

## Are Jets Narrowed or Broadened in e+A SIDIS?

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We compute the in-medium jet broadening to leading order in energy in the opacity expansion. At leading order in  $\alpha_s$  the elastic energy loss gives a jet broadening that grows with  $\ln E$ . The next-to-leading order in  $\alpha_s$  result is a jet narrowing, due to destructive LPM interference effects, that grows with  $\ln^2 E$ . We find that in the opacity expansion the jet broadening asymptotics are – unlike for the mean energy loss – extremely sensitive to the correct treatment of the finite kinematics of the problem; integrating over all emitted gluon transverse momenta leads to a prediction of jet broadening rather than narrowing. We compare the asymptotics from the opacity expansion to a recent twist-4 derivation and find a qualitative disagreement: the twist-4 derivation predicts a jet broadening rather than a narrowing. Comparison with current jet measurements cannot distinguish between the broadening or narrowing predictions. We comment on the origin of the difference between the opacity expansion and twist-4 results.

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## 1. Introduction

In the phenomenology of heavy ion collisions, equivalently high-energy nuclear physics, we're interested in the non-trivial, emergent, many-body dynamics of the strong nuclear force. The qualitative properties of this many-body physics is represented on the phase diagram of quantum chromodynamics (QCD). Heavy ion collisions probe the very low baryon chemical potential, high temperature region of this phase diagram. One of the (potentially) most precise tools to study the non-trivial dynamics of this high temperature, low baryon chemical potential region of the phase diagram is jets. In jet tomography, one hopes to place a well-controlled high-momentum parton in a nuclear medium and study the changes made to the measured jet (compared to a jet produced in vacuum, e.g. in p+p or  $e^+e^-$  collisions). The modification of the jet provides a measure of the fundamental degrees of freedom of the nuclear medium produced in a heavy ion collision and the dynamics of those degrees of freedom.

If we assume that the dynamics can be described by perturbative QCD (pQCD), then there are generally speaking two types of energy loss: collisional [1, 2] and radiative [3] (equivalently elastic and inelastic). When one uses reasonable models for these energy loss processes and a reasonable model for the nuclear medium generated in heavy ion collisions, then one finds a reasonable qualitative description of measured data over many decades of parameters [4].

Given the above success of leading order perturbative methods in describing data generated by heavy ion collisions, one may naturally ask what the next step(s) might be. One productive way forward would be to examine higher order corrections to the energy loss processes, for example higher orders in  $\alpha_s$  [5], small system size corrections [6–8], or sub-eikonal corrections [9].

Another productive way forward would be to attempt to place energy loss processes on a more rigorous footing. Factorization in QCD provides an extremely valuable framework for the rigorous consideration of high-energy nuclear processes [10]. In factorization, one has a controlled order-by-order expansion in some small momentum scale over a large momentum scale (usually  $\Lambda_{QCD}/Q$ ), which is further controlled by an order-by-order expansion in  $\alpha_s$ . There are rigorous theorems in which the expansion parameters are clear, and one can rigorously obtain error estimates from high order effects. Some processes for which factorization theorems exist include deep inelastic scattering, semi inclusive deep inelastic scattering, deeply virtual Compton scattering, and Drell-Yan [10].

Derivations of energy loss in nuclear collisions generally *assume* factorization; there are, so far, no rigorous factorization theorems associated with energy loss processes in heavy ion collisions. In the DGLV [11, 12], BDMPS-Z [13, 14], or AMY [15] approaches, the hard production process is assumed factorized from the subsequent evolution. Other energy loss derivations considered a “medium modification” of DGLAP evolution of fragmentation functions [16–19].

Recent very interesting work derived the nuclear modification to  $\langle p_T^2 \rangle$  in e+A collisions compared to e+p collisions within the twist expansion [20–22]. In this work, the production and subsequent evolution were placed on equal footing. The result is self-consistent to next-to-leading order (NLO). There is no factorization theorem yet, but we would like to answer the question: Is there an apples-to-apples comparison with the opacity expansion approach? Can one quantify the importance of neglecting the interference between production and subsequent energy loss in the opacity expansion approach through such a comparison?

