Precise measurement of the Decay $K^\pm \to \pi^0 \pi^0 \mu^\pm \nu$

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We report the first observation of the decay $K^\pm \to \pi^0 \pi^0 \mu^\pm \nu$ ($K_{\mu4}^{00}$) by the NA48/2 experiment at the CERN SPS. From 2437 detected signal candidates with a S/B ratio of about 6, the branching ratio of the decay is determined with high precision. In the region of squared dilepton mass above 0.03 GeV$^2/c^4$, the branching ratio is found to be $BR(K_{\mu4}^{00}, S_t > 0.03) = (0.65 \pm 0.03) \times 10^{-6}$. The full phase space result $BR(K_{\mu4}^{00}) = (3.4 \pm 0.2) \times 10^{-6}$, depending on the decay model extrapolation, is in a reasonable agreement with the R form factor prediction from 1-loop Chiral Perturbation Theory.

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1. Theoretical framework

The $K^+ \rightarrow \pi^0 \pi^0 \mu^+ \mu^-$ ($K_{\mu4}^{00}$) decay is usually parameterized in terms of the five Cabibbo-Maksymowicz variables [1]: $S_\pi$ – effective mass squared of the dipion system; $S_\mu$ – effective mass squared of the dilepton system; $\theta_\pi$ – the angle of a pion momentum in the dipion center-of-mass system to the dipion line of flight in the kaon rest frame; $\theta_\mu$ – the angle of the charged lepton momentum in the dilepton center-of-mass system to the dilepton line of flight in the kaon rest frame; $\phi$ – the angle between the pions plane in the kaon rest frame and the similar plane of leptons (see Fig. 1). In the case of equivalent neutral pions, there is an ambiguity in $\theta_\pi$ and $\phi$ definitions.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Cabibbo-Maksymowicz variables for $K_{\mu4}^{00}$ decay.}
\end{figure}

The phenomenological matrix element of the decay is [2]:

$$T = \frac{G_f}{\sqrt{2}} \cdot V_{us}^* \cdot \bar{u}(p_{\nu}) \cdot \gamma_\mu \cdot (1 - \gamma_5) \cdot v(p_l) \cdot (V_\mu - A_\mu),$$

(1)

where $V_{us}$ is the lepton part and $A_{\mu}$ is the hadron part.

\begin{equation}
V_\mu = -\frac{H}{M_K^3} \varepsilon_{\mu,\nu,\rho,\sigma} L^{\nu} P^{\rho} Q^{\sigma}, \quad A_\mu = -i \frac{1}{M_K^2} [P_\mu F + Q_\mu G + L_\mu R] \tag{2}
\end{equation}

$\varepsilon_{0,1,2,3}=1$ and the four-momenta are defined as $P = p_1 + p_2$, $Q = p_1 - p_2$ ($p_1$ and $p_2$ are the two pion momenta) and $L = p_1 + p_{\nu}$. The form factors $F$, $G$, $R$ and $H$ can be theoretically calculated in the framework of Chiral Perturbation Theory (ChPT). In the case of $\pi^0 \pi^0$ dipion, within S-wave approximation, decay probability does not depend on $\cos \theta_\pi$ and $\phi$, that leads to $G = 0$ and $H = 0$. After integration over these angles, the $K_{\mu4}^{00}$ differential decay width becomes a function of three kinematic variables ($S_\pi$, $S_\mu$, $\cos \theta_\mu$) [2]:

$$d\Gamma_3 = \frac{G_f^2 |V_{us}|^2 (1 - z_\mu)^2 \sigma_\pi X}{2^{11/2} \pi^5 M_K^5} (I_1 + I_2 (2 \cos \theta_\mu)^2 - 1) + I_6 \cos \theta_\mu) dS_\pi dS_\mu d\cos \theta_\mu.$$  

(3)

where $I_1 = \frac{1}{4} ((1 + z_i) |F_1|^2 + 2 z_i |F_4|^2)$, $I_2 = -\frac{1}{4} ((1 - z_i) |F_1|^2$, $I_6 = z_i R e(F_1^* F_4)$, $F_1 = X \cdot F$ and $F_4 = -(PL)F - S_i R$. Here $z_i = m_i^2 / S_i$, $\sigma_\pi = \sqrt{1 - 4M_\pi^2 / S_\pi}$ and $X = \frac{1}{2} \sqrt{\lambda(M_K^2, S_\pi, S_\mu)}$, with the $\lambda$ function defined as $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

Currently, the only source of $R$ form factor is the ChPT-based theoretical calculations [2]. But $F$ form factor measured in $K_{\mu4}^{00}$ analysis may be used for $K_{\mu4}^{00}$ decay due to the lepton universality. The measurements of $K_{\mu4}^{00}$ from NA48/2 [3] represent the best determinations of the F form factor shape $F(K_{\mu4})$ and the absolute normalization $f$ parameter ($F = f \cdot F(K_{\mu4})$, where $f = 6.079 \pm 0.012_{stat} \pm 0.027_{syst} \pm 0.046_{ext}$).

