PROCEEDINGS OF SCIENCE

A relook into the extraction of exclusive $|V_{ub}|$.

Ipsita Ray[∗] [,∗]

Department of Physics, Indian Institute of Technology Guwahati, Guwahati, Assam 781039, India.

E-mail: ray176121005@iitg.ac.in

The Cabibbo-Kobayashi-Maskawa (CKM) element V_{ub}, which is the least precisely known element till date is an important input parameter for the theoretical predictions of several observables in the flavor sector and it is responsible for the CP violating phase within the Standard Model. There exists a long standing tension between the tree-level determinations from the inclusive $B \to X_u \ell \nu$ decays (where X_u refers to sum over all final state hadrons containing an up quark) and exclusive decays like $B \to \pi \ell \nu$. We have re-analyzed all the available inputs (data and theory) on the $B \to \pi \ell \nu$ decays including the newly available inputs on the form-factors from light cone sum rule (LCSR) and Lattice QCD (LQCD) approach. We have compared the results with the procedure taken up by the Heavy Flavor Averaging Group (HFLAV), while commenting on the effect of outliers on the fits. Our best results for $|V_{ub}|^{\rm exc.}$ are consistent with the most recent estimate for $|V_{ub}|^{\rm inc.}$ from Belle within 1 σ confidence interval.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

∗I would like to thank the organisers of ICHEP 2022 conference for giving me the opportunity to present my work. I would also like to thank my collaborators Soumitra Nandi, Sunando Patra and Aritra Biswas for useful discussions.

∗Speaker

PoS(ICHEP2022)714

1. Introduction

The tree level semileptonic $b \to u\ell\nu_\ell$ ($\ell = e, \mu$) decays are useful probes for extracting the CKM element $|V_{ub}|$. In this regard, both exclusive ($B \to \pi\ell\nu$), and inclusive ($B \to X_u\ell\nu_\ell$) decays play important roles. At present, the extracted values are in mutual disagreement (by $\sim 2.2\ \sigma$). Unlike the inclusive determination of $|V_{cb}|$ from $B \to X_c\ell\nu_\ell$, the inclusive determination of $|V_{ub}|$ is not clean. The large background from the $b \to c\ell\nu$ decays necessitates experimental cuts to distinguish $b \to u$ from $b \to c$ transitions, forcing us to a corner of the phase-space region where usual Operator Product Expansion (OPE) cannot be applied. One has to rely on the non-perturbative Quantum Chromodynamics (QCD) shape functions which are modelled using various approaches, thus rendering the extracted values of $|V_{ub}|$ model dependent. Recently, Belle has extracted the value of inclusive $|V_{ub}|$ using four different methods. From an arithmetic average of these four different values, they obtain $|V_{ub}|^{\text{inc.}} = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$ which is the most precise measurement to date.

The methodology adopted by the HFLAV for the extraction of $|V_{ub}|$ from $B \to \pi l\nu$ modes involves a two-stage procedure for the extraction of $|V_{ub}|^{\text{exc.}}$. In the first stage, using the available data on the differential $B \to \pi\ell\nu$ decay rates from BaBar and Belle collaborations [1–4], they obtain an average squared four-momentum transfer (q^2) spectrum from a binned maximum-likelihood fit. As presented in their review [5], the p-value for this fit is around 6%. In the second fit, this average q^2 spectrum along with the lattice and light cone sum rule (LCSR) (at $q^2 = 0$) inputs have been used to extract $|V_{ub}|$ from a fit with a reasonably good p-value of $\sim 47\%$. After repeating a similar fit as above to obtain the average q^2 spectrum, we have arrived at an even worse quality of fit with a p value $< 1\%$. In any case, a frequentist fit with a probability of $< 5\%$ is usually considered to be of negligible significance and any further fit (in the second stage), using the outcome of this very low-significance fit may lead to biased predictions for $|V_{ub}|$. It thus becomes essential to reconsider other possible ways of analyzing the available data and pin-point the source of tension in the fits and also the reason for the discrepancy between exclusive and inclusive determinations.

