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Towards identifying the minimal flavor symmetry behind neutrino oscillations

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Current neutrino oscillation data indicate that the 3×3 Pontecorvo-Maki-Nakagawa-Sakata matrix U exhibits a μ-τ flavor interchange symmetry $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) as a good approximation. In particular, the T2K measurement implies that the maximal neutrino mixing angle θ_{23} and the CP-violating phase δ should be close to $\pi/4$ and $-\pi/2$, respectively. Behind these observations lies a minimal flavor symmetry — the effective Majorana neutrino mass term keeps invariant under the transformations $\nu_{e\rm L} \to (\nu_{e\rm L})^c$, $\nu_{\mu\rm L} \to (\nu_{\tau\rm L})^c$, $\nu_{\tau\rm L} \to (\nu_{\mu\rm L})^c$. Extending this flavor symmetry to the canonical seesaw mechanism, we find that the R-matrix describing the strength of weak charged-current interactions of heavy Majorana neutrinos satisfies $|R_{\mu i}| = |R_{\tau i}|$ as a consequence of $|U_{\mu i}| = |U_{\tau i}|$. This result can be used to set a new upper bound, which is about three orders of magnitude more stringent than before, on the flavor mixing factor associated with the charged-lepton-flavor-violating decay mode $\tau^- \to e^- + \gamma$.

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1. Introduction

Current neutrino oscillation data [1] allow us to see two salient features of the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix U. On the one hand, the observed pattern of U exhibits a μ-τ interchange symmetry $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) as a very good approximation. On the other hand, the best-fit values of the maximal neutrino mixing angle θ_{23} and the CP-violating phase δ in the standard parametrization of U are close respectively to $\pi/4$ and $-\pi/2$, as illustrated in Fig. 1 according to the recent T2K measurement [2].

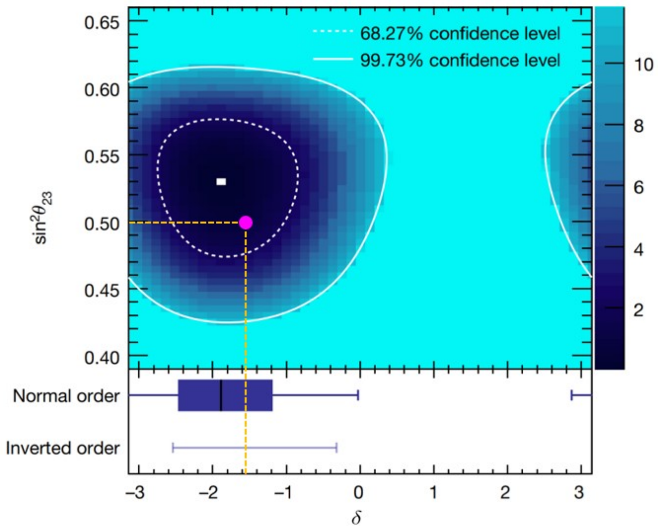

Figure 1: An illustration of the recent T2K data on the octant of θ_{23} and the quadrant of δ [2], together with their slight deviations from the μ-τ reflection symmetry predictions $\theta_{23} = \pi/4$ and $\delta = -\pi/2$.

These two observations motivate us to conjecture that behind U is most likely a kind of non-Abelian discrete flavor symmetry group, whose Clebsch-Gordan coefficients may help determine some elements of U [3]. So far a lot of flavor symmetries have been proposed and studied. At this stage one is allowed to make use of Occam's razor to shave away those complicated models and focus on the minimal flavor symmetry in the neutrino sector. Such a minimal flavor symmetry should belong to a residual flavor symmetry group which is as simple as possible, and its predictions should be as close as possible to the experimental data. We argue that the μ-τ reflection symmetry is the most appropriate example in this regard [4–6]: the effective Majorana neutrino mass term

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu (\nu_{\rm L})^c + {\rm h.c.} \tag{1}$$

keeps invariant under the transformations $\nu_{e\rm L} \to (\nu_{e\rm L})^c$, $\nu_{\mu\rm L} \to (\nu_{\tau\rm L})^c$ and $\nu_{\tau\rm L} \to (\nu_{\mu\rm L})^c$. The flavor texture of symmetric M_ν can therefore be constrained by $M_\nu = \mathcal{P} M_\nu^* \mathcal{P}$, where

$$\mathcal{P} = \mathcal{P}^T = \mathcal{P}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{2}$$

Then both $\theta_{23} = \pi/4$ and $\delta = \pm\pi/2$ can easily be achieved for the PMNS matrix U through the diagonalization $U^\dagger M_\nu U^* = D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$ with m_i being the light neutrino masses.