Before diving into the above questions, it's worth reviewing some of the key results from the twist expansion derivation of e+A SIDIS. Most important, the twist 4 derivation found that the relevant objects are modified, twist 4 parton distribution functions (PDFs). In stark contrast with the assumption of [16–19], the fragmentation functions, on the other hand, evolve as if in vacuum, with DGLAP vacuum splitting functions.

## 2. Comparison of Opacity Expansion with Twist 4 Expansion

Let us now compare order-by-order the opacity and twist expansions.

At zeroth order in opacity, there is no interaction between the produced particle (in which production is assumed factorized from subsequent evolution) and the nuclear environment. Thus  $\langle \Delta p_T^2 \rangle = 0$ , where we consider the *change* in jet broadening from e+p collisions to e+A collisions.

We take that the in medium Debye-screened scattering center is given by the Gyulassy-Wang model [23]  $\left. \frac{d^2 \sigma^{qg \rightarrow qg}}{d^2 \mathbf{q}_\perp} \right|_1 = \frac{2\alpha_s^2}{(\mathbf{q}_\perp^2 + \mu^2)^2}$ , where  $\mu \approx gT$  is the chromoelectric Debye screening mass of the medium and  $\mathbf{q}_\perp$  is the transverse momentum of the  $t$ -channel gluon exchanged with the medium [12]. Then, assuming that  $\mathbf{q}_{max}^2 \sim \mu E$ , one finds

$$\begin{aligned} \langle p_\perp^2 \rangle_{LO,1} &\equiv \frac{L}{\lambda} \int d^2 \mathbf{q}_\perp \mathbf{q}_\perp^2 \frac{d^2 \sigma^{qg \rightarrow qg}}{d^2 \mathbf{q}_\perp} \Bigg| \int d^2 \mathbf{q}_\perp \frac{d^2 \sigma^{qg \rightarrow qg}}{d^2 \mathbf{q}_\perp} \\ &\simeq \frac{L\mu^2}{\lambda} \ln\left(\frac{E}{\mu}\right) = \hat{q}L \end{aligned} \quad (1)$$

On the other hand, if we consider the twist 4 approach, then

$$\frac{d\langle \ell_\perp^2 \sigma \rangle}{dx_B dy dz_h} = \sigma_h e_q^2 \int_{x_B}^1 \frac{dx}{x} T_{qg}(x, 0, 0, \mu_f^2) \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z, \mu_f^2) \delta(1 - \hat{x}) \delta(1 - \hat{z}), \quad (2)$$

where  $\sigma_h = \frac{4\pi^2 \alpha_s z_h^2}{N_c} \sigma_0$ ,  $\sigma_0 = \frac{2\pi \alpha_{EM}^2}{Q^2} \frac{1+(1-y)^2}{y}$ ,  $\hat{x} = \frac{x_B}{x}$ , and  $\hat{z} = \frac{z_h}{z}$ .  $T_{qg}$  is the twist-4 quark-gluon correlation function, a generalization of the usual twist-2 parton distribution function. In the limit of a large and loosely bound nucleus, in which one may neglect the spatial and momentum correlations between the two nucleons, one has [22] an approximate factorization

$$\begin{aligned} T_{qg}(x_B, 0, \mu_f^2) &\approx \frac{N_c}{4\pi^2 \alpha_s} f_{q/A}(x_B, \mu_f^2) \int dy^- \hat{q}(\mu_f^2, y^-) \\ &= \frac{N_c}{4\pi^2 \alpha_s} f_{q/A}(x_B, \mu_f^2) \hat{q}(\mu_f^2) L, \end{aligned} \quad (3)$$