2. Theoretical Background

The differential decay width w.r.t. q^2 for a pseudoscalar to pseudoscalar semileptonic decay is a function of the form factors $f_{+,0}(q^2)$ and the CKM element $|V_{ub}|$ [6]. Therefore, to extract $|V_{ub}|$, we need information on the form-factors at different values of q^2 which are obtained from non-perturbative techniques like lattice-QCD and LCSR. At present the lattice estimates are available on $f_{+/0}(q^2)$ at zero and non-zero recoils from RBC-UKQCD [7] and Fermilab-MILC [8]. There is also a recent update on the values of these form-factors at zero and non-zero values of q^2 from LCSR approach [9]. It is crucial to have a parametrization of $f_{+/0}(q^2)$ satisfying the general properties of unitarity and real analyticity in the complex q^2 plane, to get the shape of the decay rate distribution in the whole q^2 region. For the form-factor parametrization, we have followed two different approaches, known as Bourrely-Caprini-Lellouch (BCL) [10] and Bharucha-Straub-Zwicky (BSZ) [11] parametrization and compared their results. According to BCL, f_+ and f_0 are as follows:

$$f_+(z) = \frac{1}{1-q^2/m^2_{B^*}} \sum_{n=0}^{N_z-1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right], \qquad f_0(z) = \sum_{n=0}^{N_z-1} b_n^0 z^n \,. \tag{1}$$

Here, $b_n^{0/+}$ are the coefficients of the expansion which are free parameters and they obey the unitarity constraint as can be seen from [8],[10]. The conformal map from q^2 to z is given by: $z(q^2) = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$ where $t_\pm \equiv (m_B \pm m_\pi)^2$ and $t_0 \equiv t_+(1-\sqrt{1-t_-/t_+})$. t_0 is a free parameter that governs the size of z in the semileptonic phase space. For BSZ, the parametrization of any form-factor reads:

$$f_i(q^2) = \frac{1}{1-q^2/m^2_{R,i}} \sum_{k=0}^{N} a_k^i \left[z(q^2) - z(0) \right]^k \,, \tag{2}$$

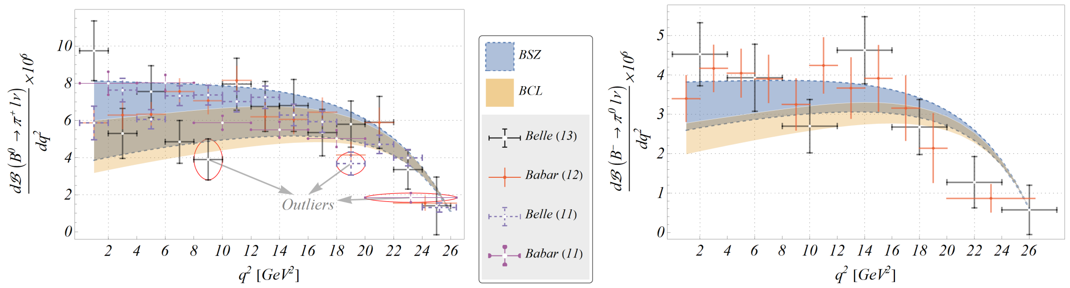
where $m_{R,i}$ denotes the masses of sub-threshold resonances compatible with the quantum numbers of the respective form factors and a_k^is are the coefficients of expansion. The details are provided in [11]. In BSZ, the kinematical constraint $f_+(q^2=0) = f_0(q^2=0)$ directly leads to the relation $a_0^+ = a_0^0$ between the coefficients, whereas in the BCL parametrization, the same kinematic constraint leads to a complex relationship between the expansion coefficients: $b_3^0 = 45.70(b_0^+ - b_0^0) - 12.78 b_1^0 - 3.58 b_2^0 + 12.85 b_1^+ + 3.44 b_2^+ + 1.21 b_3^+$. Utilising the kinematic constraint helps to reduce one parameter from the fit.