Here we are going to identify the same μ-τ reflection symmetry associated with three species of active and sterile neutrinos in the canonical seesaw mechanism by starting from the very possibility of $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) that is favored by current neutrino oscillation data. Considering that U is not exactly unitary and it is intrinsically correlated with another 3×3 flavor mixing matrix R describing the strength of weak charged-current interactions of heavy Majorana neutrinos, we show that $|U_{\mu i}| = |U_{\tau i}|$ gives rise to a novel prediction $|R_{\mu i}| = |R_{\tau i}|$ (for $i = 1, 2, 3$). We prove that behind these two sets of equalities and the preliminary experimental evidence for leptonic CP violation [2] lies an expected minimal flavor symmetry; namely, the overall neutrino mass term keeps invariant when the left-handed neutrino fields transform as $\nu_{e\rm L} \to (\nu_{e\rm L})^c$, $\nu_{\mu\rm L} \to (\nu_{\tau\rm L})^c$, $\nu_{\tau\rm L} \to (\nu_{\mu\rm L})^c$ and the right-handed neutrino fields undergo an arbitrary unitary transformation. We find that this novel result can be used to set a new upper limit, which is about three orders of magnitude more stringent than before, on the flavor mixing factor associated with the charged-lepton-flavor-violating decay mode $\tau^- \to e^- + \gamma$.

2. The canonical seesaw mechanism

To explain why the masses of three active neutrinos are tiny, a minimal extension of the standard model (SM) of particle physics is to add three right-handed neutrino fields $N_{\alpha\rm R}$ (for $\alpha = e, \mu, \tau$) and allow for lepton number violation. In this case the neutrino mass terms can be written as

$$-\mathcal{L}_\nu = \overline{\ell_{\rm L}}\, Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c}\, M_{\rm R} N_{\rm R} + {\rm h.c.}\,, \tag{3}$$

where $\ell_{\rm L}$ denotes the ${\rm SU(2)_L}$ doublet of the left-handed lepton fields, $\widetilde{H} \equiv {\rm i}\sigma_2 H^*$ with H being the Higgs doublet and σ_2 being the second Pauli matrix, $N_{\rm R} = (N_{e\rm R}, N_{\mu\rm R}, N_{\tau\rm R})^T$ is the column vector of three right-handed neutrino fields which are the ${\rm SU(2)_L}$ singlets, $(N_{\rm R})^c \equiv C\overline{N_{\rm R}}^T$ denotes the charge conjugation of $N_{\rm R}$. After spontaneous gauge symmetry breaking, Eq. (3) becomes

$$-\mathcal{L}'_\nu = \frac{1}{2}\overline{\left[\nu_{\rm L}\ \ (N_{\rm R})^c\right]}\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + {\rm h.c.}\,, \tag{4}$$

where $\nu_{\rm L} = (\nu_{e\rm L}, \nu_{\mu\rm L}, \nu_{\tau\rm L})^T$ denotes the column vector of three left-handed neutrino fields, $M_{\rm D} \equiv Y_\nu \langle H\rangle$ with $\langle H\rangle$ being the vacuum expectation value of the Higgs field. The overall symmetric 6×6 neutrino mass matrix in Eq. (4) can be diagonalized by the unitary transformation

$$\begin{pmatrix} U & R \\ S & Q\end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{pmatrix} U & R \\ S & Q\end{pmatrix}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N\end{pmatrix}\,, \tag{5}$$

where $D_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$ with M_i being the heavy Majorana neutrino masses. The 3×3 submatrices U, R, S and Q satisfy the unitarity conditions:

$$\begin{aligned} UU^\dagger + RR^\dagger = SS^\dagger + QQ^\dagger &= I\,, \\ U^\dagger U + S^\dagger S = R^\dagger R + Q^\dagger Q &= I\,, \\ US^\dagger + RQ^\dagger = U^\dagger R + S^\dagger Q &= \mathbf{0}\,; \end{aligned} \tag{6}$$

and the exact *seesaw* formula that characterizes a kind of balance between the light and heavy neutrino sectors can also be obtained from Eq. (5):

$$U D_\nu U^T + R D_N R^T = \mathbf{0} \; . \tag{7}$$

The smallness of m_i is therefore ascribed to the highly suppressed magnitude of R which signifies the largeness of M_i with respect to the electroweak scale. In this seesaw framework ν_α (for $\alpha = e, \mu, \tau$) can be expressed as a linear combination of the mass eigenstates of three active (light) neutrinos and three sterile (heavy) neutrinos (i.e., $\nu_i = (\nu_i)^c$ and $N_i = (N_i)^c$ for $i = 1, 2, 3$). The standard weak charged-current interactions of these six Majorana neutrinos is given by