where in the last line we assumed for simplicity that the parton propagates through a nucleus of constant density of thickness  $L$ . In order to most readily and clearly compare to the energy loss derivation that we will show below, we will remove the complication of the fragmentation process from the twist-4 approach by assuming exact parton-hadron duality, i.e. we will take  $D_{h/q}(z, \mu_f^2) = \delta(1 - z)$ . We then have that  $\int_0^1 dz_h \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z, \mu_f^2) \delta(1 - \hat{z}) = 1$ . Then the leading in  $\alpha_s$  contribution from the twist-4 approach is a completely factorized result  $\frac{d\langle \ell_\perp^2 \sigma \rangle}{dx_B dy} = \frac{d\sigma_0}{dx_B dy} \hat{q}(\mu_f^2) L$ , and thus the twist 4 derivation gives

$$\langle p_\perp^2 \rangle_{LO,1} \simeq \hat{q}L, \quad (4)$$

in exact agreement with the opacity expansion.

We show the full numerics of the first order in opacity, and NLO in  $\alpha_s$  in our original work [24]. Surprisingly, the numerics clearly show a jet *narrowing* in nuclear media compared to vacuum. We sought to understand these full numerics with high-energy analytics. If we assumed that the kinematic upper bound in the transverse momentum of the emitted gluon could be neglected, we found a jet broadening,

$$\langle p_{\perp}^2 \rangle_{\text{NLO},1} \approx \frac{4\alpha_s}{3\pi} \mu^2 \frac{L}{\lambda} \ln^2 \frac{E}{\mu} + \mathcal{O} \left( \alpha_s \ln \frac{E}{\mu} \right). \quad (5)$$

However, if one explicitly maintains the kinematic limits while still taking the  $E \rightarrow \infty$  limit, which is highly non-trivial [24], then one finds a jet narrowing:

$$\langle p_{\perp}^2 \rangle_{\text{NLO},1} = -\frac{C_R \alpha_s}{4} \frac{L}{\lambda} \mu^2 \left[ \ln^2 \left( \frac{4E}{\mu^2 L} \right) + \frac{5\pi^2}{12} \right]. \quad (6)$$

It's more difficult to extract the leading behavior of the twist 4 result at high energy. If one assumes that the color triviality breaking terms are small, trivializes the fragmentation functions, and makes the loosely bound nucleus approximation, then the twist 4 prediction is one of broadening,

$$\frac{\langle p_T^2 \rangle_{\text{NLO},1}}{\langle p_T^2 \rangle_{\text{LO},1}} \approx \frac{4\alpha_s}{3\pi} \ln \frac{E}{\mu} \frac{\int_{x_B}^1 \frac{dx}{x} \left[ \frac{1+\hat{x}^2}{(1-\hat{x})} + \frac{3}{2} \delta(1-\hat{x}) \right] f_{q/A}(x, \mu^2)}{f_{q/A}(x_B, \mu^2)} > 0 \quad (7)$$

Since the two approaches give qualitatively different predictions for jet  $\langle \Delta p_{\perp}^2 \rangle$ , one may ask: what do the data show? It turns out that measuring jet broadening is not an easy experimental task [25]. However, there are hints of jet narrowing from SIDIS [26].

### 3. Conclusions

We seek precision jet tomography in heavy ion collisions with which we may extract quantitative insight into many body QCD. In this work we reported on an asymptotic analysis of  $\langle \Delta p_{\perp}^2 \rangle \equiv \langle p_{\perp}^2 \rangle_{e+A} - \langle p_{\perp}^2 \rangle_{e+p}$  from the opacity expansion and twist 4 approaches. The twist 4 approach predicts a generic jet broadening, while the opacity expansion predicts a jet narrowing (due to the destructive Landau-Pomeranchuk-Migdal effect). Data from heavy ion collisions is ambiguous, with hints of narrowing from SIDIS measurements.

The opacity expansion does not capture the interference between production and subsequent evolution. The twist 4 derivation does not fully capture the LPM effect (since it only captures the leading  $1/Q$  destructive interference). Perhaps most important, the twist 4 approach is collinear: there is an integration over all  $k_{\perp}$ , which appears to yield a wrong qualitative expectation for jet broadening rather than narrowing. Future work hopefully can take the best features of these calculations.

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