3. Comparison with existing literature

As discussed in the introduction, we have repeated the binned maximum-likelihood fit to obtain the average q^2-spectrum, which is consistent with that from HFLAV within 1σ. However, our fit quality is about 1 % while that for HFLAV is about 6 %. This difference in the fit quality could be due to the non-availability of the information on the shared systematic uncertainties between measurements (like continuum subtraction, tracking efficiency, etc.) as used by HFLAV in their analysis. Thus, in order to look for a possibility of improvement in the fit-quality, one should carefully inspect all the datasets. A closer look at the data shows that BaBar(11) untagged analysis of the $B^{0,+}$ modes [1] have much lower statistics/yield than the one published in the next year: BaBar(12) [2]. Also, in BaBar(11), the event selection has been optimized over the signal-enhanced region instead of the entire fit region and this analysis uses only a subset of the full BaBar data-set. Therefore, we drop the datapoints in 6 q^2-bins from BaBar(11) as a first attempt to look for the possibility of improvement while extracting the average partial branching fraction in each q^2 interval from a binned maximum-likelihood fit to data. This leads to an improvement in the fit quality from 1% to 24.8%. This reinforces the hypothesis that the data from BaBar (11) is quite at odds with all other data-sets (Please refer to ref [12] for more details.)

4. Main results

To understand the effect of the inconsistency in data on the decay rate distributions, we have derived the $B \to \pi \ell \nu$ decay rate distributions using the form-factors extracted only from the LCSR and lattice inputs and the latest $|V_{ub}|^{\rm inc.}$ value from Belle in both the BCL [10] and BSZ [11] expansions. We have truncated both f_0 and f_+ at $N = 3$. Using the fit results for the parameters and $|V_{ub}|$ from different inclusive estimates, if we calculate the theoretical predictions of the binned branching fractions, then any large deviation of the predictions from the actual measurements could potentially diagnose the source of the apparent tension between $|V_{ub}|^{\text{inc.}}$ and $|V_{ub}|^{\text{exc.}}$. From figure 1, we observe that the q^2 distribution of the differential branching fraction in both the form-factor parametrizations can explain almost all the available data except a few which are lying entirely outside of the theoretical C.I. bands. For more details, the interested reader is referred to [12].



(a) **(b)** **(c)**

Figure 1: Differential branching fraction plots superposed on experimental data-points, with form factors fitted from lattice and LCSR, and $|V_{ub}|$ obtained from the latest Belle Inclusive Measurement [13].

In our opinion, instead of extracting $|V_{ub}|$ through a two-stage procedure for which the first fit is of very poor quality, we should directly use the individual data-points for a simultaneous extraction of $|V_{ub}|$ and the parameters corresponding to the chosen form-factor parametrization. This provides us with a single value for the fit probability to draw our inference from instead of a two stage fit. The different fit scenarios are as given below:

- ***Fit 1***: B^0 decays from Belle (2011) and Belle (2013); B^- decays from Belle(2013); the combined modes from BaBar (2011) and BaBar (2012).

- ***Fit 2***: B^0 decays from Belle (2011), BaBar (2012), and Belle (2013); B^- decays from BaBar (2012) and Belle(2013).

- ***Fit 3***: The combined modes from BaBar (2011) along with the *Fit 2* dataset.

In table 1, we have shown the extracted values of $|V_{ub}|$ in different fit scenarios with full datasets and also after dropping the data-points having pulls greater than 2, shown in the right panel of the same table.'Fit A's are with experimental data + Lattice inputs, whereas 'Fit B's are with experimental data + Lattice + LCSR inputs. In all the scenarios, the fit quality as well as the extracted $|V_{ub}|$ increases by a considerable amount when dropping a few data-points with pull > 2. This indicates that the data with large 'pull' have an impact on the extracted values of $|V_{ub}|$ too. Fig 1 shows that the partial branching fractions $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar(11)), $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle(11)) and $\mathcal{B}(B^0 \to \pi^-)^{[8,10]}$ (Belle(13)) have pull > 2. However, $\mathcal{B}(B^0 \to \pi^-)^{[8,10]}$ (Belle(13)) has a rather minor effect on $|V_{ub}|$. On the basis of these observations, we define a few additional scenarios:

- ***Fit 2B-I***: Input used in *Fit 2B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle 2011).

