PROCEEDINGS OF SCIENCE

Precise measurement of the Decay $K^{\pm} \rightarrow \pi^0\pi^0\mu^{\pm}\nu$

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We report the first observation of the decay $K^{\pm} \rightarrow \pi^0\pi^0\mu^{\pm}\nu$ ($K^{00}_{\mu 4}$) by the NA48/2 experiment at the CERN SPS. From 2437 detected signal candidates with a S/B ratio of about 6, the branching ratio of the decay is determined with high precision. In the region of squared dilepton mass above 0.03 GeV^2/c^4, the branching ratio is found to be $BR(K^{00}_{\mu 4}, S_l > 0.03) = (0.65 \pm 0.03) \times 10^{-6}$. The full phase space result $BR(K^{00}_{\mu 4}) = (3.4 \pm 0.2) \times 10^{-6}$, depending on the decay model extrapolation, is in a reasonable agreement with the R form factor prediction from 1-loop Chiral Perturbation Theory.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

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PoS(ICHEP2022)723

1. Theoretical framework

The $K^\pm \to \pi^0\pi^0\mu^\pm\nu_\mu$ ($K^{00}_{\mu 4}$) decay is usually parameterized in terms of the five Cabibbo-Maksymowicz variables [1]: S_π – effective mass squared of the dipion system; S_l – effective mass squared of the dilepton system; θ_π – the angle of a pion momentum in the dipion center-of-mass system to the dipion line of flight in the kaon rest frame; θ_l (θ_μ) – the angle of the charged lepton momentum in the dilepton center-of-mass system to the dilepton line of flight in the kaon rest frame; ϕ – the angle between the pions plane in the kaon rest frame and the similar plane of leptons (see Fig. 1). In the case of equivalent neutral pions, there is an ambiguity in θ_π and ϕ definitions.

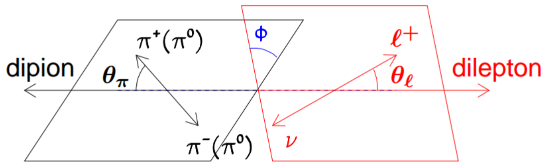

Figure 1: Cabibbo-Maksymowicz variables for K^{00}_{l4} decay.

The phenomenological matrix element of the decay is [2]:

$$T = \frac{G_f}{\sqrt{2}} \cdot V^*_{us} \cdot \underbrace{\overline{u}(p_\nu) \cdot \gamma_\mu \cdot (1-\gamma_5) \cdot v(p_l)}_{Lepton\ part} \cdot \underbrace{(V_\mu - A_\mu)}_{Hadron\ part}, \quad (1)$$

$$V_\mu = \frac{-H}{M_K^3}\varepsilon_{\mu,\nu,\rho,\sigma} L^\nu P^\rho Q^\sigma, \ A_\mu = -i\frac{1}{M_K}[P_\mu F + Q_\mu G + L_\mu R], \quad (2)$$

$\varepsilon_{0,1,2,3}$=1 and the four-momenta are defined as $P = p_1 + p_2$, $Q = p_1 - p_2$ (p_1 and p_2 are the two pion momenta) and $L = p_l + p_\nu$. The form factors F, G, R and H can be theoretically calculated in the framework of Chiral Perturbation Theory (ChPT). In the case of $\pi^0\pi^0$ dipion, within S-wave approximation, decay probability does not depend on $\cos\theta_\pi$ and ϕ, that leads to $G = 0$ and $H = 0$. After integration over these angles, the $K^{00}_{\mu 4}$ differential decay width becomes a function of three kinematic variables ($S_\pi, S_l, \cos\theta_l$) [2]:

$$d\Gamma_3 = \frac{G_f^2 |V_{us}|^2 (1-z_l)^2 \sigma_\pi X}{2^{11}\pi^5 M_K^5}(I_1 + I_2(2(\cos\theta_l)^2 - 1) + I_6 \cos\theta_l) dS_\pi dS_l d\cos\theta_l. \quad (3)$$

where $I_1 = \frac{1}{4}\{(1+z_l)|F_1|^2 + 2z_l|F_4|^2\}$, $I_2 = -\frac{1}{4}(1-z_l)|F_1|^2$, $I_6 = z_l Re(F_1^* F_4)$, $F_1 = X \cdot F$ and $F_4 = -(PL)F - S_l R$. Here $z_l = \frac{m_l^2}{S_l}$, $\sigma_\pi = \sqrt{1 - \frac{4M_\pi^2}{s_\pi}}$ and $X = \frac{1}{2}\sqrt{\lambda(M_K^2, S_\pi, S_l)}$, with the λ function defined as $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

Currently, the only source of R form factor is the ChPT-based theoretical calculations [2]. But F form factor measured in K^{00}_{e4} analysis may be used for $K^{00}_{\mu 4}$ decay due to the lepton universality. The measurements of K^{00}_{e4} from NA48/2 [3] represent the best determinations of the F form factor shape $F(K_{e4})$ and the absolute normalization f parameter ($F = f \cdot F(K_{e4})$, where $f = 6.079 \pm 0.012_{stat} \pm 0.027_{syst} \pm 0.046_{ext}$).

