PROCEEDINGS OF SCIENCE

Automation of Antenna Subtraction in Colour Space

Matteo Marcoli[a,*]

[a] *Physik-Institut, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland*

E-mail: mmarcoli@physik.uzh.ch

In the past decade, antenna subtraction has been used to compute NNLO QCD corrections to a series of phenomenologically relevant processes. However, as for other subtraction schemes at NNLO, the application of this method proceeded in a process-dependent way, with each new calculation requiring a significant amount of work. In this talk we present an improved version of antenna subtraction which aims at achieving an automated and process-independent generation of the subtraction terms required for a NNLO calculation, as well as at overcoming some intrinsic limitations present in the traditional formulation. In this new approach, a set of integrated dipoles is used to reproduce the known infrared singularity structure of one- and two-loop amplitudes in colour space. The real-virtual and double-real subtraction terms are subsequently generated inferring their structure from the corresponding integrated subtraction terms. We demonstrate the applicability of this method computing the full-colour NNLO correction to hadronic three-jet production in the gluons-only assumption.

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*Speaker

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1. Introduction

In this talk we present the colourful antenna subtraction formalism for gluonic processes, which is described in detail in [1].

In intermediate steps of higher order calculations in QCD, infrared infinities arise either from the integration over the loop momenta in virtual corrections or from soft and collinear emissions in real corrections. For sufficiently inclusive observables, called infrared-safe observables, such singularities cancel to yield the final finite result, after a proper regularization procedure is adopted. This is achieved implementing a subtraction method. The state-of-the-art of perturbative computations in QCD is represented by next-to-next-to-leading order (NNLO) calculations, with next-to-next-to-next-to-leading order (N^{3}LO) corrections available for benchmark processes.

The NNLO antenna subtraction method [2, 3] is based up to now on the identification of single and double real radiation patterns in colour-ordered subprocess contributions and has been applied successfully in computing NNLO corrections to a variety of hadron-collider processes [4–13]. However, the efficiency of the present formulation scales poorly with the number of external partons and its application to processes involving four or more external partons is extremely challenging. One reason for this is the proliferation of infrared limits in real emission corrections for high-multiplicity processes. Moreover, in the context of antenna subtraction, the treatment of contributions beyond the leading colour approximation is highly non-trivial, due to the appearance at the matrix element level of incoherent interferences between different colour orderings, which cannot be straightforwardly addressed with the traditional technique.

It is the objective of the colourful antenna subtraction method to overcome these limitations and to achieve a more general and process-independent formulation of the antenna subtraction. The primary goals are the definition of a systematic procedure for the generation of the entire subtraction infrastructure at NNLO and the more efficient treatment of colour correlations within matrix elements to directly retain the full N_c-dependence.

The main idea behind the new approach consists in relying on the predictability of the singularity structure of one- and two-loop amplitudes in colour space to automatically generate virtual subtraction terms, which cancel the explicit poles of virtual corrections. Subsequently, the one-to-one correspondence between antenna functions and their integrated counterparts is exploited to systematically infer real subtraction terms which can be used to remove the divergent behaviour of real emission corrections in the infrared limits. The colourful antenna subtraction method at NLO is briefly described in the following. The treatment of the NNLO correction is too sizeable to be discussed here and it is described in detail in [1]. Nevertheless, the underlying logic is completely analogous to the NLO case, with natural complications due to the more involved infrared structure of the NNLO scenario.