BSZ Parametrization								
Run Name	Full				Dropped Pull > 2			
	$\chi^2_{\min}$/DOF	p-value(%)	$V_{ub}\times 10^3$		$\chi^2_{\min}$/DOF	p-value(%)	$V_{ub}\times 10^3$	
			Frequentist	Bayesian			Freq.	Bayes
Fit 1A	73.4/56	5.92	3.69(14)	3.67(14)	46.6/52	68.68	3.79(15)	$3.77\binom{15}{16}$
Fit 1B	77./65	14.57	3.74(13)	$3.73\binom{13}{14}$	49.3/61	85.77	3.83(14)	$3.82\binom{14}{16}$
Fit 2A	59.5/61	53.17	3.81(14)	3.79(15)	46./59	89.26	3.86(15)	$3.85\binom{15}{16}$
Fit 2B	62./70	74.23	3.85(14)	$3.83\binom{13}{15}$	48.3/68	96.63	3.91(14)	$3.89\binom{14}{15}$
Fit 3A	82.2/67	9.98	3.70(14)	3.69(14)	53.3/62	77.56	3.76(14)	$3.76\binom{15}{14}$
Fit 3B	85.9/76	20.54	3.75(13)	$3.74\binom{13}{14}$	62./73	81.79	3.84(14)	3.83(14)

BCL Parametrization								
Run Name	Full				Dropped Pull > 2			
	$\chi^2_{\min}$/DOF	p-value(%)	$V_{ub}\times 10^3$		$\chi^2_{\min}$/DOF	p-value(%)	$V_{ub}\times 10^3$	
			Freq.	Bayes			Freq.	Bayes
Fit 1A	73.5/56	5.84	3.69(14)	$3.67\binom{13}{15}$	46.7/52	68.34	3.79(15)	3.78(15)
Fit 1B	92.1/65	1.51	3.79(13)	$3.78\binom{14}{13}$	63.2/61	39.84	3.89(14)	$3.87\binom{14}{15}$
Fit 2A	60.1/61	50.8	3.81(14)	3.81(15)	46.5/59	88.19	3.87(15)	$3.85\binom{14}{15}$
Fit 2B	75.9/70	29.42	3.91(14)	3.90(15)	58.3/67	76.64	3.96(14)	$3.96\binom{16}{14}$
Fit 3A	82.7/67	9.35	3.70(14)	$3.69\binom{13}{14}$	57.8./63	66.09	3.77(14)	3.76(15)
Fit 3B	101.4/76	2.73	3.80(13)	$3.79\binom{13}{15}$	76.3/73	37.27	3.90(14)	$3.89\binom{14}{15}$

Table 1: Freq. and Bayesian

- ***Fit 3B-I***: Input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar 2011).
- ***Fit 3B-II***: Input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle 2011) and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar 2011).

Fit Scenario	BSZ				BCL			
	χ^2/DOF	p-value(%)	$V_{ub}\times 10^3$		χ^2/DOF	p-value(%)	$V_{ub}\times 10^3$	
			Frequentist	Bayesian			Frequentist	Bayesian
F2B-I	55.4/69	88.14	3.90(14)	$3.89^{+0.14}_{-0.15}$	68.85/69	48.25	3.96(14)	$3.95^{+0.14}_{-0.15}$
F3B-I	78.86/75	35.8	3.83(14)	3.83(13)	93.6/75	7.19	3.89(14)	3.89(14)
F3B-II	72.96/74	51.25	3.88(14)	$3.87^{+0.14}_{-0.15}$	87.2/74	13.99	3.94(14)	$3.93^{+0.14}_{-0.15}$

Table 2: Final table of comparison for $|V_{ub}|^{\text{exc.}}$ obtained in this work.