Preliminary results of the first observation and branching fraction measurement of $K^{00}_{\mu 4}$ decay are presented below.

2. Beams and detectors

The NA48/2 detector and beams at the CERN SPS were designed to search for direct CP violation in $K^{\pm} \to 3\pi$ decays [4, 5]. In 2003 and 2004, the NA48/2 beam particles were produced by 400 GeV protons colliding with a beryllium target. Two beams of oppositely charged kaons, with a central momentum of 60 GeV/c and a momentum band of ±3.8% (RMS), were formed by a system of magnets and collimators. The beam particles were measured by KAon Beam Spectrometer (KABES) with a spatial, momentum and time resolutions of about 800μm, 1% and 600 ps, respectively. The two beams were superimposed in the decay volume inside a 114 m long vacuum tank.

Charged particles from the $K^{\pm}$ decays were detected using a magnetic spectrometer (DCH) located in a tank filled with helium at atmospheric pressure. The magnetic spectrometer was followed by a scintillator hodoscope (HOD) with a time resolution of about 150 ps.

A liquid Krypton calorimeter (LKr), located behind the hodoscope, was used to reconstruct photons. It was an ionization chamber, segmented transversely into 13248 projective cells, 2×2 cm^2 each. The calorimeter energy resolution was $\sigma(E)/E = 0.032/\sqrt{E} \pm 0.09/E \pm 0.0042$ (E in GeV). Muon Veto system (MUV), consisting of three scintillator layers and 80 cm thick iron walls, was used for muons detection.

3. Reconstruction and selection of events

Only events with at least one DCH track and at least one KABES track, both consistent with a kaon decay vertex, are considered for the selection. Longitudinal position of the 'charged' decay vertex Z_c is defined from the two tracks vertex fit.

Each combination of four time-consistent photon candidates with LKr energy $E_{LKr} > 3$ GeV/c is considered as a possible result of two π^0 decays. For each two-π^0 decay vertex, the distances to LKr face, Z_1 and Z_2, are calculated under the assumption that each pair of photons is produced in $\pi^0 \to \gamma\gamma$ decay. Only the pairs of pion candidates with $|Z_1 - Z_2| < 500$ cm are considered further. For each two-π^0 combination, a 'neutral' vertex longitudinal position $Z_n = Z_{LKR} - (Z_1 + Z_2)/2$ is defined, where Z_{LKR} is the position of LKr. Only combinations of a π^0 pair and a charged vertex consistent in time and with $|Z_n - Z_c| < 600$ cm are considered further. Nearly 3.8% of signal $K^{00}_{\mu 4}$ events contain more than one candidate combination. The 'best' combinations are selected using a special criterion minimizing both $|Z_1 - Z_2|$ and $|Z_n - Z_c|$.

The normalization $K^{00}_{3\pi}$ decay candidates are required to pass the above common selection, and the DCH track momentum, P_{DCH}, is required to be above 5 GeV/c. In the plane $M_{3\pi}$ (invariant mass of the $\pi^{\pm}\pi^0\pi^0$ system) vs P_t (transverse momentum of the registered particles), the events within the ellipse centered at kaon PDG mass M_K^{PDG} [6] and $P_t = 5$ MeV/c, with semi-axes $\Delta M(K_{3\pi}) = 10$ MeV/c^2 and $\Delta P_t = 20$ MeV/c are taken as $K^{00}_{3\pi}$ candidates.

Signal mode $K^{00}_{\mu 4}$ candidates are selected among the preliminary selected combinations, that do not pass the above $K^{00}_{3\pi}$ elliptic cut. For $K^{00}_{\mu 4}$ selection, minimum P_{DCH} is increased to 10 GeV/c to ameliorate MUV efficiency. The track is required to be within the restricted geometrical acceptance of MUV ensuring high efficiency, and only tracks with associated reconstructed muon were selected.