Preliminary results of the first observation and branching fraction measurement of $K_{\mu4}^{00}$ decay are presented below.
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2. Beams and detectors

The NA48/2 detector and beams at the CERN SPS were designed to search for direct CP violation in $K^+ \rightarrow 3\pi$ decays [4, 5]. In 2003 and 2004, the NA48/2 beam particles were produced by 400 GeV protons colliding with a beryllium target. Two beams of oppositely charged kaons, with a central momentum of 60 GeV/c and a momentum band of ±3.8% (RMS), were formed by a system of magnets and collimators. The beam particles were measured by KAk Beam Spectrometer (KABES) with a spatial, momentum and time resolutions of about 800 $\mu$m, 1% and 600 ps, respectively. The two beams were superimposed in the decay volume inside a 114 m long vacuum tank.

Charged particles from the $K^\pm$ decays were detected using a magnetic spectrometer (DCH) located in a tank filled with helium at atmospheric pressure. The magnetic spectrometer was followed by a scintillator hodoscope (HOD) with a time resolution of about 150 ps.

A liquid Krypton calorimeter (LKr), located behind the hodoscope, was used to reconstruct photons. It was an ionization chamber, segmented transversely into 13248 projective cells, $2 \times 2$ cm$^2$ each. The calorimeter energy resolution was $\sigma(E)/E = 0.032/\sqrt{E} \pm 0.09/E \pm 0.0042$ (E in GeV).

Muon Veto system (MUV), consisting of three scintillator layers and 80 cm thick iron walls, was used for muons detection.

3. Reconstruction and selection of events

Only events with at least one DCH track and at least one KABES track, both consistent with a kaon decay vertex, are considered for the selection. Longitudinal position of the ‘charged’ decay vertex $Z_c$ is defined from the two tracks vertex fit.

Each combination of four time-consistent photon candidates with LKr energy $E_{LKr} > 3$ GeV/c is considered as a possible result of two $\pi^0$ decays. For each two-$\pi^0$ decay vertex, the distances to LKr face, $Z_1$ and $Z_2$, are calculated under the assumption that each pair of photons is produced in $\pi^0 \rightarrow \gamma\gamma$ decay. Only the pairs of pion candidates with $|Z_1 - Z_2| < 500$ cm are considered further. For each two-$\pi^0$ combination, a ‘neutral’ vertex longitudinal position $Z_n = Z_{LKr} - (Z_1 + Z_2)/2$ is defined, where $Z_{LKr}$ is the position of LKr. Only combinations of a $\pi^0$ pair and a charged vertex consistent in time and with $|Z_n - Z_c| < 600$ cm are considered further. Nearly 3.8% of signal $K^{00}_{\mu4}$ events contain more than one candidate combination. The ’best’ combinations are selected using a special criterion minimizing both $|Z_1 - Z_2|$ and $|Z_n - Z_c|$.

The normalization $K^{00}_{3n}$ decay candidates are required to pass the above common selection, and the DCH track momentum, $P_{DCH}$, is required to be above 5 GeV/c. In the plane $M_{3\pi}$ (invariant mass of the $\pi^+\pi^-\pi^0$ system) vs $P_{t}$ (transverse momentum of the registered particles), the events within the ellipse centered at kaon PDG mass $M^{PDG}_{K}$ [6] and $P_{t} = 5$ MeV/c, with semi-axes $\Delta M(K_{3\pi}) = 10$ MeV/c$^2$ and $\Delta P_{t} = 20$ MeV/c are taken as $K^{00}_{3n}$ candidates.

Signal mode $K^{00}_{\mu4}$ candidates are selected among the preliminary selected combinations, that do not pass the above $K^{00}_{3n}$ elliptic cut. For $K^{00}_{\mu4}$ selection, minimum $P_{DCH}$ is increased to 10 GeV/c to ameliorate MUV efficiency. The track is required to be within the restricted geometrical acceptance of MUV ensuring high efficiency, and only tracks with associated reconstructed muon were selected.
Thenormalization events selection resultedin
\( N \) factor usage instead of
\( M \)on
\( \text{form factor} \)
background.