2. Colourful antenna subtraction at NLO

The NLO QCD correction to an n-jet partonic cross section with parton species a and b in the initial state is given by [3]:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n \left(\mathrm{d}\hat{\sigma}^V_{ab,NLO} + \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}\right) + \int_{n+1} \mathrm{d}\hat{\sigma}^R_{ab,NLO}, \tag{1}$$

where the symbol $\int_n$ indicates an integration over the n final state particles. $\mathrm{d}\hat{\sigma}^V_{ab,NLO}$ and $\mathrm{d}\hat{\sigma}^R_{ab,NLO}$ respectively represent the virtual and real corrections, while $\mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}$ is the NLO mass factorization counterterm. Due to the emergence of infrared divergences in both the virtual and real corrections, a subtraction procedure is needed to numerically evaluate (1). In the context of antenna subtraction, this is achieved constructing a real subtraction term $\mathrm{d}\hat{\sigma}^S_{ab,NLO}$ [3], which locally removes the singular behaviour of $\mathrm{d}\hat{\sigma}^R_{ab,NLO}$ in the IR limits and can be analytically integrated over the phase space of the unresolved radiation. This latter feature is required to obtain from $\mathrm{d}\hat{\sigma}^S_{ab,NLO}$ the virtual subtraction term $\mathrm{d}\hat{\sigma}^T_{ab,NLO}$, which cancels the explicit poles of the virtual correction and contains the mass factorization contribution. The NLO cross section can then be reformulated as [3]:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n \left[\mathrm{d}\hat{\sigma}^V_{ab,NLO} - \mathrm{d}\hat{\sigma}^T_{ab,NLO}\right] + \int_{n+1} \left[\mathrm{d}\hat{\sigma}^R_{ab,NLO} - \mathrm{d}\hat{\sigma}^S_{ab,NLO}\right], \tag{2}$$

with

$$\mathrm{d}\hat{\sigma}^T_{ab,NLO} = -\int_1 \mathrm{d}\hat{\sigma}^S_{ab,NLO} - \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}. \tag{3}$$

The singularity structure of renormalized $(n+2)$-parton one-loop amplitudes in QCD can be described in colour space with [14]:

$$|\mathcal{A}^1_{n+2}\rangle = \boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right)|\mathcal{A}^0_{n+2}\rangle + |\mathcal{A}^{1,\mathrm{fin}}_{n+2}(\mu_r^2)\rangle\,, \tag{4}$$

where μ_r is the renormalization scale, $|\mathcal{A}^{1,\mathrm{fin}}_{n+2}(\mu_r^2)\rangle$ is a finite remainder and $\boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right)$ is Catani's IR insertion operator [14], which can be rewritten as

$$\boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right) = \sum_{(i,j)} \left(\boldsymbol{T}_i\cdot\boldsymbol{T}_j\right)\mathcal{I}^{(1)}_{ij}\left(\epsilon,\mu_r^2\right), \tag{5}$$

where in the last line the sum runs over pairs of partons. For the gluons-only case that is considered in this talk, we only need the expression of $\mathcal{I}^{(1)}_{i_g j_g}\left(\epsilon,\mu_r^2\right)$ at $N_f=0$:

$$\mathcal{I}^{(1)}_{i_g j_g}\left(\epsilon,\mu_r^2\right) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon^2}+\frac{b_0}{\epsilon}\right]\left(\frac{-s_{ij}}{\mu_r^2}\right)^{-\epsilon}, \qquad b_0=\frac{11}{6}. \tag{6}$$

Using (5) it is possible to write down the IR singularity structure of the virtual correction in the following general way:

$$\begin{aligned}\mathcal{P}oles\left(\hat{\sigma}^V_{gg,NLO}\right) \;&=\; \mathcal{N}^V_{NLO}\int \mathrm{d}\Phi_n(p_3,\ldots,p_{n+2};p_1,p_2)\,J^{(n)}_n(\{p\}_n)\\ &\times\mathcal{P}oles\left[\sum_{(i_g,j_g)}\langle\mathcal{A}^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|\mathcal{A}^0_{n+2}\rangle\,2\,\mathrm{Re}\left(\mathcal{I}^{(1)}_{i_g j_g}\left(\epsilon,\mu_r^2\right)\right)\right], \end{aligned}\tag{7}$$

where the factor $\mathcal{N}^V_{NLO}$ is an appropriate overall normalization. In the colourful antenna subtraction approach, we exploit the previous result to directly construct the NLO virtual subtraction term. To do so, we define a NLO singularity dipole operator in colour space for an $(n+2)$-parton process:

