

## A novel approach to semileptonic heavy-to-light $B$ decays through the Dispersive Matrix method

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In this contribution we analyse the heavy-to-light  $B$  decays through the Dispersive Matrix method, which can be applied to any semileptonic decays of hadrons once lattice QCD computations of the hadronic Form Factors and of the relevant susceptibilities are available. We will explicitly discuss the application of the Dispersive Matrix approach to both  $B \rightarrow \pi \ell \nu_\ell$  and  $B_s \rightarrow K \ell \nu_\ell$  decays. As usual in our analysis strategy, only LQCD computations of the FFs at high values of the momentum transfer will be used to determine the shape of the FFs in the whole kinematical range without making any assumption on their momentum dependence. Then, the experimental data will be used only to obtain our final exclusive determinations of  $|V_{ub}|$ . In this way, our calculation of the FFs allows pure theoretical estimates of several quantities of phenomenological interest to be obtained, for instance the  $\tau/\mu$  ratio of the differential decay rates  $R_{\pi(K)}^{\tau/\mu}$ , which is an important tool for testing Lepton Flavour Universality. We will also present a summary of all the results obtained so far for semileptonic  $B$  decays within the Dispersive Matrix approach.

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## 1. Introduction

Since many years, the heavy-to-light semileptonic  $B$ -meson transitions are intriguing processes mainly because of a long-standing tension between the inclusive and the exclusive determinations of the CKM matrix element  $|V_{ub}|$ . The most recent version of the FLAG report [1] quotes for the exclusive estimate of  $|V_{ub}|$  the value  $|V_{ub}|_{excl} \cdot 10^3 = 3.74(17)$  from  $B \rightarrow \pi \ell \nu_\ell$  decays. This value is well compatible with other recent analyses present in literature, see for instance [2]. For what concerns the inclusive determinations, many averages have been recently computed. HFLAV Collaboration [3] quotes the value  $|V_{ub}|_{incl} \cdot 10^3 = 4.19(12)_{(-12)}^{(+11)}$ , where the errors correspond to experimental and theoretical uncertainties, respectively. In the last FLAG review [1] one finds  $|V_{ub}|_{incl} \cdot 10^3 = 4.32(29)$ . Finally, the last PDG review [4] quotes  $|V_{ub}|_{incl} \cdot 10^3 = 4.13(26)$ .

By comparing the FLAG exclusive determination with each of the inclusive values, we find a difference that varies in the  $1.5 - 2\sigma$  range. The present state-of-the-art thus lightens the  $|V_{ub}|$  puzzle with respect to the past years. However, it remains important to address the problem of an accurate determination of  $|V_{ub}|$  from the relevant exclusive channels.

## 2. The Dispersion Matrix approach

For both semileptonic  $B \rightarrow \pi$  and  $B_s \rightarrow K$  decays, the differential decay width reads

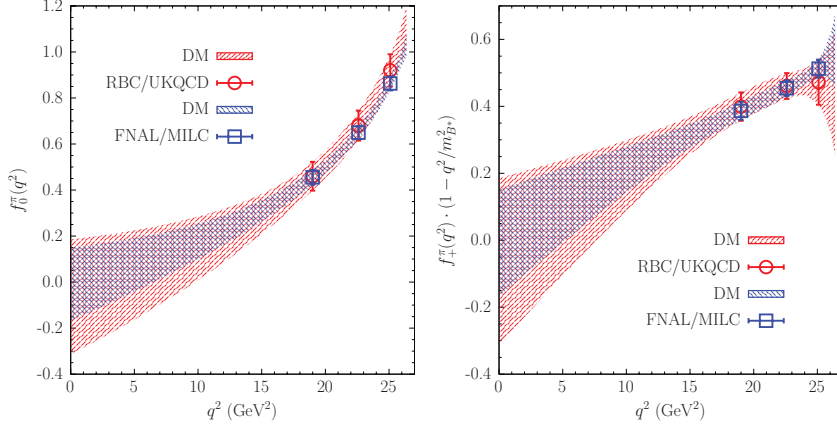
$$\frac{d\Gamma(B_{(s)} \rightarrow \pi(K)\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left[ |\vec{p}_{\pi(K)}|^3 \left(1 + \frac{m_\ell^2}{2q^2}\right) |f_+^{\pi(K)}(q^2)|^2 + m_{B_{(s)}}^2 |\vec{p}_{\pi(K)}| \left(1 - \frac{m_{\pi(K)}^2}{m_{B_{(s)}}^2}\right)^2 \frac{3m_\ell^2}{8q^2} |f_0^{\pi(K)}(q^2)|^2 \right], \quad (1)$$

where  $G_F$  is the Fermi constant,  $\vec{p}_{\pi(K)}$  the 3-momentum of the  $\pi(K)$  meson and  $m_\ell$  the mass of the produced lepton. From the theoretical point of view, a central role is then played by the hadronic Form Factors (FFs)  $f_+^{\pi(K)}(q^2)$ ,  $f_0^{\pi(K)}(q^2)$ , which can be directly computed on the lattice with high precision *only* at high values of the momentum transfer  $q^2$ .

