spectrum of a GUT.

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