$K^{00}_{\mu 4}$ squared missing mass M^2_{miss} is defined with the PDG muon mass attributed to the charged decay product. In a similar way one can define $M^2_{miss}(\pi^{\pm})$ as a squared missing mass reconstructed using the charged pion PDG mass instead of the muon mass. Small absolute values of $M^2_{miss}(\pi^{\pm})$ correspond to $K^{00}_{3\pi}$ decay kinematics, so we reject them for the signal selection. Due to the correlation between the reconstructed $M^2_{miss}(\pi^{\pm})$ and M^2_{miss}, the applied cut depends on M^2_{miss}: $M^2_{miss}(\pi^{\pm}) < 0.5M^2_{miss} - 0.0008$ GeV$^2/c^4$. Background is suppressed further by the $cos(\theta_\mu) < 0.6$ requirement. A cut on reconstructed dilepton mass $S_l = M^2(\mu\nu)$ is applied in order to exclude the region of large background from $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}(\pi^{\pm} \rightarrow \mu\nu)$. The quantity S_l is required to be greater than 0.03 GeV$^2/c^4$, well above the squared PDG value of $\pi^{\pm}$ mass.

After the selection, 3718 $K^{00}_{\mu 4}$ signal candidate events remain, without applying any restriction on M^2_{miss}. In the M^2_{miss} signal region [-0.002,0.002] GeV$^2/c^4$, $N_S = 2437$ candidates are observed. The normalization events selection resulted in $N_N = 72.99 \times 10^6$ $K^{00}_{3\pi}$ data events with a negligible background.

4. Acceptances and signal background

For the signal MC sample production, the generator in Ref. [2] is used with ChPT one-loop form factor $R(1loop)$, $H = 0, G = 0$ and zero complex phase between F and R. The experimental $F(K_{e4})$ form factor shape from Ref. [3] is used with the normalization $F = 5 \cdot F(K_{e4})$ providing a reasonable agreement with one-loop F calculation. Signal acceptance A_S is defined for the signal missing mass region, while the normalization acceptance A_N is calculated for all the selected events.

The uncertainty due to the modelling of form factors is estimated by the MC events weighting according to the following modifications of the matrix element: 1) implementation of the small complex phase between F and R, that is calculated in 1-loop approximation [2]; 2) $R(tree)$ form factor usage instead of $R(1loop)$ [2]; 3) 20% change of the general relation between F and R, covering the difference between the MC generator and the measured $F(K_{e4})$ [3]; 4) shifts of the measured $F(K_{e4})$ parameters by their total uncertainties [3].

The available fast MC model of the detector responses simulates the main contribution to the background, that is related to the charged pion decays upstream the LKr. The muons from hadron showers and from $\pi^{\pm} \rightarrow \mu\nu_\mu$ decays inside the LKr and downstream it (late muons) are not simulated. Shape of the M^2_{miss} spectrum for the late muon background is built using a special data sample, $\pi^{\pm}(LKr)$, representing the cases when charged pions do not decay up to the LKr and interact inside it with a considerable energy deposit. The probability of $\pi^{\pm} \rightarrow \mu^{\pm}\nu$ decay instead of interaction leading to the MUV response is approximately inversely proportional to the pion momentum. A weight $w_P = \frac{10\text{ GeV}/c}{P_{DCH}}$ is applied to each event of the $\pi^{\pm}(LKr)$ sample to represent the late muons background.

The $K^{00}_{\mu 4}$ signal is observed as a peak in the distribution of the missing mass squared of selected events (Fig. 2). In order to extract the signal size from the data spectrum, a fit outside the signal region in the M^2_{miss} interval [-0.003,0.006] GeV$^2/c^4$ is performed, taking into account the tails of MC simulated signal and two background contributions multiplied by free parameters of the fit. The background-related systematic uncertainty is determined by varying the way the background is estimated. The resulting number of background events is $354 \pm 33_{stat} \pm 62_{syst}$, the signal to background ratio is $5.9 \pm 1.4_{tot}$.

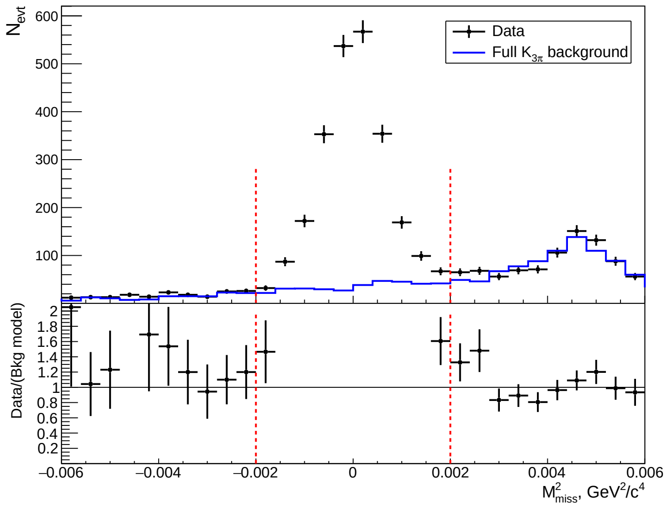

Figure 2: Distribution of the missing mass squared for the signal data events shown together with the results of the background fit ignoring the signal region.