Now, the goal of this proceedings is to describe the FFs entering in Eq. (1) by using the novel Dispersion Matrix (DM) method [5]. The DM method, in fact, allows us to study the FFs in the whole kinematical region in a non-perturbative and model-independent way. To be more specific, starting from the available lattice QCD (LQCD) computations of the FFs at high momentum transfer, we can extrapolate their behaviour in the opposite kinematical region without assuming any functional dependence of the FFs on  $q^2$  and using only non-perturbative inputs.

From the mathematical point of view, the idea is to obtain bounds on the hadronic FFs by using the computations of the FFs on the lattice and of the derivatives of suitable Green functions of bilinear quark operators, that hereafter we will refer to as *susceptibilities*. To be more specific, the susceptibilities follow from the dispersion relation associated to a particular spin-parity quantum channel and have been computed for the first time on the lattice in [6, 7] for  $b \rightarrow u$  and  $b \rightarrow c$  quark transitions, respectively. Using the DM method, one can obtain from first principles the lower and the upper bounds of a generic FF  $F$  for each generic value of  $q^2$ , *i.e.*

$$F_{lo}(q^2) \leq F(q^2) \leq F_{up}(q^2). \quad (2)$$



**Figure 1:** The scalar  $f_0^\pi(q^2)$  (left panel) and vector  $f_+^\pi(q^2)$  (right panel) FFs entering the semileptonic  $B \rightarrow \pi \ell \nu_\ell$  decays computed by the DM method as a function of the 4-momentum transfer  $q^2$  using the computations of the FFs by RBC/UKQCD (red points) and FNAL/MILC (blue squares) Collaborations. In the right panel, the vector FF is multiplied by the factor  $(1 - q^2/m_{B^*}^2)$  with  $m_{B^*} = 5.325$  GeV.

The explicit expressions of  $F_{\text{lo,up}}(q^2)$  can be found in [5].

### 3. The Dispersion Matrix applied to semileptonic $B \rightarrow \pi \ell \nu$ decays

Let us discuss the application of the Dispersive Matrix method to the  $B \rightarrow \pi \ell \nu$  decays [6]. The FFs  $f_+^\pi(q^2)$ ,  $f_0^\pi(q^2)$  have been studied by the RBC/UKQCD [8] and the FNAL/MILC [9] Collaborations (the results of a new computation of these hadronic FFs have been recently published also by the JLQCD Collaboration [10]). The lattice computations of the FFs  $f_+^\pi(q^2)$ ,  $f_0^\pi(q^2)$  have been considered in the large- $q^2$  region, namely at  $q^2 = \{19.0, 22.6, 25.1\}$  GeV<sup>2</sup>. In Fig. 1 we show the red (blue) DM bands coming from the RBC/UKQCD (FNAL/MILC) data, respectively.

For the extraction of the CKM matrix element we compute bin-per-bin values of  $|V_{ub}|$  for each  $q^2$ -bin of each available experiment. At present, several experiments [11–14] have measured the differential branching fractions of the semileptonic  $B \rightarrow \pi$  transition. All the details of this study can be found in [6]. Our final result for  $|V_{ub}|$  reads

$$|V_{ub}|^{B\pi} \cdot 10^3 = 3.62 \pm 0.47. \quad (3)$$

Let us mention here that we are currently investigating new strategies to improve our precision on the determination of  $|V_{ub}|$  within the DM approach. Some results can be found in [15], where our improved determination of the CKM matrix element  $|V_{ub}|$  from semileptonic  $B \rightarrow \pi$  decays reads