\( M_\mu > 0.03 \text{ GeV} \)
complex phase between
\( \rightarrow \)
according to the following modifications of the matrix element:
\( 1) \) implementation of the small
\( \text{background ratio is} \)
\( \text{data sample, } \)
\( \text{not simulated.} \)
\( \text{shape of the} \)
\( \text{measured} \)
\( \text{F} \)
\( \text{covering the difference between the MC generator and the measured} \)
\( \text{reasonable agreement with one-loop} \)
\( \text{MUV response is approximately inversely proportional to the pion} \)
\( \text{momentum. A weight} \)
\( \text{of interaction leading to the MUV response is approximately inversely proportional to the pion} \)
\( \text{decay kinematics, so we reject them for the signal selection. Due to the} \)
\( \text{correlation between the reconstructed} \)
\( \text{missing mass region, while the normalization acceptance} \)
\( \text{after the selection, } 3718 \)
\( \text{K}^0 \) signal candidate events remain, without applying any restriction
\( \text{on} \)
\( \text{signal region [-0.002,0.002] GeV}^2/c^4, N_S = 2437 \) candidates are observed.

The normalization events selection resulted in
\( \times 10^6 K^{00}_{3\pi} \) data events with a negligible background.

4. Acceptances and signal background

For the signal MC sample production, the generator in Ref. [2] is used with ChPT one-loop
form factor \( R(1\text{loop}) \), \( H = 0, G = 0 \) and zero complex phase between \( F \) and \( R \). The experimental
\( F(K_{e4}) \) form factor shape from Ref. [3] is used with the normalization
\( F = 5 \cdot F(K_{e4}) \) providing a reasonable agreement with one-loop \( F \) calculation. Signal acceptance \( A_S \) is defined for the signal
missing mass region, while the normalization acceptance \( A_N \) is calculated for all the selected events.

The uncertainty due to the modelling of form factors is estimated by the MC events weighting
according to the following modifications of the matrix element:
\( 1) \) implementation of the small
\( \text{complex phase between} \)
\( \text{and} \)
\( \text{is required to be} \)
\( \text{late muons background.} \)
\( \text{K}^0 \)
\( \text{signal is observed as a peak in the distribution of the missing mass squared of selected} \)
\( \text{events (Fig. 2). In order to extract the signal size from the data spectrum, a fit outside the signal} \)
\( \text{in the M}^2_{\text{miss}} \) interval [-0.003,0.006] GeV\(^2/c^4 \) is performed, taking into account the tails of
\( \text{MC simulated signal and two background contributions multiplied by free parameters of the fit.} \)
\( \text{The background-related systematic uncertainty is determined by varying the way the background} \)
\( \text{is estimated. The resulting number of background events is} \)
\( \text{signal to background ratio is} \)

\( K^{00}_{\mu4} \) squared missing mass \( M^2_{\text{miss}} \) is defined with the PDG muon mass attributed to the charged
decay product. In a similar way one can define \( M^2_{\text{miss}}(\pi^\pm) \) as a squared missing mass reconstructed
using the charged pion PDG mass instead of the muon mass. Small absolute values of \( M^2_{\text{miss}}(\pi^\pm) \)
correspond to \( K^{00}_{3\pi} \) decay kinematics, so we reject them for the signal selection. Due to the

correlation between the reconstructed \( M^2_{\text{miss}}(\pi^\pm) \) and \( M^2_{\text{miss}} \), the applied cut depends on \( M^2_{\text{miss}} \):
\( M^2_{\text{miss}}(\pi^\pm) < 0.5M^2_{\text{miss}} - 0.0008 \text{ GeV}^2/c^4 \). Background is suppressed further by the \( \cos(\theta_\mu) < 0.6 \)
requirement. A cut on reconstructed dilepton mass \( S_I = M^2(\mu\nu) \) is applied in order to exclude
the region of large background from
\( \rightarrow \)
\( > \)
\( \rightarrow \)
\( \mu \nu \). The quantity \( S_I \) is required to be greater than 0.03 GeV\(^2/c^4 \), well above the squared PDG value of \( \pi^\pm \) mass.