$$\begin{aligned}\boldsymbol{\mathcal{J}}^{(1)}(\epsilon) \;&=\; \sum_{(i,j)\geq 3}\left(\boldsymbol{T}_i\cdot\boldsymbol{T}_j\right)\mathcal{J}^{(1)}_2(i_g,j_g) + \sum_{i\neq 1,2}\left(\boldsymbol{T}_1\cdot\boldsymbol{T}_i\right)\mathcal{J}^{(1)}_2(1_g,i_g)\\ &+\sum_{i\neq 1,2}\left(\boldsymbol{T}_2\cdot\boldsymbol{T}_i\right)\mathcal{J}^{(1)}_2(2_g,i_g) + \left(\boldsymbol{T}_1\cdot\boldsymbol{T}_2\right)\mathcal{J}^{(1)}_2(1_g,2_g)\,. \end{aligned}\tag{8}$$

The first sum in the previous formula runs over all pairs of gluons in the final state, the second and the third sums include all pairs with an initial-state gluon (respectively 1_g and 2_g) and a final-state one and the last term addresses the configuration where both gluons are in the initial state. The scalar functions $\mathcal{J}_2^{(1)}(i,j)$ are colour stripped one-loop integrated dipoles [3, 15], given by a combination of integrated three-parton tree-level antenna functions and NLO mass factorization kernels. The explicit expressions of the gluon-gluon integrated dipoles for final-final (FF), initial-final (IF) and initial-initial (II) configurations are the following:

$$\begin{aligned}\mathcal{J}_2^{(1)}(i_g,j_g) &= \frac{1}{3}\mathcal{F}_3^0(s_{ij}), \quad \mathcal{J}_2^{(1)}(1_g,j_g) = \frac{1}{2}\mathcal{F}_{3,g}^0(s_{1j}) - \frac{1}{2}\Gamma_{gg}^{(1)}(x_1)\,, \\ \mathcal{J}_2^{(1)}(1_g,2_g) &= \mathcal{F}_{3,gg}^0(s_{12}) - \frac{1}{2}\Gamma_{gg}^{(1)}(x_1)\,\delta_2 - \frac{1}{2}\Gamma_{gg}^{(1)}(x_2)\,\delta_1,\end{aligned} \tag{9}$$

where $\delta_i = \delta(1-x_i)$. The functions $\mathcal{F}_3^0$, $\mathcal{F}_{3,g}^0$ and $\mathcal{F}_{3,gg}^0$ are gluon-gluon three-parton integrated antenna functions [2, 16]. The integrated dipoles in (9) incorporate the mass factorization counterterm, as indicated by the presence of the gluon-gluon splitting kernels $\Gamma_{gg}^{(1)}$. The poles carried by the mass factorization kernels cancel with poles in the integrated initial-final and initial-initial antenna functions associated with initial-state collinear divergences. The remaining ϵ-poles exactly match the ones of the virtual matrix element, once the operator in (8) is evaluated on the corresponding Born-level amplitude in colour space. In particular, at one loop the following relation holds:

$$\mathcal{P}oles\left[\mathcal{J}_2^{(1)}(i_g,j_g)\right] = \mathcal{P}oles\left[\mathrm{Re}\left(\mathcal{I}_{i_g j_g}^{(1)}\left(\epsilon,\mu_r^2\right)\right)\right]. \tag{10}$$

It is then possible to express the NLO virtual subtraction term as

$$\begin{aligned}\mathrm{d}\hat{\sigma}_{gg,NLO}^T &= \mathcal{N}_{NLO}^V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n(p_3,\ldots,p_{n+2};x_1p_1,x_2p_2)J_n^{(n)}(\{p\}_n) \\ &\quad \times 2\,\langle \mathcal{A}_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{A}_{n+2}^0\rangle\,.\end{aligned} \tag{11}$$

We remark that (11) is a completely general result in the case of gluon scattering: it is valid for any number of external legs and retains the full N_c dependence.

