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Aspects of an $\mathcal{N}=1$ split-like supersymmetric model resulting from the dimensional reduction of an $\mathcal{N}=1$, $10D$, E_8 gauge theory over a modified flag manifold

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We review a supersymmetric extension of the Standard Model which originates from a $10D$, $\mathcal{N}=1$, E_8 gauge theory. The original theory is dimensionally reduced over the $SU(3)/U(1)\times U(1)\times\mathbb{Z}_3$ space and, after making use of the Wilson breaking mechanism, the resulting $4D$ theory is an $\mathcal{N}=1$, $SU(3)^3\times U(1)^2$ Grand Unified Theory. Below the unification scale the remaining model is viewed as a split-like supersymmetric version of the Standard Model with two global $U(1)$s. The unification scale is predicted at $\sim 10^{16}$GeV, the model is proton-decay safe and the lightest supersymmetric particles acquire masses at the region of a few TeV.

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1. Introduction

Our study is a realistic example of the fundamental works of Forgacs-Manton (F-M), the Coset Space Dimensional Reduction (CSDR) [1–3] and Scherk-Schwartz (S-S) [4], the group manifold reduction. In our work on CSDR taking into account the number of dimensions and starting gauge group as predicted by the heterotic string, we share with the latter the common ground that they might lead to promising Grand Unified Theories (GUTs). A few remarkable features of the CSDR are that the resulting $4D$ theory is totally calculable before spontaneous symmetry breakings. In particular the kinetic fermion terms lead also to to $4D$ Yukawa interactions and the theory can be chiral [5]. When the reduction is done over non-symmetric coset spaces, the CSDR leads to softly broken $N=1$, at least if the higher-dimensional theory is defined in $10D$ [6–10].

In this particular model, the initial, higher-dimensional theory is a $10D$, $\mathcal{N}=1$, E_8 gauge theory whose spectrum is minimal, consisting solely of a vector supermultiplet. The CSDR mechanism (and a breaking by Wilson lines) is performed over the $SU(3)/U(1)^2\times\mathbb{Z}_3$ and the remaining $4D$ GUT is a softly broken $\mathcal{N}=1$ supersymmetric $SU(3)^3$ [2, 7–9, 11].

A specific choice of small compactification space radii breaks the $SU(3)^3\times U(1)^2$ gauge group at a unification scale $\sim 10^{16}$ GeV, resulting in a split-like supersymmetric scenario, in which gauginos, Higgs fields and Higgsinos (of the third generation) and a singlet field that originates from the higher-energy theory all acquire masses at the TeV scale, and the rest supesymmetric spectrum is superheavy. The heavy states are integrated out many orders of magnitude above the TeV scale, leading to additional interaction between the light states which will be taken into account. The full analysis of a version of this model can be found in our recent work [12, 13].

2. Dimensional Reduction of E_8 over $SU(3)/U(1)\times U(1)$

When we apply directly the CSDR in our specific case, that is the $10D$, $\mathcal{N}=1$, E_8 Yang-Mills-Dirac theory with Weyl-Majorana fermions over the non-symmetric coset space $SU(3)/U(1)^2$ [2, 7], the produced $4D$ action is:

$$S = C\int d^4x\,\mathrm{tr}\left[-\frac{1}{8}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}(D\phi_a)^2\right]+V(\phi)+\frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi-\frac{i}{2}\bar{\psi}\Gamma^a D_a\psi\,, \tag{1}$$

where $V(\phi)$ is given as:

$$V(\phi) = -\frac{1}{8}g^{ac}g^{bd}\mathrm{tr}\left(f_{ab}^{\ \ C}\phi_C - ig[\phi_a,\phi_b])(f_{cd}^{\ \ D}\phi_D - ig[\phi_c,\phi_d]\right) \tag{2}$$