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \, \overline{(e \quad \mu \quad \tau)_{\rm L}} \, \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.} \; . \tag{8}$$

So U describes flavor mixing and CP violation of three active neutrinos in neutrino oscillations, and R measures the strength of weak charged-current interactions of three heavy neutrinos in precision collider physics and thermal leptogenesis. Note that U must be non-unitary, but its deviation from unitarity is at most of $\mathcal{O}(1\%)$ and hence is insensitive to all the present experiments.

3. The minimal flavor symmetry

As pointed out in Ref. [7], the possibilities of $|U_{\mu i}| = |U_{\tau i}|$ and $\delta = \pm\pi/2$ inspire us to conjecture how $U_{\mu i}$ is directly related to $U^*_{\tau i}$ under a simple flavor symmetry. In the same flavor symmetry $|U_{ei}|$ should keep unchanged. We find that $U = \mathcal{P} U^* \zeta$ with $\zeta = {\rm Diag}\{\eta_1, \eta_2, \eta_3\}$ and $\eta_i = \pm 1$, where $\mathcal{P}$ has been given in Eq. (2), satisfies our requirements and allows the PMNS matrix elements $U_{\alpha i}$ (for $\alpha = e, \mu, \tau$) to transform together. Inserting $U = \mathcal{P} U^* \zeta$ into Eq. (7) and taking the complex conjugate for the whole equation, we arrive at

$$U D_\nu U^T + \mathcal{P} R^* D_N (\mathcal{P} R^*)^T = \mathbf{0} \; . \tag{9}$$

Comparing this equation with Eq. (7), we immediately get $R = \mathcal{P} R^* \zeta'$ with $\zeta' = {\rm Diag}\{\eta'_1, \eta'_2, \eta'_3\}$ and $\eta'_i = \pm 1$. This result leads us to a novel and rephasing-invariant prediction $|R_{\mu i}| = |R_{\tau i}|$, which is actually a natural consequence of $|U_{\mu i}| = |U_{\tau i}|$ in the canonical seesaw mechanism.

Substituting $U = \mathcal{P} U^* \zeta$ and $R = \mathcal{P} R^* \zeta'$ into Eq. (6), we find $S = \mathcal{T} S^* \zeta$ and $Q = \mathcal{T} Q^* \zeta'$ with $\mathcal{T}$ being an arbitrary unitary matrix. Let us proceed to insert all these four relations into Eq. (5) and then take the complex conjugate for the whole equation. We are therefore left with

$$\begin{pmatrix} U & R \\ S & Q \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & \mathcal{P} M^*_{\rm D} \mathcal{T} \\ \mathcal{T}^T M^\dagger_{\rm D} \mathcal{P} & \mathcal{T}^T M^*_{\rm R} \mathcal{T} \end{pmatrix} \begin{pmatrix} U & R \\ S & Q \end{pmatrix}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix} \; . \tag{10}$$

A comparison between Eqs. (5) and (10) yields

$$M_{\rm D} = \mathcal{P} M^*_{\rm D} \mathcal{T} \; , \quad M_{\rm R} = \mathcal{T}^T M^*_{\rm R} \mathcal{T} \; . \tag{11}$$

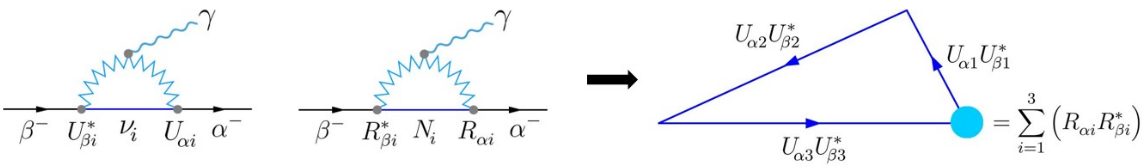

Figure 2: The charged-lepton-flavor-violating $\beta^- \to \alpha^- + \gamma$ decays mediated by the light Majorana neutrinos ν_i and the heavy Majorana neutrinos N_i (for $i = 1, 2, 3$), where α and β run over the e, μ and τ flavors and $m_\beta > m_\alpha$ holds. The related unitarity hexagons, reduced as the effective U-dominated unitarity triangles corrected by the effective R-dominated vertices, are also shown.