The real subtraction term at NLO is systematically obtained from (11) relying on the one-to-one correspondence between integrated and unintegrated antenna functions:

$$\mathcal{X}_3^0(s_{ij}) \leftrightarrow X_3^0(i,k,j), \tag{12}$$

where $\mathcal{X}_3^0(s_{ij})$ is the integrated antenna function obtained integrating the tree-level three-parton antenna function $X_3^0(i,k,j)$ over the phase space of the unresolved parton k. Due to this relation, once the virtual subtraction term is obtained, the structure of the real subtraction term can be completely determined by inserting an unresolved gluon between each pair of hard radiators appearing in the integrated dipoles. The procedure to obtain $\mathrm{d}\hat{\sigma}_{NLO,gg}^S$ from $\mathrm{d}\hat{\sigma}_{NLO,gg}^T$ can be then formulated as follows:

1. Removal of the splitting kernels from the integrated dipoles;
2. Transition from integrated three-parton antenna functions to unintegrated ones:

$$\begin{aligned}\text{FF:}\quad & \mathcal{F}_3^0(s_{ij}) \;\to\; 3\,f_3^0(i,k,j), \\ \text{IF:}\quad & \mathcal{F}_{3,g}^0(s_{1i}) \;\to\; 2\,f_{3,g}^0(1,k,i), \\ \text{II:}\quad & \mathcal{F}_{3,gg}^0(s_{12}) \;\to\; F_{3,gg}^0(1,k,2),\end{aligned} \tag{13}$$

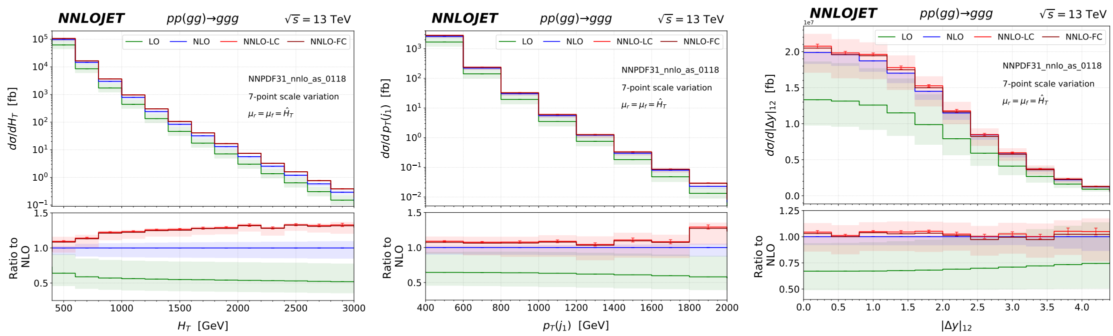

Figure 1: Differential distributions in $H_T = \sum_{j\in jets} p_{T,j}$ (left), transverse momentum of the leading jet (centre) and rapidity difference between the two leading jets (right). NNLO-LC and NNLO-FC respectively indicate the NNLO correction in the leading colour approximation and in full colour.

3. Momenta relabeling within colour interferences and jet functions according to the accompanying antenna function;

4. Sum over permutations of the $n + 3$ momenta to cover all possible IR limits and symmetry factor adjustments;

3. Results and conclusions

We implemented the colourful antenna subtraction method to construct the subtraction infrastructure required for the calculation of the NNLO correction to the gluons-only process $gg \to ggg$. This computation is part of the NNLO correction to 3-jet production, recently presented in [17], and demonstrates the applicability of the colourful antenna approach to the construction of NNLO subtraction terms for a highly non-trivial high-multiplicity process. A selection of results, obtained in the NNLOJet framework, is presented in Figure 1 to illustrate the quality of numerical convergence that can be obtained with the generated subtraction terms.

The natural next step for the development of the described approach is the extension to subprocesses involving quarks. Consistent work has already been performed in this direction and the goal remains the definition of a complete, process-independent and systematic procedure for the generation of the subtraction terms in the context of antenna subtraction.

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