and $\mathrm{tr}(T^iT^j)=2\delta^{ij}$, where T^i are the E_8 generators. Also, g is the coupling constant, C is the coset volume, $D_\mu=\partial_\mu - igA_\mu$, D_a are the $4D$ covariant derivative and the coset space covariant derivative, respectively and, last, g_{ab} is the metric of the coset space, given by $g_{\alpha\beta}=\mathrm{diag}(R_1^2,R_1^2,R_2^2,R_2^2,R_3^2,R_3^2)$. $V(\phi)$ is only formal since ϕ must satisfy $f_{ai}^D\phi_D-[\phi_a,\phi_i]=0$. The $4D$ gauge group is determined by the centralizer of $U(1)\times U(1)$ in E_8:

$$H = C_{E_8}(U(1)_A\times U(1)_B) = E_6\times U(1)_A\times U(1)_B\,.$$

Moreover, the CSDR rules determine the representations of the particles that consist the particle spectrum of the $4D$ theory (details in [2, 6, 7, 11]). Specifically the surviving gauge fields (of $E_6 \times U(1)_A \times U(1)_B$) fall into $\mathcal{N}=1$ vector supermultiplets whereas the matter fields fall into six $\mathcal{N}=1$ chiral ones, where three of the latter are E_6 singlets carrying $U(1)_A \times U(1)_B$ charges, while the rest have non-trivial transformation properties under the whole $4D$ gauge group. In particular, the matter fields transform under $E_6 \times U(1)_A \times U(1)_B$ as:

$$\alpha_i \sim 27_{(3,\frac{1}{2})},\ \beta_i \sim 27_{(-3,\frac{1}{2})},\ \gamma_i \sim 27_{(0,-1)},\ \alpha \sim 1_{(3,\frac{1}{2})},\ \beta \sim 1_{(-3,\frac{1}{2})},\ \gamma \sim 1_{(0,-1)}\,. \tag{3}$$

Regarding the potential of the theory, besides the terms identified as F- and D-terms, the rest are interpreted as soft scalar masses and trilinear soft terms. The gaugino mass is of geometrical origin and, for the appropriate choice of torsion, is at the TeV region.

3. Wilson flux breaking

The resulting $4D$ gauge group, $E_6 \times U(1)^2$, of CSDR on an E_8 gauge theory over the coset space $SU(3)/U(1)^2$ cannot be broken down to the gauge group of the Standard Model (SM). For that reason the Wilson flux breaking mechanism is introduced [14–16]. Since we need a multiply connected coset space, the freely-acting discrete symmetry $\mathbb{Z}_3$ on $SU(3)/U(1)^2$ is employed, therefore the space on which the reduction is performed is now the $SU(3)/U(1)^2 \times \mathbb{Z}_3$ and the resulting gauge group is the $SU(3)^3$. For the trivial case, out of the three E_6 singlets α, β, γ of eq.(3) only one survives, specifically the $\alpha \equiv \theta_{(3,\frac{1}{2})}$. In turn, the $SU(3)^3$ representations of the non-trivial surviving matter fields are obtained by the decomposition $E_6 \supset SU(3)^3$, that is $27 = (1,3,\bar{3}) \oplus (\bar{3},1,3) \oplus (3,\bar{3},1)$ and are found to be the following:

$$\alpha_1 \equiv \Psi_1 \sim (1,3,\bar{3})_{(3,\frac{1}{2})}, \beta_3 \equiv \Psi_2 \sim (\bar{3},1,3)_{(-3,\frac{1}{2})}, \gamma_2 \equiv \Psi_3 \sim (3,\bar{3},1)_{(0,-1)}, \tag{4}$$

where the above are the parts of the three 27 chiral multiplets of $\alpha_i, \beta_i, \gamma_i$ of eq.(3) and combined they form one complete generation. In order to have three generations once more, non-trivial monopole charges in the $U(1) \times U(1)$ part of the coset needs to be introduced [17].