The uncertainty due to the modelling of form factors is estimated by the MC events weighting
according to the following modifications of the matrix element:
\( 1) \) implementation of the small
\( \text{complex phase between} \)
\( \text{and} \)
\( \text{is required to be} \)
\( \text{late muons background.} \)
\( \text{K}^0 \)
\( \text{signal is observed as a peak in the distribution of the missing mass squared of selected} \)
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\( \text{in the M}^2_{\text{miss}} \) interval [-0.003,0.006] GeV\(^2/c^4 \) is performed, taking into account the tails of
\( \text{MC simulated signal and two background contributions multiplied by free parameters of the fit.} \)
\( \text{The background-related systematic uncertainty is determined by varying the way the background} \)
\( \text{is estimated. The resulting number of background events is} \)
\( \text{signal to background ratio is} \)
5. Results

Resulting $K^{00}_{\mu4}$ branching ratio is calculated as $BR(K^{00}_{\mu4}) = \frac{N_S}{N_N} \cdot \frac{A_N}{A_S} \cdot K_{\text{trig}} \cdot BR(K^{00}_{3\pi})$, where $K_{\text{trig}} = 0.999 \pm 0.002$ is the trigger correction estimated using control triggers and MC simulation. $BR(K^{00}_{3\pi})$ is the normalization mode branching ratio ((1.760 ± 0.023)%)[6].

<table>
<thead>
<tr>
<th>$BR(K^{00}_{\mu4})$ central value [$10^{-6}$]</th>
<th>Full phase space</th>
<th>$S_l &gt; 0.03$ GeV²/c⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta BR [10^{-6}]$</td>
<td>$\delta BR/BR$</td>
<td>$\delta BR [10^{-6}]$</td>
</tr>
<tr>
<td>Data stat. error</td>
<td>0.10</td>
<td>2.85%</td>
</tr>
<tr>
<td>MC stat. error</td>
<td>0.01</td>
<td>0.21%</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.01</td>
<td>0.18%</td>
</tr>
<tr>
<td>Background</td>
<td>0.10</td>
<td>2.96%</td>
</tr>
<tr>
<td>Accidentals</td>
<td>0.01</td>
<td>0.32%</td>
</tr>
<tr>
<td>MUV inefficiency</td>
<td>0.06</td>
<td>1.65%</td>
</tr>
<tr>
<td>Form factors modeling</td>
<td>0.05</td>
<td>1.37%</td>
</tr>
<tr>
<td>$BR(K^{00}_{3\pi})$ error (external)</td>
<td>0.05</td>
<td>1.31%</td>
</tr>
<tr>
<td>Total error</td>
<td>0.17</td>
<td>4.83%</td>
</tr>
</tbody>
</table>

Table 1: $BR(K^{00}_{\mu4})$ and its error components

The results are shown in the Table 1. Accidental activity contributions to the systematic uncertainty were estimated as the effects of doubling the time windows used for different timing cuts. The systematic uncertainty related to MUV residual inefficiency is conservatively taken as 100% of the inefficiency size estimated from data. As a main result, the branching ratio for the restricted phase space with $S_l^{true} > 0.03$, characterised by small uncertainty due to form factors modelling, is presented.
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**Figure 3:** Comparison of the present NA48/2 measurement result with the theoretical predictions.

Comparison of the results with theoretical predictions is shown in Fig. 3. The left three theoretical points correspond to the tree approximation, the one-loop calculation and the 'beyond 1-loop' result [2] based on experimental inputs available in 1994, respectively. The last column of theoretical points in the Fig. 3 is calculated using the last measured $K_{\mu4}^{(0)}$ form factor $F$ [3] and one-loop $R$ form factor [2] ($R_1 = R(1\text{loop}), 2R_1 = 2 \times R(1\text{loop})$).

The preliminary result for restricted phase space ($S_l > 0.03$) is $BR(K_{\mu4}^{(0)}, S_l > 0.03) = (0.65 \pm 0.019_{\text{stat}} \pm 0.024_{\text{syst}}) \times 10^{-6} = (0.65 \pm 0.03) \times 10^{-6}$. The preliminary full phase space result is $BR(K_{\mu4}^{(0)}) = (3.4 \pm 0.10_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-6} = (3.4 \pm 0.2) \times 10^{-6}$. Both results are consistent with a contribution of the R form factor as computed at 1-loop ChPT.

References


[3] NA48/2 collaboration, *Detailed study of the K$^\pm \rightarrow \pi^0 \pi^0 e^+\nu$ (K$^{(0)}_{\mu4}$) decay properties*, *JHEP* **08** (2014) 159 [1406.4749].

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