$$|V_{ub}|_{\text{impr}}^{B\pi} \cdot 10^3 = 3.88 \pm 0.32. \quad (4)$$

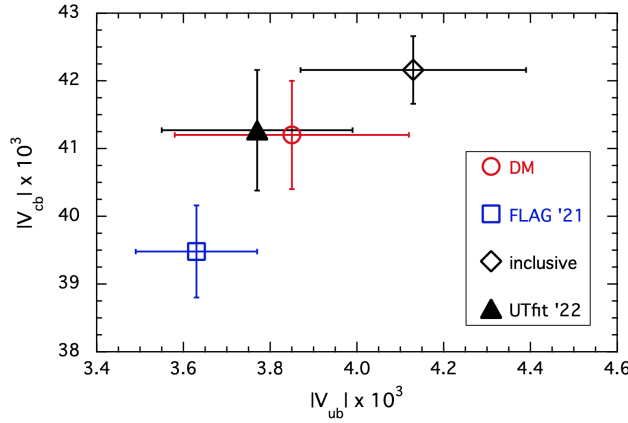
Moreover, the DM bands of the FFs  $f_{+,0}^\pi(q^2)$  can be used to compute fully-theoretical estimates of the  $\tau/\mu$  ratio  $R_\pi^{\tau/\mu} \equiv \Gamma(B \rightarrow \pi \tau \nu_\tau)/\Gamma(B \rightarrow \pi \mu \nu_\mu)$ . Our DM estimate reads  $R_\pi^{\tau/\mu} = 0.793(118)$ , which is well compatible with the experimental value  $R_\pi^{\tau/\mu}|_{\text{exp}} = 1.05 \pm 0.51$  by Belle [16].

#### 4. The Dispersion Matrix applied to semileptonic $B_s \rightarrow K\ell\nu$ decays

In the case of the semileptonic  $B_s \rightarrow K$  transition, the FFs  $f_+^K(q^2)$ ,  $f_0^K(q^2)$  have been computed on the lattice by the RBC/UKQCD [8], HPQCD [17] and FNAL/MILC [18] Collaborations. Also in this case the LQCD values of the FFs have been considered at three values of the momentum transfer in the large- $q^2$  regime, namely  $q^2 = \{17.6, 20.8, 23.4\}$  GeV<sup>2</sup>. We have then implemented the DM method to obtain unitary bands for the FFs (see all the details in [6]).

Our goal is now to determine  $|V_{ub}|$  from semileptonic  $B_s \rightarrow K$  decays. The LHCb Collaboration has recently observed for the first time these decays [19] by measuring the ratio  $R_{BF} \equiv \mathcal{B}(B_s^0 \rightarrow K^- \mu^+ \nu_\mu) / \mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)$  in two different  $q^2$ -bins, namely  $q^2 \leq 7$  GeV<sup>2</sup> and  $q^2 \geq 7$  GeV<sup>2</sup>. In order to obtain an exclusive estimate of  $|V_{ub}|$ , we made use of the lifetime of the  $B_s$ -meson,  $\tau_{B_s^0} = (1.516 \pm 0.006) \cdot 10^{-12}$  s [4], and of the experimental value of the branching ratio  $\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)$  measured by the LHCb Collaboration in [20]. Our final result, resulting from a weighted average of the two  $q^2$ -bins, reads

$$|V_{ub}|_{\text{combined}}^{B_s K} \cdot 10^3 = 3.77 \pm 0.48. \quad (5)$$



**Figure 2:**  $|V_{cb}|$  vs  $|V_{ub}|$  correlation plot. The numerical values of the various entries (DM estimates, FLAG 2021 values, inclusive estimates [4, 21], UTfit determinations [22, 23]) can be found in Table 1.

#### 5. Conclusions

In this contribution we have reviewed the results of the application of the Dispersion Matrix approach to semileptonic  $B \rightarrow \pi$  and  $B_s \rightarrow K$  decays [6]. In Figure 2 we have condensed the results obtained so far from the application of the DM method, enlarging the discussion also to the semileptonic  $B \rightarrow D^{(*)}$  [24, 25] and  $B_s \rightarrow D_s^{(*)}$  [26] transitions. The DM values of the CKM matrix elements represent the averages of all the DM determinations of  $|V_{cb}|$  and  $|V_{ub}|$  from the various decay channels, which are also presented for clarity in Table 1. The value of  $|V_{ub}|$  corresponds to the average of the values in Eqs. (4) and (5). For both the CKM matrix elements, the DM determinations are compatible with the corresponding inclusive values within the  $1\sigma$  level. Furthermore, the DM values are practically identical to the indirect determinations coming for the latest analysis by the UTfit Collaboration [22, 23].

**Table 1:** Numerical values of the CKM matrix elements  $|V_{cb}|$  and  $|V_{ub}|$  plotted in Figure 2.

	Decay channel	DM values	FLAG '21	Inclusive	UTfit '22
$ V_{cb}  \times 10^3$	$B_{(s)} \rightarrow D_{(s)}^{(*)}$	41.2 (8)	39.48 (68)	42.16 (50)	41.27 (89)
$ V_{ub}  \times 10^3$	$B_{(s)} \rightarrow \pi(K)$	3.85 (27)	3.63 (14)	4.13 (26)	3.77 (22)

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