The employment of the Wilson flux breaking mechanism affects the scalar potential as well, in the sense that it can be rewritten from the E_6 language to the $SU(3)_c \times SU(3)_L \times SU(3)_R$ one (simplified to one generation) as $V = C + V_{susy} + V_{soft} = C + V_D + V_F + V_{soft}$ [11], where C is a constant of order 1/R. The F-terms derive from the superpotential which is given by $\mathcal{W} = \sqrt{40} d_{abc} \Psi_1^a \Psi_2^b \Psi_3^c$, the various D-terms are written as:

$$D^A = \frac{1}{\sqrt{3}} \langle \Psi_i | G^A | \Psi_i \rangle, \quad D_1 = 3\sqrt{\frac{10}{3}} (\langle \Psi_1 | \Psi_1 \rangle - \langle \Psi_2 | \Psi_2 \rangle), \tag{5}$$

$$D_2 = \sqrt{\frac{10}{3}} (\langle \Psi_1 | \Psi_1 \rangle + \langle \Psi_2 | \Psi_2 \rangle - 2\langle \Psi_3 | \Psi_3 \rangle - 2|\theta|^2) \tag{6}$$

and, last, the soft supersymmetry breaking terms are:

$$\begin{aligned} V_{soft} =& \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2}\right) \langle \Psi_1 | \Psi_1 \rangle + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2}\right) \langle \Psi_2 | \Psi_2 \rangle + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2}\right) (\langle \Psi_3 | \Psi_3 \rangle + |\theta|^2) \\ &+ 80\sqrt{2} \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_1 R_2}\right) (d_{abc} \Psi_1^a \Psi_2^b \Psi_3^c + h.c) \\ =& m_1^2 \langle \Psi_1 | \Psi_1 \rangle + m_2^2 \langle \Psi_2 | \Psi_2 \rangle + m_3^2 \Big(\langle \Psi_3 | \Psi_3 \rangle + |\theta|^2 \Big) + (\alpha_{abc} \Psi_1^a \Psi_2^b \Psi_3^c + h.c)\,. \end{aligned} \tag{7}$$

The multiplets of the fields found in (4) can be nicely expressed as:

$$\Psi_2 \to q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \; \Psi_3 \to Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}, \; \Psi_1 \to L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}.$$

4. Specification of parameters

Now that the theoretical framework has been established, one has to make two important assumptions. First, the compactification scale is considered to be high[1] and second, the compactification and unification scales coincide, $M_C = M_{GUT}$, which means that the scale of the three radii of the compactification scale is $R_l \sim \frac{1}{M_{GUT}}$. Without any further assumption this would lead to a superheavy supersymmetric spectrum (of $O(M_{GUT})$) and soft trilinear couplings. However, we can treat one of the radii (R_3) as slightly different than the others. This assumption leads to a split-like scenario, in which the squarks and sleptons being superheavy but the different radii yield a relation among the soft Higgs masses and the squark masses such that the 2-loop soft Higgs masses expressions undergo a partial cancellation and the end up in the $O(M_{TeV})$ region.

The breaking of the trinification group to the SM gauge group parts of the gauge group [18] can be triggered by $\langle L_s^{(3)}\rangle = \mathrm{diag}(0,0,V)$, $\langle L_s^{(2)}\rangle = \mathrm{anti-diag}(0,0,V)$, where the s index designates the scalar component of the supermultiplet. According to the configuration of the scalar potential, the above breaking gives vevs to the singlet of each family (not necessarily to all three), specifically in our case, $\langle\theta^{(3)}\rangle \sim O(TeV)$, $\langle\theta^{(1,2)}\rangle \sim O(M_{GUT})$. As far as the two abelian symmetries are concerned, they break due to $\langle\theta^{(1,2)}\rangle$ (in addition to $\langle L_s^{(2,3)}\rangle$), but their global versions remain in the theory. The electroweak breaking proceeds by the following vev configuration, $\langle L_s^{(3)}\rangle = \mathrm{diag}(v_d, v_u, 0)$.