Inserting Eq. (11) into Eq. (4), the overall neutrino mass term $\mathcal{L}'_\nu$ reads as

$$-\mathcal{L}''_\nu = \frac{1}{2}\overline{\left[\mathcal{P}(\nu_{\rm L})^c \quad \mathcal{T}N_{\rm R}\right]}\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{bmatrix} \mathcal{P}\nu_{\rm L} \\ \mathcal{T}^*(N_{\rm R})^c\end{bmatrix} + {\rm h.c.}\,. \tag{12}$$

Comparing between Eqs. (4) and (12), we find that $\mathcal{L}''_\nu = \mathcal{L}'_\nu$ holds under the transformations [7]

$$\nu_{\rm L} \to \mathcal{P}(\nu_{\rm L})^c\,, \quad N_{\rm R} \to \mathcal{T}^*(N_{\rm R})^c\,, \tag{13}$$

no matter what specific form of $\mathcal{T}$ is taken.

If the heavy degrees of freedom in the canonical seesaw mechanism are integrated out, one may obtain the effective Majorana neutrino mass term in Eq. (1). The corresponding effective mass matrix M_ν can be given by the approximate seesaw relation $M_\nu \simeq -M_{\rm D}M_{\rm R}^{-1}M_{\rm D}^T$. Inserting Eq. (11) into this formula and take the complex conjugate, we arrive at $M_\nu = \mathcal{P}M_\nu^*\mathcal{P}$ (namely, M_ν respects the μ-τ reflection symmetry although it originates from the seesaw mechanism).

4. Charged lepton flavor violation

Now we show that the μ-τ reflection symmetry in the canonical seesaw framework can help constrain the unitarity of U through the charged-lepton-flavor-violating decay modes $\beta^- \to \alpha^- + \gamma$ (for $\alpha, \beta = e, \mu, \tau$ and $m_\beta > m_\alpha$) as illustrated in Fig. 2, which are mediated by both the light Majorana neutrino ν_i and the heavy Majorana neutrino N_i (for $i = 1, 2, 3$). For this purpose, we are more interested in the following ratios in the natural case of $m_i \ll M_W \ll M_i$ [8, 9]:

$$\begin{aligned} \xi_{\alpha\beta} &\equiv \frac{\Gamma(\beta^- \to \alpha^- + \gamma)}{\Gamma(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta)} \\ &\simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6} + \frac{1}{4}\cdot\frac{m_i^2}{M_W^2}\right) - \frac{1}{3}\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right|^2 \\ &\simeq \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right|^2 = \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right|^2\,, \end{aligned} \tag{14}$$

where $\alpha_{\rm em}$ is the fine structure constant of electromagnetic interactions, and $UU^\dagger + RR^\dagger = I$ has been used. Then we obtain

$$\left|\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right| = \left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right| = \sqrt{\frac{8\pi}{3\alpha_{\rm em}}\xi_{\alpha\beta}} \simeq 33.88\sqrt{\xi_{\alpha\beta}}\,, \tag{15}$$

where $\alpha_{\rm em} \simeq 1/137$ has been input. Taking account of current experimental upper bounds on the branching ratios of $\beta^- \to \alpha^- + \gamma$ [1], we have $\xi_{e\mu} < 4.20 \times 10^{-13}$, $\xi_{e\tau} < 1.85 \times 10^{-7}$ and $\xi_{\mu\tau} < 2.42 \times 10^{-7}$. So Eq. (15) give the constraints

$$\left|\sum_{i=1}^{3} R_{ei} R^*_{\mu i}\right| < 2.20 \times 10^{-5} \; , \quad \left|\sum_{i=1}^{3} R_{ei} R^*_{\tau i}\right| < 1.46 \times 10^{-2} \; , \quad \left|\sum_{i=1}^{3} R_{\mu i} R^*_{\tau i}\right| < 1.66 \times 10^{-2} \; . \quad (16)$$

These results mean that the effective R-dominated vertices of the effective U-dominated unitarity triangles in Fig. 2 are really small, at most at the $\mathcal{O}(1\%)$ level.

As we have shown in section 3, R respects the μ-τ reflection symmetry as U does. In this case we can achieve a new upper bound on the flavor mixing factor associated with $\tau^- \to e^- + \gamma$, which is about three orders of magnitude more stringent than that obtained in Eq. (16):

$$\left|\sum_{i=1}^{3} R_{ei} R^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei} R^*_{\mu i}\right| < 2.20 \times 10^{-5} \; . \quad (17)$$

Some other important applications of the μ-τ reflection symmetry in neutrino phenomenology have recently been reviewed in Refs. [5, 6]. A further study of its implications is well worth the wait.

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