From table 2, we notice that even in the presence of other outliers, the most influential data-points in determining the estimate of $|V_{ub}|^{\text{exc.}}$ are the partial branching fractions $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle(11)) and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar(11)).

5. Summary

We have extracted $|V_{ub}|$ utilizing all the available inputs on the exclusive $B \to \pi l \nu$ decays which include the data on the partial branching fractions and inputs from lattice and LCSR. After repeating a similar analysis as HFLAV, we have arrived at a fit with very low probability for the average q^2 spectrum at the first stage. We have identified BaBar(11) data (at least a part of it) as a probable source of such a bad quality fit. We simultaneously fit all the data (instead of a two-stage fit) after defining different fit scenarios. In this process, we have identified outliers i.e. data-points which do not fit comfortably with other data. The goal is to check if some of these outliers are also influential in the extraction of $|V_{ub}|$. We have found a few data-points that compromise the fit-quality, and at the same time, influence the extraction of $|V_{ub}|$. Our best result $|V_{ub}| = (3.94(14)) \times 10^{-3}$ is consistent with the one extracted from inclusive $B \to X_u \ell \nu_\ell$ decays from Belle within 1 σ.

References

[1] P. del Amo Sanchez *et al.* [BaBar], Phys. Rev. D **83** (2011), 032007 doi:10.1103/PhysRevD.83.032007 [arXiv:1005.3288 [hep-ex]].

[2] J. P. Lees *et al.* [BaBar], Phys. Rev. D **86** (2012), 092004 doi:10.1103/PhysRevD.86.092004 [arXiv:1208.1253 [hep-ex]].

[3] H. Ha *et al.* [Belle], Phys. Rev. D **83** (2011), 071101 doi:10.1103/PhysRevD.83.071101 [arXiv:1012.0090 [hep-ex]].

[4] A. Sibidanov *et al.* [Belle], Phys. Rev. D **88** (2013) no.3, 032005 doi:10.1103/PhysRevD.88.032005 [arXiv:1306.2781 [hep-ex]].

[5] Y. S. Amhis *et al.* [HFLAV], Eur. Phys. J. C **81** (2021) no.3, 226 doi:10.1140/epjc/s10052-020-8156-7 [arXiv:1909.12524 [hep-ex]].

[6] Y. Sakaki, M. Tanaka, A. Tayduganov and R. Watanabe, Phys. Rev. D **88** (2013) no.9, 094012 doi:10.1103/PhysRevD.88.094012 [arXiv:1309.0301 [hep-ph]].

[7] J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water and O. Witzel, Phys. Rev. D **91** (2015) no.7, 074510 doi:10.1103/PhysRevD.91.074510 [arXiv:1501.05373 [hep-lat]].

[8] J. A. Bailey *et al.* [Fermilab Lattice and MILC], Phys. Rev. D **92** (2015) no.1, 014024 doi:10.1103/PhysRevD.92.014024 [arXiv:1503.07839 [hep-lat]].

[9] N. Gubernari, A. Kokulu and D. van Dyk, JHEP **01** (2019), 150 doi:10.1007/JHEP01(2019)150 [arXiv:1811.00983 [hep-ph]].

[10] C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D **79** (2009), 013008 [erratum: Phys. Rev. D **82** (2010), 099902] doi:10.1103/PhysRevD.82.099902 [arXiv:0807.2722 [hep-ph]].

[11] A. Bharucha, D. M. Straub and R. Zwicky, JHEP **08** (2016), 098 doi:10.1007/JHEP08(2016)098 [arXiv:1503.05534 [hep-ph]].

[12] A. Biswas, S. Nandi, S. K. Patra and I. Ray, JHEP **07** (2021), 082 doi:10.1007/JHEP07(2021)082 [arXiv:2103.01809 [hep-ph]].

[13] L. Cao *et al.* [Belle], Phys. Rev. D **104** (2021) no.1, 012008 doi:10.1103/PhysRevD.104.012008 [arXiv:2102.00020 [hep-ex]].