Due to the presence of the global symmetries, invariant lepton Yukawa terms are not allowed in the Yukawa sector. Nevertheless, the $4D$ theory can be considered as renormalizable, therefore below the unification scale an effective term can emerge in the form of higher-dimensional operator $L\overline{e}H_d\left(\frac{\overline{K}}{M}\right)^3$ [11], where $\overline{K}$ denotes the vev of the conjugate scalar component of any combination of $S^{(i)}$, $\nu_R^{(i)}$ and $\theta^{(i)}$. Similar argumentation may also allow mass terms for $S^{(i)}$ and $\nu_R^{(i)}$, ending up to be superheavy. Moreover, appropriate higher-dimensional operators can be employed for the emergence of the μ-term, one for each family $H_uH_d\overline{\theta}\frac{\overline{K}}{M}$. Due to the vev configuration, it is understood that the μ terms corresponding to the Higgs doublets of the $l=1,2$ generations will be supermassive, while that of the $l=3$ generation will be at the TeV scale. Thus, for consistency reasons we include all operators of dimension 5,6 and 7.

5. Gauge Unification and Proton Decay

The first test for each GUT is to produce the prediction of the unification scale, M_{GUT}. We follow the straightforward methodology, namely the $a_{1,2}$ are used for the M_{GUT} calculation and the a_3 is used for confirmation. The 1-loop gauge β-functions are given by $2\pi\beta_i = b_i\alpha_i^2$, where the b_i coefficients vary for each of the three energy regions according to the corresponding particle

[1]Working with high compactification scale, any Kaluza-Klein excitations can be ignored.

spectrum [12]. We consider all superheavy particles to decouple at an intermediate scale M_{int}, which is considered a few orders of magnitude below the unification scale. Taking into account an uncertainty of 0.3% at the boundary of M_{GUT}, the various scales of our model are obtained: $M_{GUT} \sim 10^{16} GeV$, $M_{int} \sim 10^{14} GeV$, $M_{TeV} \sim 10^{3} GeV$. The calculation of the α_s finds

$$a_s(M_Z) = 0.1155 , \tag{8}$$

which is within 2σ of the experimental value, $a_s(M_Z) = 0.1187 \pm 0.0016$ [19].

Concerning the proton decay, the dangerous processes that could fall under the experimental limits of the proton halflife are the decays to $K^+\overline{\nu}$, $\pi^0\mu^+$, $\pi^0 e^+$, $\pi^+\overline{\nu}$ and $K^0\mu^+$. In superfield notation the terms that can account for all the above processes are:

$$\bullet Lq^cQ + QLq^c + h.c. \qquad \bullet\, QQQ + q^cq^cq^c + h.c. \tag{9}$$

From the combination of terms in the above expressions we can get several proton-decay dangerous processes. However, the presence of the two abelian symmetries forbids the terms of the second expression. Thus, proton decay cannot occur from such processes.

Since the model does not feature an R-parity, one can have superfast proton decay (from the process of the above diagram) if $L_iQ_j\overline{d}_k$ and $\overline{u}_i\overline{d}_j\overline{d}_k$ are *both* present. However, neither term exists in the model, since the former does not appear in the superpotential (that is derived from the initial theory) and cannot appear as a higher dimensional operator because of restrictions by the abelian symmetries, while the latter is forbidden by the abelian symmetries and cannot appear as a higher dimensional operator for the same reason. Consequently, the proton is stable in our model.

6. Conclusions

We considered a $10D$, $\mathcal{N} = 1$, E_8 Yang-Mills-Dirac theory with Weyl-Majorana fermions, constructed on the compactified spacetime of the form $M_4 \times B_0/\mathbb{Z}_3$, where B_0 is the coset space $SU(3)/U(1) \times U(1)$ and $\mathbb{Z}_3$ is a discrete group which acts freely on B_0. In order to result with the promising $4D$ (softly broken) $\mathcal{N} = 1$, $SU(3)^3$ GUT (plus two $U(1)$s), we employed two mechanisms: the CSDR and the Wilson flux breaking. The GUT breaking along with the assumption of a slight discrepancy between the radii of the coset led to a split-like supersymmetric scenario with the unification scale $\sim 10^{16}$GeV. The model is proton-decay protected from its abelian global symmetries.

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