5. Results

Resulting $K^{00}_{\mu4}$ branching ratio is calculated as $BR(K^{00}_{\mu4}) = \frac{N_S}{N_N} \cdot \frac{A_N}{A_S} \cdot K_{trig} \cdot BR(K^{00}_{3\pi})$, where $K_{trig} = 0.999 \pm 0.002$ is the trigger correction estimated using control triggers and MC simulation. $BR(K^{00}_{3\pi})$ is the normalization mode branching ratio ($(1.760 \pm 0.023)\%$)[6].

	Full phase space		$S_l > 0.03$ GeV^2/c^4	
$BR(K^{00}_{\mu4})$ central value $[10^{-6}]$	3.45		0.651	
	$\delta BR[10^{-6}]$	$\delta BR/BR$	$\delta BR[10^{-6}]$	$\delta BR/BR$
Data stat. error	0.10	2.85%	0.019	2.85%
MC stat. error	0.01	0.21%	0.001	0.21%
Trigger	0.01	0.18%	0.001	0.18%
Background	0.10	2.96%	0.019	2.96%
Accidentals	0.01	0.32%	0.002	0.32%
MUV inefficiency	0.06	1.65%	0.011	1.65%
Form factors modeling	0.05	1.37%	0.001	0.14%
$BR(K^{00}_{3\pi})$ error (external)	0.05	1.31%	0.009	1.31%
Total error	0.17	4.83%	0.030	4.64%

Table 1: $BR(K^{00}_{\mu4})$ and its error components

The results are shown in the Table 1. Accidental activity contributions to the systematic uncertainty were estimated as the effects of doubling the time windows used for different timing cuts. The systematic uncertainty related to MUV residual inefficiency is conservatively taken as 100% of the inefficiency size estimated from data. As a main result, the branching ratio for the restricted phase space with $S_l^{true} > 0.03$, characterised by small uncertainty due to form factors modelling, is presented.

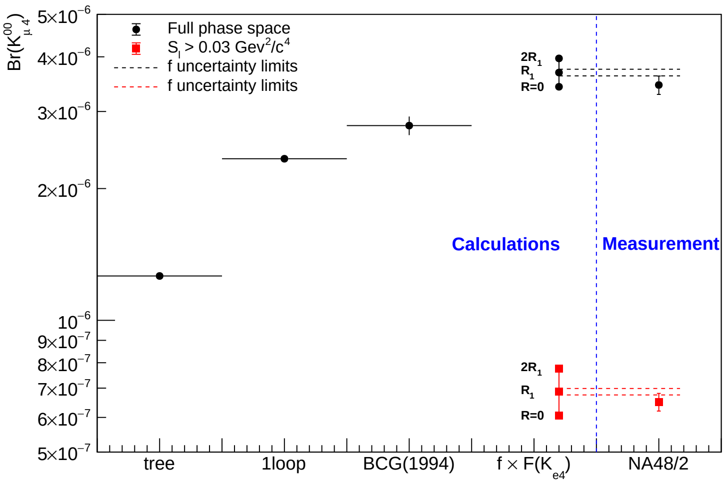

Figure 3: Comparison of the present NA48/2 measurement result with the theoretical predictions.

Comparison of the results with theoretical predictions is shown in Fig. 3. The left three theoretical points correspond to the tree approximation, the one-loop calculation and the 'beyond 1-loop' result [2] based on experimental inputs available in 1994, respectively. The last column of theoretical points in the Fig. 3 is calculated using the last measured K^{00}_{e4} form factor F [3] and one-loop R form factor [2] ($R_1 = R(1loop), 2R_1 = 2 \times R(1loop)$).

The preliminary result for restricted phase space ($S_l > 0.03$) is $BR(K^{00}_{\mu 4}, S_l > 0.03) = (0.65 \pm 0.019_{stat} \pm 0.024_{syst}) \times 10^{-6} = (0.65 \pm 0.03) \times 10^{-6}$. The preliminary full phase space result is $BR(K^{00}_{\mu 4}) = (3.4 \pm 0.10_{stat} \pm 0.13_{syst}) \times 10^{-6} = (3.4 \pm 0.2) \times 10^{-6}$. Both results are consistent with a contribution of the R form factor as computed at 1-loop ChPT.

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